GCSE Maths – Algebra

Generating a Sequence

Notes

WORKSHEET

Sequences

A sequence is a **series of numbers** that follow a particular **rule** or **pattern**.

An example of a sequence is **0, 2, 4, 6, 8, 10….** This sequence follows the rule of adding 2 each time. If the rule involves **adding** or **subtracting** the same amount each time, the sequence is called an **arithmetic** sequence.

Furthermore, if the difference between each number in the sequence is the **same each time**, the sequence is said to be **linear**. Other types of sequence include quadratic and exponential.

Alternatively, the rule for the sequence could be **multiplication** or **division**. In this case, the sequence is called a **geometric** sequence.

The numbers in the sequence are called **terms**, so the first number in the sequence is the first term, the second number is the second term, and so on.

To work out the terms of a sequence, we can use two approaches:
- **Term-to-term** rule
- **Position-to-term** rule

Term-to-Term Rule

The term-to-term rule shows us how to get from one term of the sequence to the next. To do this, we need to work out the **difference** between the terms.

For example, consider the sequence

$$2, 5, 8, 11…$$

Looking at the first two terms of this sequence, 2 and 5, we need to calculate the difference between them, which is 3. Therefore, the term-to-term rule is '**add 3**'.

Now that we know the term-to-term rule, we can calculate the **next terms** in the sequence:

$$…11, 14, 17, 20…$$

Example: The term-to-term rule of a sequence is '*subtract 2*'.
The starting value is 13. Calculate the first 5 terms of this sequence.

The first term of the sequence is 13, *and the term-to-term rule is 'subtract 2'.*

Second term: $13 - 2 = \mathbf{11}$
Third term: $11 - 2 = \mathbf{9}$
Fourth term: $9 - 2 = \mathbf{7}$
Fifth term: $7 - 2 = \mathbf{5}$

Position-to-Term Rule

Using the position-to-term rule requires **mapping each term to its position** in the sequence, where the first term is in position 1, the second term is in position 2, and so on.

Position-to-term rules involve **algebra** – we are working out how to get from position n to the value of the term. Consider the following sequence:

12, 13, 14, 15…

We need to map each term to its position:

- The first term, **12**, is in position **1**.
- The second term, **13**, is in position **2**.
- The third term, **14**, is in position **3**.
- The fourth term, **15**, is in position **4**.

With each of these terms and positions, the difference is **11**. This means that for any position in the sequence, n, we add 11 to n to generate the term, i.e. $n^{th} \, term = n + 11$

The next terms in the sequence, using the position-to-term rule, are:

- Position 5: 5 + 11 = **16**
- Position 6: 6 + 11 = **17**
- Position 7: 7 + 11 = **18**

….

Example: The position-to-term rule of a sequence is 2n.
Work out the first 5 terms of a sequence.

We have been told the position-to-term rule of the sequence, which is $2n$.

The first 5 positions are represented by $n = 1$, $n = 2$, $n = 3$, $n = 4$ and $n = 5$. We therefore perform the operation (multiplication by 2) for each position:

$$Position \, 1: \quad 2 \times 1 \, = \, \mathbf{2}$$
$$Position \, 2: \quad 2 \times 2 \, = \, \mathbf{4}$$
$$Position \, 3: \quad 2 \times 3 \, = \, \mathbf{6}$$
$$Position \, 4: \quad 2 \times 4 \, = \, \mathbf{8}$$
$$Position \, 5: \quad 2 \times 5 \, = \, \mathbf{10}$$

*The first 5 terms are **2, 4, 6, 8, 10**.*

Once we know the position-to-term rule, we can calculate the term of **any position**. Sequences with a position-to-term rules are represented by formulas of **nth term**. This is covered in more detail in the nth term revision notes.

Generating a Sequence – Practice Questions

1. Calculate the term-to-term rule and write the next 3 terms for the following sequences:

   a) -12, -8, -4, 0….

   b) 1.25, 1.5, 1.75, 2, 2.25…

   c) 6, 16, 26, 36, 46…

2. Calculate the position-to-term rule for the following sequences and work out the term for the position given.

   a) -8, -7, -6, -5, -4… and position 9

   b) 7, 14, 21, 28, 35… and position 7

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.