GCSE Maths - Algebra

Common Sequences

Notes

WORKSHEET

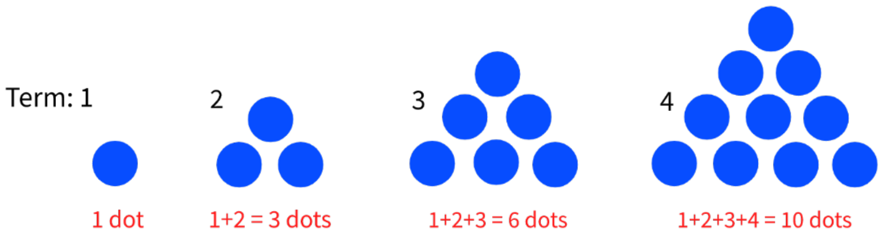
Common Sequences

It is important to be able to identify common mathematical sequences. Here is a list of sequences you should be able to recognise.

Triangular Number Sequence

1, 3, 6, 10, 15, 21, 28, 36, 45, …

Triangular numbers can be arranged into a triangle when displayed as dots.



To find the next term, add another row of dots and then count up all of the dots.

Let's look at the sequence of differences between each term: 2, 3, 4, ...
We can see the difference increases by 1 each time.

Example: What is the next term in the sequence $10, 15, 21, 28, \ldots$?

1. Identify the triangular sequence.

   We see it is a triangular sequence because we could arrange the numbers as dots into triangles, with another row being added for each term of the sequence.

2. Find the difference between previous terms.

$$Previous\ difference = 28 - 21 = \mathbf{7}$$

3. Use the previous difference to work out the next term.

   For a triangular sequence, the difference between terms increases by 1 each time. Using the previous difference,

$$Next\ term = 28 + (7 + 1) = \mathbf{36}$$

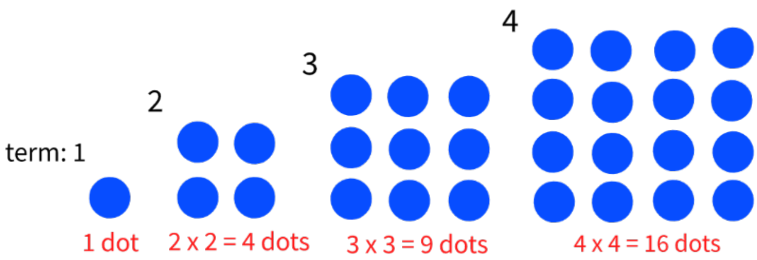
Square Number Sequence

1, 4, 9, 16, 25, 36 ...

To square a number, multiply the number by itself. For example, $3^2 = 3 \times 3 = 9$.
The nth term for the square number sequence is n^2.

Example: What is the 9th term of the square number sequence?
Remember the nth term is n^2.   *For* $n = 9$*:* $9^2 = 9 \times 9 = \mathbf{81}$

To visualise the square number sequence, draw a pattern of dots:



Cube Number Sequence

1, 8, 27, 64, 125, 216, 343, …

To cube a number, multiply the number by itself three times.
For example, $2^3 = 2 \times 2 \times 2 = 8$.
The nth term for the cube number sequence is n^3. It is similar to the square number sequence, but each number is cubed not squared.

Example: What is the 4th term of the cube number sequence?
Remember the nth term is n^3.   *So, for* $n = 4$, $4^3 = 4 \times 4 \times 4 =$ ***64.***

Example: Is 216 part of the cube number sequence?
216 *is part of the cube number sequence if there is a positive integer* n *such that* $n^3 = 216$*. So, we need to see if we can solve* $\mathbf{n^3 = 216}$.   ***Cube root 216 to find n:*** $n = \sqrt[3]{216} = 6$ *Since* $6^3 = 216$*,* 216 *is the* ***6th term*** *in the sequence.*

Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144…

The first two terms are set as 0 and 1.

To find subsequent terms, add the previous two terms together.

For example, the third term is $0 + 1 = 1$, and fourth term is $1 + 1 = 2$.

Example: What is the next term in this part of the Fibonacci sequence?
$\mathbf{21, 34, 55, 89, 144, \ldots}$

Fibonacci rule: Add the two previous terms together to get the next term. You can check this works across all the terms to confirm you should use the Fibonacci rule.

So, for the next term:

$$89 + 144 = \mathbf{233}$$

Fibonacci numbers crop up regularly in **nature**. For example, it is seen in the number of petals on a flower.

Arithmetic Sequence

nth term: $an + b$

The nth term of the arithmetic sequence shows that the rule is to add or subtract the same value each time. In other words, the difference between subsequent terms is always the same. We call this the **common difference**.

Both ***a*** and ***b*** are constant values.

An arithmetic sequence is also called a **linear sequence**. If we plotted the term '*n*' on the x-axis and the value of each '*nth term*' on the y-axis, it would form a straight-line graph.

Example: Identify if the following sequences are arithmetic

a) 14, 12, 10, 8, …
Yes, it has common difference -2.

b) -30, -27, -24, -21, ...
Yes, it has common difference 3.

c) 2, 4, 8, 14, ...
No, the difference between terms increases each time and is not constant.

Geometric Sequence

To get from one term to the next, you **multiply (or divide) by a constant value**. The concept is similar to adding or subtracting a common difference like in arithmetic sequences, except multiplication (or division) is used. To check if a sequence is geometric, you can find the **ratio** between two consecutive terms:

If the ratio between consecutive terms is the same for all the terms, the sequence is geometric.

Example: Is *1, 2, 4, 8, 16, 32, …* a geometric sequence?
If it is, find the next term in the sequence.

1. ***Identify if there is a common ratio between terms.***
   To get from one term to the next, multiply by 2. So, the common ratio is ***2****.*
   You can also find the common ratio by checking the ratio between consecutive terms.

2. ***Use the common ratio to find the next term in the sequence.***
   The last term given is 32. Since the common ratio is 2, the next term is $\mathbf{32 \times 2 = 64}$.

Example: What is the common ratio of the sequence *28, 14, 7, 3.5, …*?

1. ***Identify the common ratio between terms.***
   By inspection, each term is ½ of the one before it. You can also check the common ratio between consecutive terms: $14/28 = 1/2, 7/14 = 1/2, 3.5/7 = 1/2$.
   So, the common ratio is ***½*** .

Higher Only: You may come across geometric sequences where the common ratio is a surd, for example $\sqrt{3}$.

Quadratic Sequence

$$\text{n}^{\text{th}}\textbf{ term: } an^2 + bn + c$$

If you find the sequence of differences between terms of a quadratic sequence, the sequence of differences changes by the same amount each time (see below).

'*a*', '*b*', and '*c*' are constant and '*a*' is generally not 0. If '*a*' is 0 then the sequence is an arithmetic sequence.

Example: Show that the sequence 3, 6, 12, 21, 33, … is a quadratic sequence

The differences between terms are 3, 6, 9, 12, …
The difference changes by 3 each time so the sequence is quadratic.

	3		6		12		21		33
first difference:		3		6		9		12	
second difference:			3		3		3		

See the notes on nth term for further detailed examples.

Common Sequences - Practice Questions

1. Identify the following sequences.

    a) 2, 3, 5, 8, 13, 21...

    b) 2, 6, 10, 14, 18…

    c) 1, 4, 9, 16, 25…

    d) 8, 27, 64, 125…

2. Continue the following sequences by finding the next two terms.

    a) 5, 12, 25, 44…

    b) 1, 6, 36, 216…

    c) 21, 26, 31, 36…

    d) 28, 36, 45, 55…

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.