GCSE Maths – Algebra

Solving Linear Inequalities

Notes

WORKSHEET

Solving Linear Inequalities

Inequalities

Inequalities are equations which contain **signs such as <, >, $\leq$ and $\geq$**. The meaning of the inequality signs are as below:

- > means **greater than**
- < means **less than**
- $\geq$ means **greater than or equal to**
- $\leq$ means **less than or equal to**

For example,

$$5 < 7$$

$$-3 > -4$$

$$2 \geq 0$$

$$-9 \leq 3$$

$$6 \leq 6.$$

The difference between $<$ and $\leq$ is whether the starting number is included.

For example,

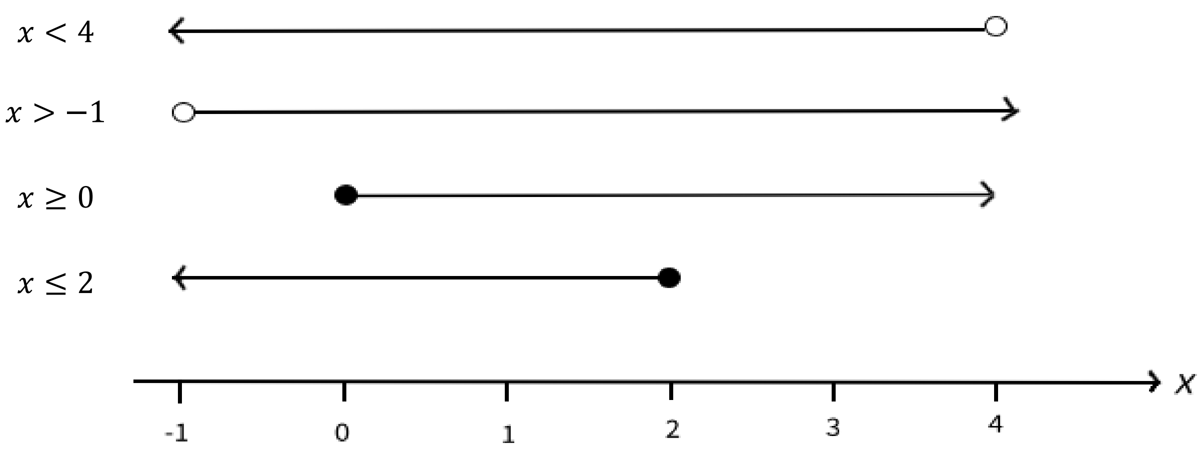
$x < 4$ means x can take values $3, 2, 1, 0, -1, \ldots$.
$x \leq 4$ means x can take values $4, 3, 2, 1, 0, -1, \ldots$

Inequalities can be expressed on a **number line**.

- A **solid circle** is used to represent inequalities with $\leq$ and $\geq$ signs. Solid circles mean the number indicated is **included** within the answer range.

- An **open circle** is used to represent inequalities with $>$ and $<$ signs. Open circles mean the number indicated is **excluded** from the answer range.

Inequality



Using Set Notation to Present Inequalities (Higher Only)

You may be asked to present your answers in set notation rather than on a number line. In this case, you need to use the curly bracket to include your answer. Below are two examples on how to write set notations.

Inequality: $x \geq 7$
Set notation: $\{\, x : x \geq 7 \,\}$

Inequality: $y \leq 14$
Set notation: $\{\, y : y \leq 14 \,\}$

Note, the colon represents "such as". So, the first curly bracket set should be read as "x such that x is greater than or equal to 7".

Solving linear inequalities

Solving linear inequalities is quite similar to solving linear equations. However, there are several basic principles:

- The inequality sign should be **reversed** when it is **divided** or **multiplied** by a **negative** integer.

  For example, if $-x < 2$ then dividing by -1 gives $x > -2$ since we must flip the direction of the inequality sign. You can think of it as adding x to both sides of the inequality and then subtracting 2 from both sides of the inequality.

Example: Solve the inequality $6 - 4x \geq 18$. Present your answer in a number line.

1. **Ensure** only the unknown is present on **one side** of the inequality.

$$6 - 4x \geq 18$$

Subtract 6 from both sides of the equation:

$$-4x \geq 12$$

2. **Solve for x,** ensuring x has a **positive** sign.

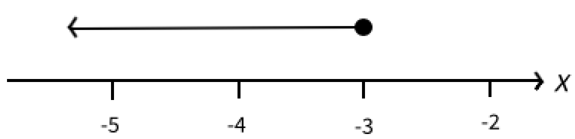
$$-4x \geq 12$$

Divide both sides of the equation by -4, remembering to flip the direction of the inequality sign:

$$x \leq -3$$

Hence, the final answer is $\boldsymbol{x \leq -3}$.

3. **Draw** a **number line** to illustrate the answer. Since the sign used here is $\leq$, a solid circle should be used.



Compound inequalities

Compound inequalities are statements which contain a combination of **two inequalities**. They can be solved by splitting the inequalities into two parts, solving each part separately, and then finding the values which satisfy both results.

Example: Solve the inequality $6x - 5 \leq 5x - 4 > 4(x - 2)$.
Present your answer in a number line and list down the integer solutions.

1. **Split** the inequality into two parts.

a) $6x - 5 \leq 5x - 4$
b) $5x - 4 > 4(x - 2)$

2. **Solve** the inequality separately.

a) $6x - 5 \leq 5x - 4$

Subtract $5x$ from both sides of the inequality:

$$x - 5 \leq -4$$

Add 5 to both sides of the equation:

$$\boldsymbol{x \leq 1}$$

b) $5x - 4 > 4(x - 2)$

Expand the bracket:

$$5x - 4 > 4x - 8$$

Subtract $4x$ from both sides of the inequality:

$$x - 4 > -8$$

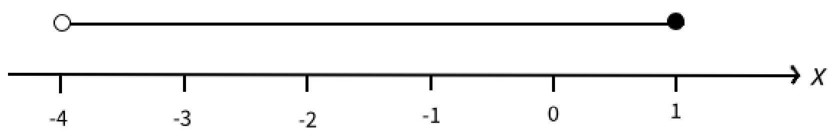
Add 4 to both sides of the inequality:

$$\boldsymbol{x > -4}$$

Putting $x \leq 1$ together with $x > -4$, we obtain $\boldsymbol{-4 < x \leq 1}$*.*

3. **Draw** a **number line** to illustrate the answer for both parts.

A solid circle is drawn at $x = 1$ since the inequality at $x = 1$ is inclusive.



4. **List** the **set of integers** which satisfy the number line.

The values which satisfy $\boldsymbol{-4 < x \leq 1}$ *are* $\boldsymbol{x = -3, -2, -1,\ 0,\ 1}$.

Solving linear inequalities with two variables (Higher only)

Some inequalities may have two variables such as x and y. These inequalities require us to sketch a graph.

Some key important points when sketching a graph for inequalities:

- Treat the inequality just like a normal equation when initially sketching the graph.
- If the inequality has a $\geq$ or $\leq$ sign, a solid line should be drawn. This represents the fact that values on the line are included.
- If the inequality has a $>$ or $<$ sign, a dashed line should be drawn. This represents the fact that values on the line are not included.

Example: Solve the inequality $x + y > 3x + 3$

1. **Ensure** only y is present on the left-hand side and x is on the right hand side of the equation.

$$x + y > 3x + 3$$

Subtract x from both sides:

$$y > 2x + 3$$

2. Find the $\boldsymbol{x}$ **−intercept** and the $\boldsymbol{y}$ **−intercept** to find coordinates on the line.

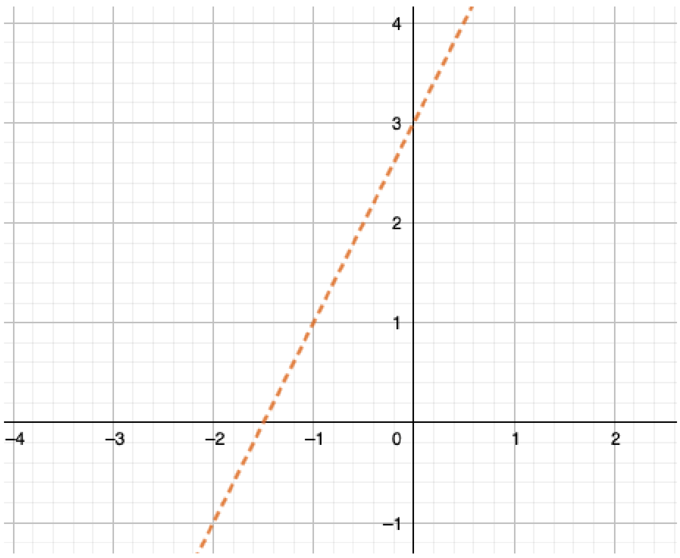
To find the x −intercept, y is set equal to 0:

$$y = 2x + 3$$
$$0 = 2x + 3$$
$$x = -1.5$$

To find the y −intercept, x is set equal to 0:

$$y = 2(0) + 3$$
$$y = 3$$

5. **Plot** both the x −intercept and the y −intercept and **draw** the line which passes through these points. Since our inequality has a $>$ sign, the line drawn should be a dotted line.



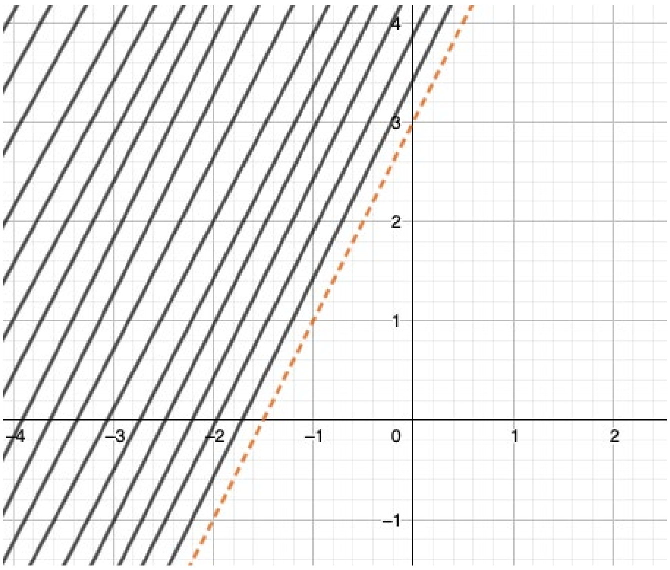
6. **Shade** the correct region which satisfies the inequality. You could **choose a coordinate in a region** and **substitute it** into the inequality. If the number satisfies the inequality, the region where the coordinate lies should be shaded.

   For $y > 2x + 3$, this means the value of y should always be greater than the dotted line. Since y is greater in the upper region of the graph, that region should be shaded.

   Alternatively, we can also choose a point in the upper region to check the answer. For instance, if we choose a point $(-2,2)$ and substitute it into the inequality, as shown below, we will get a correct statement. This means that the upper region should be shaded.

$$(2) > 2\,(-2) + 3$$

$$2 > -1$$



Example: Solve the inequality $3x + y < -8x - 10$

1. **Ensure** only y is present on the left-hand side and x is on the right-hand side.

$$3x + y < -8x - 10$$

   Subtract $3x$ from both sides of the equation:

$$y < -11x - 10$$

7. Find the $\boldsymbol{x}$ **–intercept** and the $\boldsymbol{y}$ **–intercept** to find coordinates on the line.

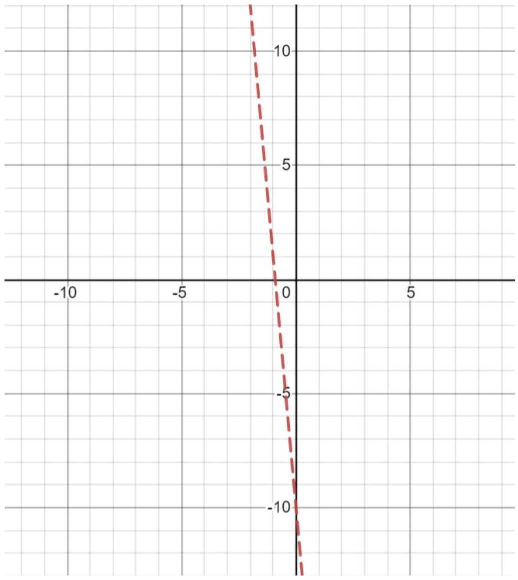
   To find the x –intercept, y should be equal to 0:

$$y = -11x - 10$$
$$0 = -11x - 10$$
$$x = -\frac{10}{11}$$

   To find the y –intercept, x should be equal to 0:

$$y = -11(0) - 10$$
$$y = -10$$

8. **Plot** both the x −intercept and the y −intercept and **draw** the line connecting the two points. Since our inequality has a < sign, the line drawn should be a dotted line.

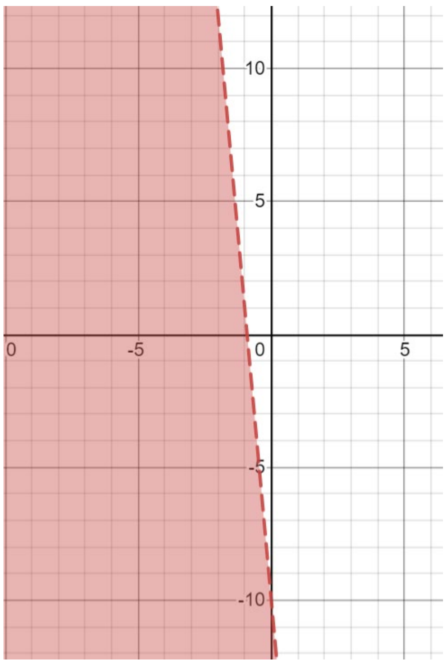


9. **Shade** the correct region which satisfies the inequality. You could **choose a coordinate in a region** and **substitute it** into the inequality. If the number satisfies the inequality, the region where the coordinate lies should be shaded.

   For $y < -11x - 10$, *we can also choose a point in the upper region to check our answer. For instance, if we choose a point* $(-3,0)$ *and substitute it in the inequality (as shown below), we get a correct statement. Therefore, region containing this point should be shaded.*

$$(0) < -11(-3) - 10$$

$$0 < 23$$



Solving Linear Inequalities – Practice Questions

1. Solve the following inequalities and present your answer in a number line:

   a) $2x + 1 \geq 5 + x$

   b) $2(x + 2) < -14 - x$

   c) $x - 6 \geq 4x + 3$

   d) $-4(x - 5) \leq -3(2x - 7)$

2. Solve the following inequalities. List the integers in each solution set.

   a) $1 \leq 2y - 1 \leq 6$

   b) $-6 < 3(p - 1) \geq 4p - 9$

(Higher only) – Practice Questions

3. Solve the following inequalities and present your answers in a graph.

   a) $3x - y < 8y + 2$

   b) $2g + 2m \geq 7g - 10$

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.