GCSE Maths – Geometry and Measures

Arc Length and Area of Sector

Notes

WORKSHEET

Arc Lengths and Sectors

An arc length is a **portion of the circumference** of a circle. A sector is a **"pie-shaped" portion of the area** of a circle. For example, of you think of a slice of pizza, the sector is the pizza slice, and the arc length is the crust.
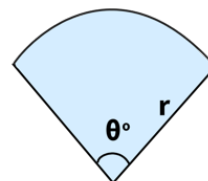
Arc Length

An arc length is the length between two points on the circumference of a circle.

- The **minor** arc length is the **shorter length** between the two points.
- The **major** arc length is the **longer length** between the two points.
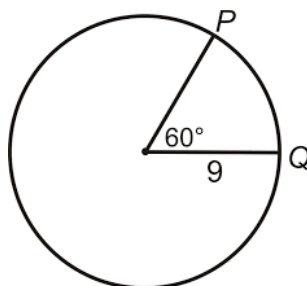
Arc length is calculated using the formula

$$Arc\ length = 2\pi r \times \frac{\theta}{360}$$

where θ is the angle between the two radius lines and r is the radius.

When calculating the major arc length, you must use the angle θ which falls in the range $180 \leq \theta \leq 360$.

Example: Calculate the minor arc length PQ in terms of π

1. Work out numerical values for r and θ.

   For minor arc length we use the angle that is less than 180°. This means we use the given angle for θ:

   $$r = 9 \text{ cm}$$
   $$\theta = 60°$$

2. Substitute the values into the equation to calculate arc length.

   $$Arc\ length = 2\pi r \times \frac{\theta}{360} = 2\pi(9) \times \frac{60}{360}$$

   Simplify the solution, leaving your answer in terms of π:

   $$Arc\ length = \left(2(9) \times \frac{60}{360}\right)\pi = 3\pi$$

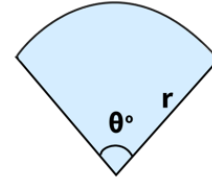   *The minor arc length is 3π cm.*

Area of an Arc Sector

An arc sector is the pie-shaped area between two points on the circumference of a circle.

- A **minor** arc sector is smaller than a semi-circle.
- A **major** arc sector is bigger than a semi-circle.

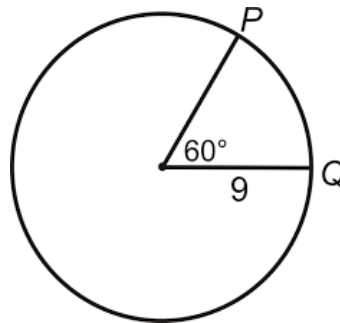The arc sector area is calculated by using the formula

$$Arc\ sector\ area = \pi r^2 \times \frac{\theta}{360}$$

where θ is the angle between the two radius lines and r is the radius.

When calculating the **major** arc sector area, you must use the angle θ which falls in the range $180 \leq \theta \leq 360$.

Example: Calculate the major arc sector area PQ in terms of π

1. Work out numerical values for r and θ.

$$r = 9 \text{ cm}$$

For major arc, we use the angle that is more than 180°.

$$\theta = 360° - 60° = 300°$$

2. Substitute values into the equation.

$$Area = \pi r^2 \times \frac{\theta}{360} = \pi(9)^2 \times \frac{300}{360}$$

3. Calculate the numerical value of the arc area in terms of π.

$$Area = \pi(9)^2 \times \frac{300}{360} = 67.5\pi$$

The area of the major arc sector is **67.5π cm^2**.

# Calculating angle θ

Some questions may ask you to find the value of θ. In this case, we can use the previous formulas but **rearrange** them to find θ.

However, we must pay attention to **which angle is being asked to calculate**. In minor arcs we use the angle in the range $0 \leq \theta \leq 180$ and in major arcs we use the angle in the range $180 \leq \theta \leq 360$.

Example: Calculate angle y, where the minor arc length is 0.7π
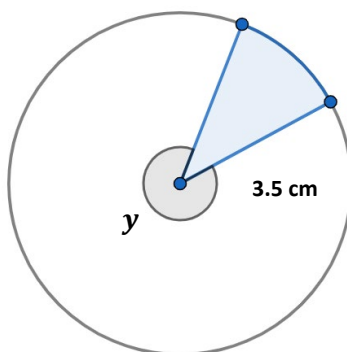
1. Work out numerical values for r.

$$r = 3.5 \text{ cm}$$

For minor arc length we use the angle that is less than 180°. This means for minor arc we use the angle x where $x = 360 - y$.

2. **Substitute** known values into the equation.

$$Arc\ Length = 2\pi r \times \frac{\theta}{360}$$

$$0.7\pi = 2\pi(3.5) \times \frac{x}{360}$$

$$0.7\pi = \frac{7x}{360}\pi$$

3. **Solve for x.**

$$0.7 = \frac{7x}{360}$$

$$x = 0.7 \div \frac{7}{360} = 36°$$

4. **Calculate angle y.**
   *Refer to the question, which is looking for the **major angle y**.*
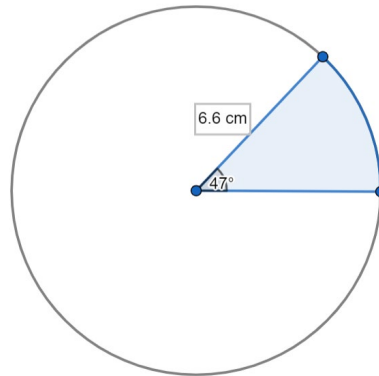
   Angles around a point equal 360°:

$$y = 360° - x$$
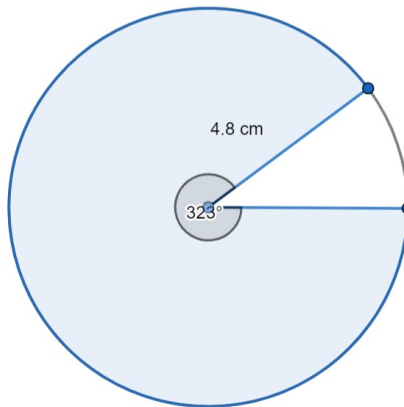$$y = 360° - 36° = \mathbf{324°}$$
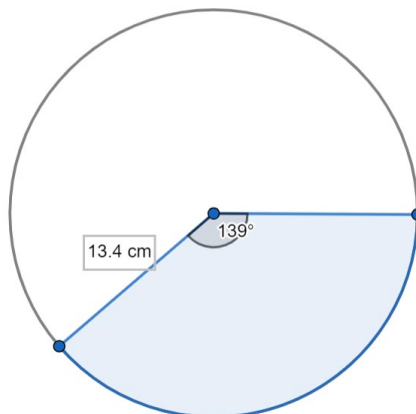
Arc Length and Area of Sector – Practice Questions
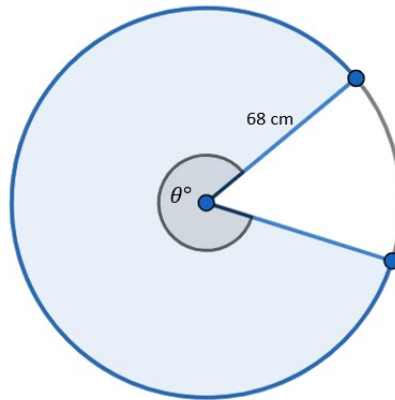
1. Calculate the minor arc length to 1 decimal place

   6.6 cm
   47°

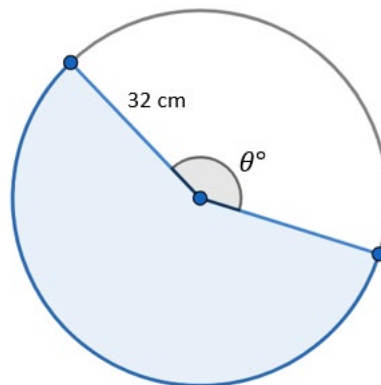2. Calculate the area of the major arc sector to 1 decimal place.

   4.8 cm
   323°

3. Calculate the area of the major arc sector to 1 decimal place

   139°
   13.4 cm

4. Calculate θ to 1 decimal place when the major arc length is 341.3 cm

5. Calculate θ to 1 decimal place when the area of the major sector is 1876 cm^2

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.