GCSE Maths – Algebra

Solving Linear Equations

Notes

WORKSHEET

Solving Linear Equations

Linear equations

Linear equations are equations with only **one** type of unknown. This unknown can be on either or both sides of the equation.

For an equation to be called linear, the power of the unknown variable is to the power of 1 (e.g. x or y). To solve linear equations, you must ensure the unknowns are put together on only **one** side of the equation.

Example: Solve $2x + 6 = 10$

1. **Eliminate** the numbers which are present on the same side as the unknown. This can be done by **subtraction or addition**. Make sure that any operation is performed on **both sides** of the equation.

$$2x + 6 = 10$$

Subtract 6 from both sides of the equation:

$$2x = 4$$

2. **Eliminate** the coefficient of the unknown by **dividing** or **multiplying** the unknown by its own coefficient.

$$2x = 4$$

Divide both sides of the equation by 2:

$$x = 2$$

3. Check the answer by **substituting** the value of the unknown back into the original equation. The value of the expression on the left-hand side of the equation should be equal to the value of the expression on the right-hand side of the equation.

$$2(2) + 6 = 10$$
$$10 = 10$$

Hence, the final answer is indeed $\boldsymbol{x = 2}$.

Solving equations with unknowns on both sides

There will be cases where the unknown variable, say x, is present on **both sides** of the equation.

- In this case, first we need to collect the unknowns together on one side of the equation.
- Once the unknowns are together, we can solve the equation by following the same method as is done in the example above.

Example: Solve $4x - 3 = 2x + 5$

1. Ensure the unknowns are collected together on **one side** of the equation. To do this, we can subtract the unknown with the **smaller coefficient** from both sides of the equation.

$$4x - 3 = 2x + 5$$

Subtract $2x$ from both sides of the equation:

$$2x - 3 = 5$$

2. **Eliminate** the whole number on the same side with the unknown.

$$2x - 3 = 5$$

Add 3 to both sides of the equation:

$$2x = 8$$

3. **Eliminate** the coefficient of the unknown variable x.

$$2x = 8$$

Divide both sides of the equation by 2:

$$x = 4$$

4. **Substitute** the value of the unknown back to the original equation to **check** the answer.

$$4(4) - 3 = 2(4) + 5$$
$$13 = 13$$

Hence, the final answer is indeed $\boldsymbol{x = 4}$.

Solving linear equations involving fractions

For linear equations with fractions, we need to eliminate the denominator of the fraction. To do this, we multiply both sides with the denominator value. The equation can then be solved using the same steps as the other linear equations previously seen.

Example: Solve for x when $\frac{1}{4}(9x - 2) = 2x + 4$

1. **Convert** the equation into a fraction.

$$\frac{9x - 2}{4} = 2x + 4$$

2. **Multiply** both sides of the equation by the denominator of the fraction.

$$\frac{9x - 2}{4} = 2x + 4$$

Multiply both sides of the equation by 4*:*

$$9x - 2 = 8x + 16$$

3. **Place** the unknowns on one side of the equation.

$$9x - 2 = 8x + 16$$

Subtract $8x$ *from both sides of the equation:*

$$x - 2 = 16$$

4. **Eliminate** the whole number on the same side of the equation as the unknown.

$$x - 2 = 16$$

Add 2 *to both sides of the equation:*

$$x = 18$$

5. **Substitute** the value of the unknown back to the original equation to check the answer.

$$\frac{1}{4}(9\,(18) - 2) = 2\,(18) + 4$$
$$40 = 40$$

Hence, the final answer is indeed $\boldsymbol{x = 18}$.

Solving linear equations with brackets

For linear equations involving brackets, the brackets need to be expanded before the equation can be solved.

Example: Solve $4(x-2)-3(x+1)=5$

1. **Expand** the brackets.

$$4(x-2)-3(x+1)=5$$
$$4x-8-3x-3=5$$

2. **Simplify** the equation by collecting like terms.

$$4x-8-3x-3=5$$
$$x-11=5$$

3. **Eliminate** the whole number on the same side of the unknown.

$$x-11=5$$

Add 11 *to both sides of the equation:*

$$x=16$$

4. **Substitute** the value of the unknown back to the original equation to check the answer.

$$4(16-2)-3(16+1)=5$$
$$5=5$$

Hence, the final answer is indeed $\boldsymbol{x=16}$.

Find approximate solutions using a graph

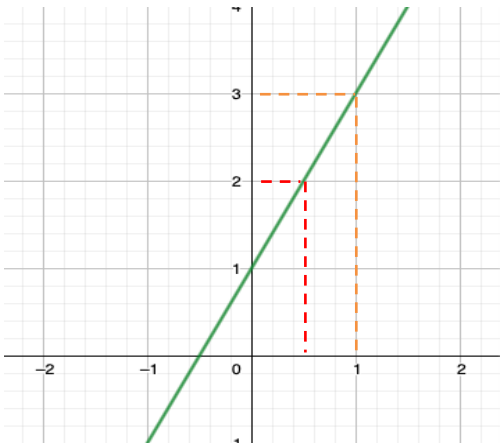
The value of an unknown in a linear equation can be determined using a graph. A linear equation corresponds to a straight line on a graph. Using the line, we can calculate the value when we input a particular value for x or y.

Example: Using the given graph, determine the values of x when $y=3$ and $y=2$.
The equation of the graph is $y=2x+1$.

1. Using the given line, we can estimate the value of x by looking at the point on the line where $y=3$. From the orange line, we can determine that $y=3$ when $x=1$.

2. Next, we estimate the value of x when $y=2$. From the red line, we can estimate that the value of x is approximately 0.5 when $y=2$.

3. *Hence, the final answer is* $x=1$ *when* $y=3$ *and* $x=0.5$ *when* $y=2$.



Solving Linear Equations – Practice Questions

1. Solve the following linear equation for x:

$$3x - 4 = 11$$

2. Solve the following linear equation for x:

$$10 - x = 4$$

3. Solve the following linear equation for x:

$$2x - 15 = 5x - 21$$

4. Solve the following linear equation for x:

$$-8 - 9x = 28 - 3x$$

5. Solve the following linear equation for x:

$$\frac{1}{5}x = x - 2$$

6. Solve the following linear equation for x:

$$\frac{(4 + 3x)}{5} - \frac{(x + 4)}{2} = \frac{3}{2}$$

7. A line has an equation of $y = 3x + \frac{1}{2}$. By sketching the graph, determine the approximate value of y when $x = 1$.

8. A line has an equation of $y - 6 = 2 - 4x$. By sketching the graph, estimate the value of x when $y = 2$.

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.