GCSE Maths – Algebra

Translations and Reflections (Higher Only)

Notes

WORKSHEET

Translations

A translation is a type of transformation of a function in which the curve or line **moves** in the **vertical** or **horizontal** direction. Before we think about translations, we need to know the correct notation.

Curves represented by equations such as $\boldsymbol{y = x^2}$ are the same as $\boldsymbol{y = f(x)}$, meaning that y is equal to a **function** of x. Therefore, we can also write this equation as $\boldsymbol{f(x) = x^2}$. Writing our curves as functions like this will make translations and reflections easier to understand.

Vertical Translations

If our curve is being translated in the vertical direction (i.e. it moves **up** or **down**), the number of units the curve moves will be represented by a value **after the function**, in the form

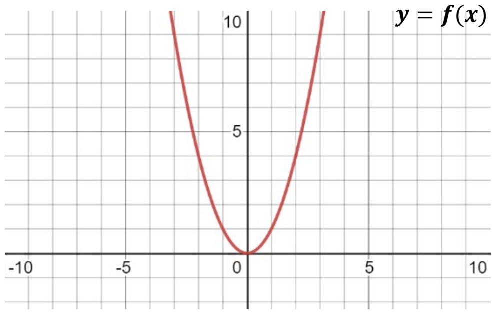
$$\boldsymbol{f(x) + a,}$$

where $\boldsymbol{a}$ represents the number of units the function moves up or down.

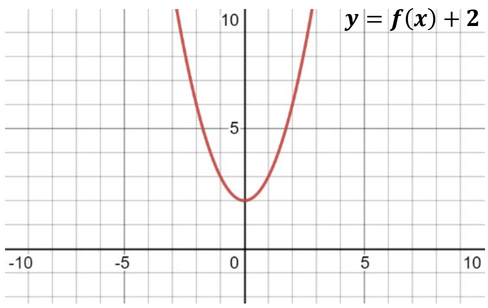
We see that the sign of a in $f(x) + a$ determines whether the curve moves up or down:

- If $\boldsymbol{a > 0}$, the curve moves in the **upwards** direction.
- If $\boldsymbol{a < 0}$, the curve moves in the **downwards** direction.

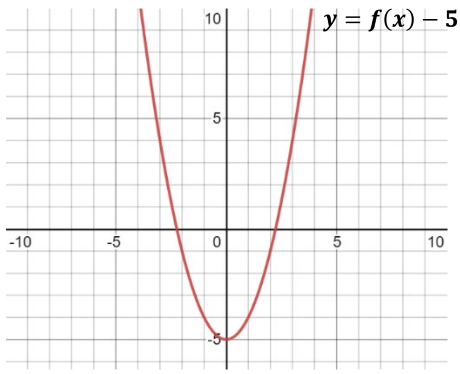
For example, consider the graph represented by the function $\boldsymbol{f(x) = x^2}$:



If this function is translated **up** by 2 units, then we write this as $\boldsymbol{f(x) + 2 = x^2 + 2}$ and the curve moves 2 units in the upwards direction, parallel to the y-axis.



Equally, if the curve were to be **translated down** by 5 units, it would be written as $\boldsymbol{f(x) - 5 = x^2 - 5}$ and the curve moves 5 units in the downwards direction.



Horizontal Translations

Translations in the horizontal direction are slightly different. To show a horizontal translation, we add the numbers of units of translation **within the bracket** so that they have the general form

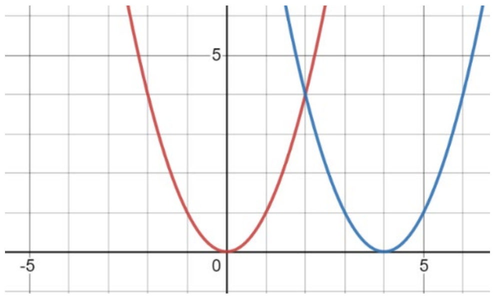
$$\boldsymbol{f(x+a)},$$

where $\boldsymbol{a}$ is the number of units the function moves parallel to the x-axis.

- If $\boldsymbol{a > 0}$, then the curve shifts to the **left**.
- If $\boldsymbol{a < 0}$, then the curve shifts to the **right.**

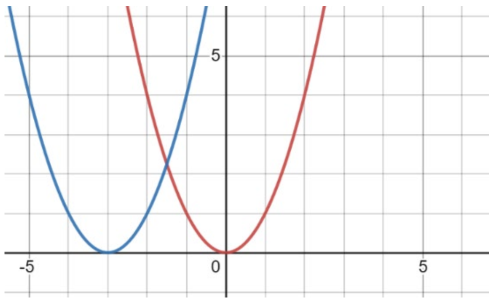
For example, consider the graph opposite. The red curve has the equation $\boldsymbol{f(x) = x^2}$. The blue curve is $\boldsymbol{f(x-4) = (x-4)^2}$.

The red curve shifts **4 units to the right** since $a = -4$ which is negative.



A translation of the red curve to the **left** will be shown by a **positive number** within the bracket

For example, here the blue curve represents the translation $\boldsymbol{f(x+3) = (x+3)^2}$ which has $a = 3$ which is positive.



Example: A function $f(x)$ passes through the point $(4, 6)$. What are the new coordinates of this point if the function is transformed by:

a) $f(x) + 5$

b)$f(x-5)$

a) *This is a translation in the vertical direction. The curve is shifted up by 5 units. This means the x coordinate remains the same and the y coordinate increases by 5 units. Therefore, the point $(4, 6)$ becomes $(4, 11)$.*

b) *This is a translation in the horizontal direction. Since the translation has a negative value, the curve will shift to the right by 5 units. This means the y coordinate remains the same and the x coordinate increases by 5 units. Therefore, the point $(4, 6)$ becomes $(9, 6)$.*

Reflections

A curve can be reflected in the **x** or **y-axis**.

- A reflection in the **x-axis** is indicated by a **negative sign outside the bracket**. It has general form

$$\boldsymbol{y = -f(x)}.$$

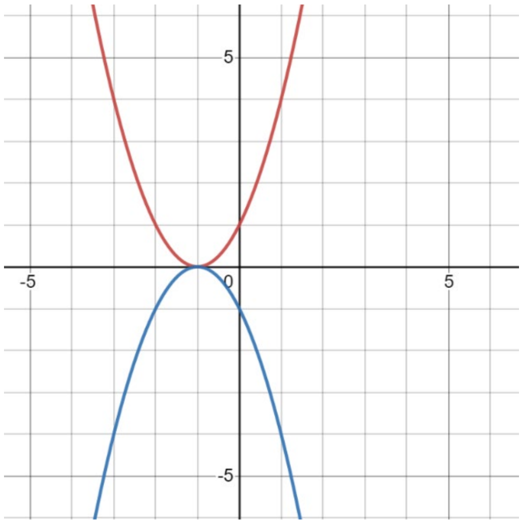
- A reflection in the **y-axis** is indicated by a **negative sign inside the bracket**. It has general form

$$\boldsymbol{y = f(-x)}.$$

For example, consider the graph opposite.

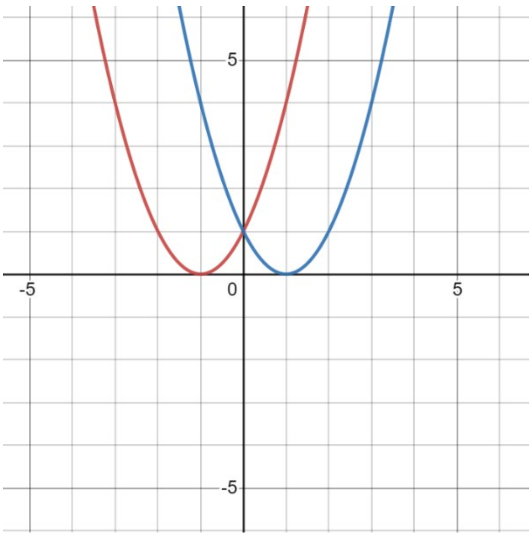
The red curve is $f(x) = (x + 1)^2$.
The blue curve is $-f(x) = -(x + 1)^2$.

This shows a reflection in the x-axis.



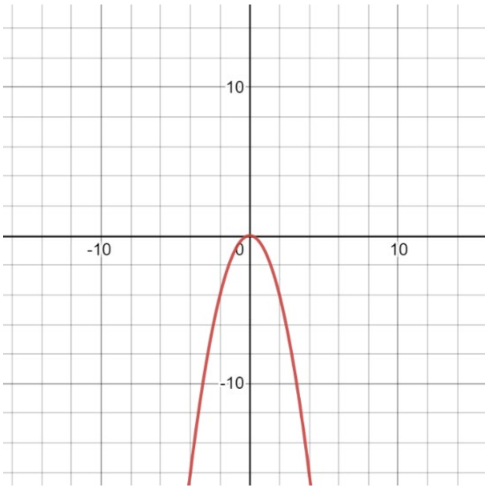
The following graph shows a reflection in the y-axis.

The red curve is $f(x) = (x + 1)^2$.
The blue curve is $f(-x) = (-x + 1)^2$.

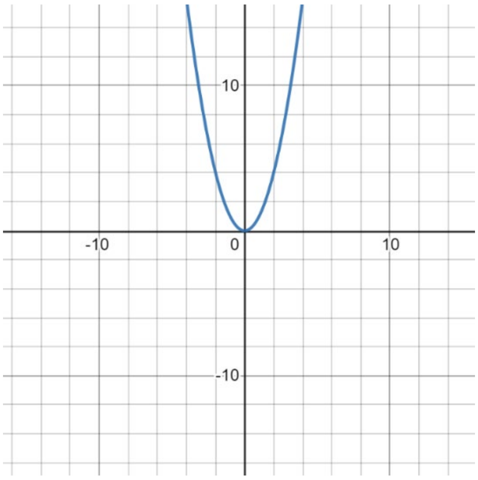


If there is **already a negative sign** inside or outside the bracket before the curve is reflected, then adding another negative sign will **cancel this out** and make a positive.

For example, take the curve given by the function $f(x) = -(x)^2$, which looks like this:



If we were to reflect this curve in the **x-axis**, we would need to add a **negative sign outside the bracket**. As there already is a negative sign outside the bracket, the two **cancel out** to make a **positive**. This means the function becomes $-f(x) = x^2$:



Example: A function $f(x)$ passes through the point $(-1, 3)$. What are the new coordinates of this point if the function is transformed by:

a) $-f(x)$

b)$f(-x)$

a) *This is a reflection in the x-axis. Therefore, the y-coordinate is multiplied by* -1 *and the x-coordinate remains the same. So, this point will become* $(-1, -3)$.

b) *This is a reflection in the y-axis. The x-coordinate will be multiplied by* -1 *and the y-coordinate remains the same. So, this point will become* $(1, 3)$.

Translations and Reflections – Practice Questions

1. If $f(x) = x^2$, sketch $f(x+2)$

2. $f(x) = x^2$, sketch $f(x) - 1$

3. A curve is described by the function $f(x) = (x-4)^3$. Sketch the curve and write the function if the curve is reflected in the x-axis.

4. A curve is described by the function $f(x) = (x-4)^3$. Sketch the curve and write the function if the curve is reflected in the y-axis.

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.