GCSE Maths – Geometry and Measures

Congruence – Lengths, Areas and Volumes

Notes

WORKSHEET

Congruence

Congruent shapes are **identical in shape and size** but may have undergone transformations such as **reflections and rotations**.

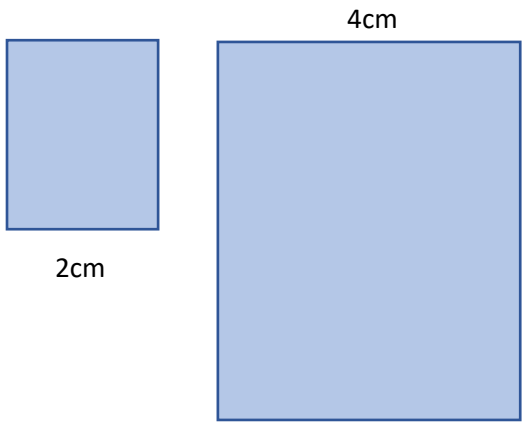
Similar shapes are shapes that have undergone **enlargements**. When a shape is enlarged, **scale factors** show how much larger or smaller the new shape has become. These shapes are similar in image but a **different size.**

It is important that you know the difference between congruent shapes and similar shapes. Similar shapes are not congruent but congruent shapes could be described as being similar.

Length

When a shape is enlarged, it is enlarged by a **certain value** called the **scale factor.** All the lengths of the shape are enlarged by the **same ratio.**

Example: The rectangles below are similar. Calculate the scale factor between these two images.



The width of the smaller rectangle is 2 cm.
The width of the larger rectangle is 4 cm.
The scale factor determines how much the shape has enlarged by. Divide the larger value by the smaller value to find the scale factor:

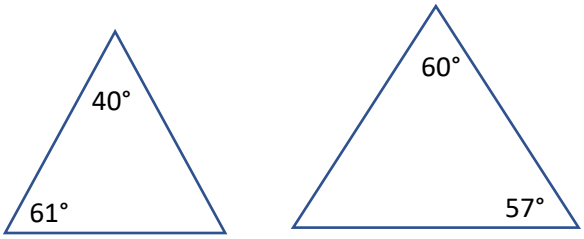
$$Scale\ factor = 4 \div 2 = 2$$

***The scale factor is* 2.**

Triangles are **similar** when the corresponding **angles** in both images are **equal.** Even though the lengths may be enlarged the **angles will always be the same**.

Another way to prove that triangles are equal is if two lengths are enlarged by the same scale and the angle between them is equal.

Example: Prove that the triangles below are not similar.



*Triangles are similar if all angles are equal. We can see that these triangles will not have equal angles so they **cannot be similar**.*

The missing angle in the smaller triangle is

$$180 - 40 - 61 = \mathbf{79}°$$

The missing angle in the larger triangle is

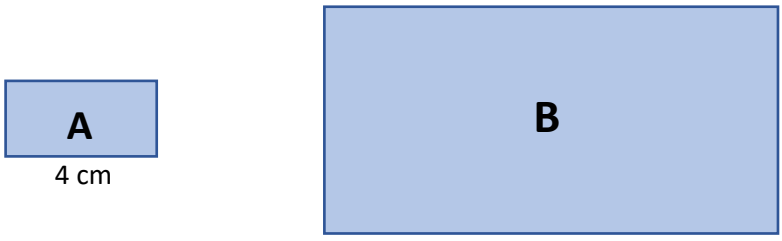
$$180 - 60 - 57 = \mathbf{63}°$$

So, we confirm that the angles are not the same and so the triangles are not similar.

Area

When two shapes are similar, the area can be calculated using the scale factor. If the scale factor for length is $\boldsymbol{k}$, the scale factor for area is $\boldsymbol{k^2}$.

Example: Given that the rectangles below are similar and the area of A is 8 cm, what is the area of Rectangle B?



1. Calculate the linear scale factor.

$$Linear\ scale\ factor = 12 \div 4 = \mathbf{3}$$

2. Calculate the area scale factor.

$$Area\ scale\ factor = (Linear\ scale\ factor)^2 = 3^2 = 9$$

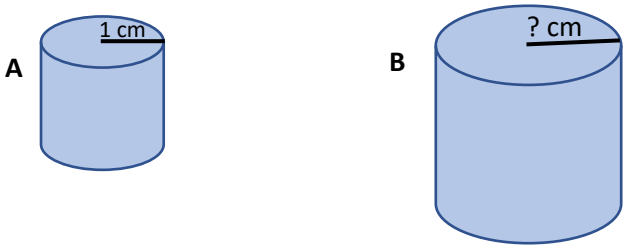
3. Calculate the area of B.

$$Area\ of\ B = Area\ A \times 9 = 8 \times 9 = \mathbf{72\ cm^2}$$

Volume

When two shapes are similar, the volume can also be calculated using the scale factor. If the scale factor for length is $\boldsymbol{k}$, the scale factor for volume is $\boldsymbol{k^3}$.

Example: For cylinder A, the volume is $4\ \text{cm}^3$ and the radius of the cross section is $1\ \text{cm}$. The volume of cylinder B is $108\ \text{cm}^3$. Given that the cylinders are similar, what is the radius of cylinder B?



1. Calculate the volume scale factor.

$$Volume\ scale\ factor = Volume\ B \div Volume\ A = 108 \div 4 = 27$$

2. Calculate the linear scale factor.

$$Volume\ scale\ factor = k^3 = 27$$
$$Linear\ scale\ factor = k$$

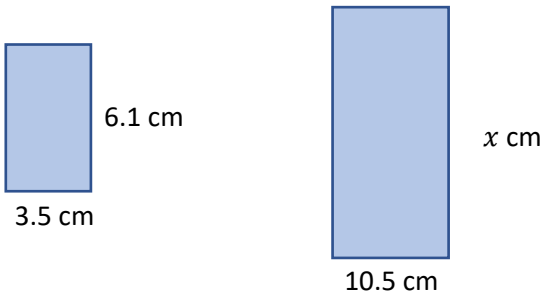
$$Linear\ scale\ factor = \sqrt[3]{27} = 3$$

3. Calculate the radius of cylinder B.

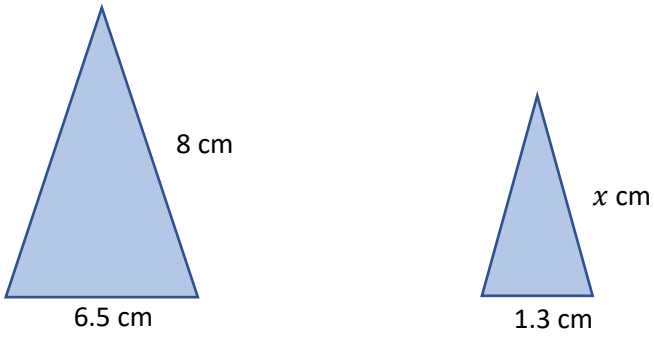
$$\boldsymbol{Radius\ B} = Radius\ A \times Linear\ scale\ factor = 1 \times 3 = \mathbf{3\ cm}$$

Congruence – Practice Questions

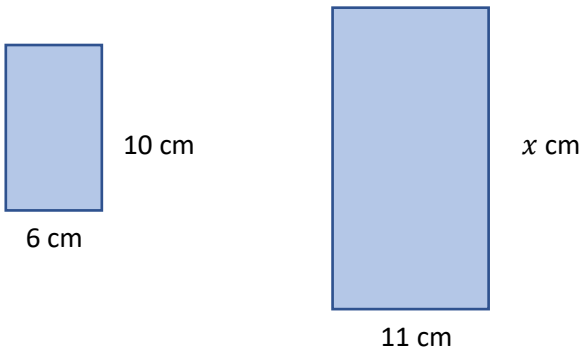
1. Given that the rectangles are similar, calculate length x cm.



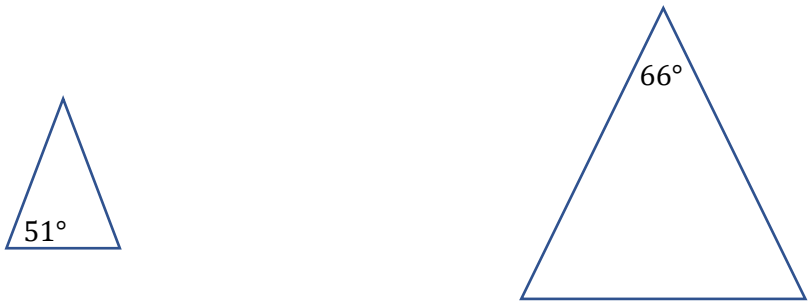
2. Given that the triangles are similar, calculate length x cm.



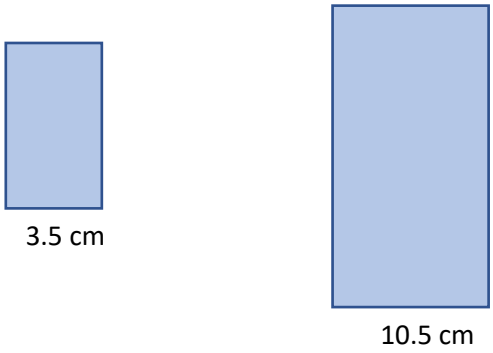
3. Given that the rectangles are similar, calculate length x cm.



4. Both triangles below are isosceles. Are they similar?



5. Given that the rectangles are similar, calculate the area scale factor.



6. The surface area of the cube A is $24\ \text{cm}^2$. The surface area of cube B is $54\ \text{cm}^2$ and the volume is $27\ \text{cm}^3$. Calculate the volume of cube A.

7. The volume of the smaller prism is $25\ \text{cm}^3$ and the volume of the larger prism is $200\ \text{cm}^3$. Find the value of x.

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.