1. Malik is playing a game in which he has to throw a 6 on a fair six-sided die to start the game. Find the probability that

   i. Malik throws a 6 for the first time on his third attempt,

   [3]

   ii. Malik needs at most ten attempts to throw a 6.

   [2]

2. The weights of bags of a particular brand of flour are quoted as 1.5 kg. In fact, on average 10% of bags are underweight.

   i. Find the probability that, in a random sample of 50 bags, there are exactly 5 bags which are underweight.

   [3]

   ii. Bags are randomly chosen and packed into boxes of 20. Find the probability that there is at least one underweight bag in a box.

   [2]

   iii. A crate contains 48 boxes. Find the expected number of boxes in the crate which contain at least one underweight bag.

   [2]

3. Every evening, 5 men and 5 women are chosen to take part in a phone-in competition. Of these 10 people, exactly 3 will win a prize. These 3 prize-winners are chosen at random.

   i. Find the probability that, on a particular evening, 2 of the prize-winners are women and the other is a man. Give your answer as a fraction in its simplest form.

   [4]

   ii. Four evenings are selected at random. Find the probability that, on at least three of the four evenings, 2 of the prize-winners are women and the other is a man.

   [4]

4. In a particular country, 8% of the population has blue eyes. Find the probability that, in a random sample of 20 people, exactly two have blue eyes. [2]

5. A rugby team of 15 people is to be selected from a squad of 25 players.

i. How many different teams are possible?

[2]

ii. In fact the team has to consist of 8 forwards and 7 backs. If 13 of the squad are forwards and the other 12 are backs, how many different teams are now possible?

[2]

iii. Find the probability that, if the team is selected at random from the squad of 25 players, it contains the correct numbers of forwards and backs.

[2]

6. The discrete random variable X takes the values 0, 1, 2 and 3 only. You are given that

$$\mathrm{P}(X = r) \; = \; k\,r!, \text{ where } k \text{ is a positive constant.}$$

(a) Show that $k = 0.1$. [2]

(b) Find $\mathrm{P}(X = 3)$. [1]

(c) Calculate the probability of obtaining the value 3 exactly 16 times when 32 independent observations of X are made. [2]

7. Every evening at bedtime my cat Arthur decides whether to spend the night inside or outside the house.

If Arthur spends a night inside, the probability that he will spend the next night outside is 0.6.

If Arthur spends a night outside, the probability that he will spend the next night inside is 0.9.

Arthur spends a Saturday night inside.

(a) Calculate the probability that Arthur will decide to spend the next Monday night outside. [3]

(b) Show that it is extremely likely that Arthur will spend at least one out of the next seven nights (Sunday to Saturday) inside. [3]

8. Every morning before breakfast Laura and Mike play a game of chess. The probability that Laura wins is 0.7. The outcome of any particular game is independent of the outcome of other games. Calculate the probability that, in the next 20 games,

(a) Laura wins exactly 14 games, [2]

(b) Laura wins at least 14 games. [2]

9. The discrete random variable X takes the values 0, 1, 2, 3, 4 and 5 with probabilities given by the formula

$$P(X = x) = k(x + 1)(6 - x) .$$

(a) Find the value of k. [2]

In one half-term Ben attends school on 40 days. The probability distribution above is used to model X, the number of lessons per day in which Ben receives a gold star for excellent work.

(b) Find the probability that Ben receives no gold stars on each of the first 3 days of the half-term and two gold stars on each of the next 2 days. [2]

(c) Find the expected number of days in the half-term on which Ben receives no gold stars. [2]

10. The 24 members of staff at the Blackley branch of the Midshires Bank decide to start a lottery. They will each pay £5 per week to enter the lottery.

Each week, three of the numbers 1, 2, 3 and 4 will be placed in a random order by a computer. Each member of staff will make 3 different attempts, entered into their computer workstation, to guess what the computer has chosen.

(a) Show that the probability that a particular member of staff enters the correct numbers in the correct order in any week is $\frac{1}{8}$. [3]

If exactly one person chooses the right numbers in the right order, that person will win a prize of all £120 paid by staff to enter the lottery. If **no-one** chooses the right numbers, or **if more than one person** chooses the right numbers, the prize will be 'rolled over' to the next week.

(b) Calculate the probability that someone will win the prize in the first week. [2]

(c) Calculate the probability that the customer services manager, Marak, will win the prize in each of the first two weeks. [2]

In fact, when the lottery started, nobody won until the prize reached £6000. At that point Milena, the chief cashier, won. Marak commented that there was a one in a million chance of this happening to Milena.

(d) Determine whether Marak's comment is correct. [4]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$\left(\frac{5}{6}\right)^2 \times \frac{1}{6} = \frac{25}{216}$ (= 0.116)	M1	For 5/6 (or 1 – 1/6) seen	If extra term or whole number factor present give M1M0A0
		i		M1	For whole product	
		i		A1	cao **Examiner's Comments** This was well answered with the most candidates gaining all 3 marks. Some calculated $(1/6)^3$ and so scored 0 and some just scored 1 for ⅚.	Allow 0.12 with working
		ii	$1-\left(\frac{5}{6}\right)^{10} = 1-0.1615 = 0.8385$	M1	For $(5/6)^{10}$(without extra terms)	Allow 0.838 or 0.839 without working and 0.84 with working. For addition $P(X = 1) + \ldots + P(X = 10)$ give M1A1 for 0.84 or better, otherwise M0A0
		ii		A1	cao **Examiner's Comments** Only about 25% of candidates gained marks in this part - most either misread or misinterpreted the question and calculated the probability of needing exactly 10 attempts. Some candidates read the question correctly but spent time calculating all the probabilities from 1 to 10 and summing them, usually successfully. Very few used the method stated in the mark scheme.	
			Total	5		

2		i	X~ B(50, 0.1)	M1	For $0.1^5 \times 0.9^{45}$	
		i	$P(5\text{ underweight}) = \binom{50}{5} \times 0.1^5 \times 0.9^{45} = 0.1849$	M1	For $\binom{50}{5} \times p^5 \times q^{45}$	With $p + q = 1$ Also for $2118760 \times 8.73 \times 10^{-8}$
		i		A1	CAO Examiner's Comments This question was very well answered, with most candidates scoring all 3 marks. However, a few candidates seemed to have no idea about the binomial distribution.	Allow 0.185 or better NB 0.18 gets A0
		ii	X~ B(20, 0.1)			
		ii	$P(X \geq 1) = 1 - P(X = 0)$	M1	For 0.1216	Allow M1 for 0.9^{20}
		ii	$= 1 - 0.1216 = 0.8784$	A1	CAO Examiner's Comments Again another well answered question, although occasionally candidates did not read the question carefully and continued to use $n = 50$ in their calculation.	Allow 0.878 or better See tables at the website http:/www.mei.org.uk/files/pdf/formula_book_mf2.pdf
		iii	$E(X) = 48 \times 0.8784 = 42.16$ (= 42.2)	M1		
		iii		A1	FT their probability from part (ii) Examiner's Comments Full marks were available here for a correct follow through from part (ii), so many candidates managed to recover from an incorrect answer. However a large proportion of candidates rounded their answer to the	If any indication of rounding to 42 or 43 or to another integer on FT allow M1A0 SC1 for 48 × their p giving an integer answer. NB 0.6083 in (ii) leads to 29.20 NOTE RE OVER-SPECIFICATION OF ANSWERS If answers are grossly over-specified, deduct the final

					nearest whole number, thus losing a mark. Others over-specified their final answer, again losing a mark. Other common errors were to use $p = 0.1$, rather than their answer to part (ii), or to use $n = 48 \times 20$.	answer mark in every case. Probabilities should also be rounded to a sensible degree of accuracy. In general final non probability answers should not be given to more than 4 significant figures. Allow probabilities given to 5 sig fig. PLEASE HIGHLIGHT ANY OVER-SPECIFICATION Please note that there are no G or E marks in scoris, so use B instead
			Total	7		
3		i	$3 \times \frac{5}{10} \times \frac{4}{9} \times \frac{5}{8} = \frac{300}{720} = \frac{5}{12} = (0.4167)$	M1	For 5/10 × 4/9	Correct working but then multiplied or divided by some factor scores M1M1M0A0 Zero for binomial
		i		M1	For × 5/8	Allow M2 for equivalent triple such as $\frac{5}{10} \times \frac{5}{9} \times \frac{4}{8}$
		i		M1	For 3 × triple product	Or 3 separate equal triplets added
		i		A1	CAO (Fully simplified)	Answer must be a fraction
		i	**Or** $\frac{\binom{5}{2} \times \binom{5}{1}}{\binom{10}{3}} = \frac{10 \times 5}{120} = \frac{5}{12}$	M1*	**For** $\binom{5}{2} \times \binom{5}{1}$	Seen
		i		M1*	**For** $\binom{10}{3}$	Seen

		i		M1*dep	For whole fraction	Correct working but then multiplied or divided by some factor scores M1M1M0A0
		i		A1	CAO (Fully simplified) Examiner's Comments Candidates using the ^nC_r method tended to be more successful, as when using the product of 3 fractions method many did not realise that they needed to multiply the final product by 3. A small minority of candidates did not follow instructions and either left a fraction in unsimplified form (usually 15/36) or gave the answer as a decimal.	
		ii	$4\times\frac{7}{12}\times\left(\frac{5}{12}\right)^3+\left(\frac{5}{12}\right)^4$	M1FT	For first probability	Allow $^4C_3$
		ii		M1FT	For $(5/12)^4$	
		ii		M1FT	For sum of both correct probabilities	Provided sum <1
		ii	= 0.169 + 0.030 × 0.199 Or = $\frac{875}{5184}+\frac{625}{20736}=\frac{1375}{6912}$	A1	CAO Do not allow 0.2, unless fuller answer seen first Examiner's Comments Most candidates made a reasonable start in this part, using their answer from part (i). However, many only calculated one probability, or missed the coefficient of 4 when calculating the probability of 3 evenings, not realising this was a binomial situation. Some candidates calculated the probability of 3, rather than *at least* 3, and thus only gained 1 mark. A small minority of candidates used statistical functions on graphical calculators to just write down an answer – this was a risky strategy, as a slip in copying the	Alternative for 1- (P(0) + P(1) + P(2)) allow M1FT for two 'correct' probs, M1 for sum of three 'correct', M1 for 1 – answer, A1 CAO NOTE RE OVER-SPECIFICATION OF ANSWERS If answers are grossly over-specified, deduct the final answer mark in every case. Probabilities should also be rounded to a sensible degree of accuracy. In general final non probability answers should not be given to more than 4 significant figures. Allow probabilities given to 5 sig fig. PLEASE HIGHLIGHT ANY OVER-SPECIFICATION

					answer was heavily penalised, since no method was shown.	Please note that there are no G or E marks in scoris, so use B instead
			Total	8		
4			Binomial(20, 0.08) P(2 blue) = 0.27[11]	M1(AO3.3) A1(AO1.1) [2]	BC	
			Total	2		
5		i	$\binom{25}{15}$ $= 3268760$	M1		Accept $^{25}C_{15}$ or $^{25!}/_{(15!10!)}$ or equivalent for M1 No marks for permutations
		i		A1	**Examiner's Comments** This was very well-answered.	Exact answer required
		ii	$\binom{13}{8} \times \binom{12}{7} = 1287 \times 792$ $= 1019304$	M1	For product of both correct combinations	No marks for permutations
		ii		A1	CAO **Examiner's Comments** This was again usually answered well although a fairly common error was to add the two combinations rather than to multiply them.	Exact answer required

		iii	1019304/3268760	M1	For their (ii) divided by their (i)	
		iii	= 0.312 (0.311832) Allow fully simplified fraction11583/37145	A1FT		Allow 0.31 with working
		iii	OR			
		iii	$\binom{15}{8}\times\frac{13}{25}\times\frac{12}{24}\times\frac{11}{23}\times\frac{10}{22}\times\frac{9}{21}\times\frac{8}{20}\times\frac{7}{19}\times\frac{6}{18}\times\frac{12}{17}\times\frac{11}{16}\times\frac{10}{15}\times\frac{9}{14}\times\frac{8}{13}\times\frac{7}{12}\times\frac{6}{11}$	M1	For product of fractions with coefficient	SC1 for $\binom{15}{8}\times\left(\frac{13}{25}\right)^8\times\left(\frac{12}{25}\right)^7$ Allow $\binom{15}{8}$ or $\binom{15}{7}$
		iii	= 0.312	A1	**Examiner's Comments** This was another well-answered question with even those who had added in part (ii) still usually scoring both marks on follow through. Candidates who tried to use a probability method (instead of simply dividing their answer to part (i) by their answer to part (ii)) were rarely successful, and even if they did have the correct product of 15 probabilities, they rarely multiplied this by any, let alone the correct combination.	
			Total	6		
6		a	k[P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)] =1 k (1 + 1 + 2 + 6) = 1 so k = 0.1	M1(AO1.1) A1(AO3.1a) [2]		
		b	P(X = 3) = 0.6	B1(AO1.1) [1]		

		c	$X \sim$ B (32, 0.6) 0.073	M1(AO3.3) A1(AO3.4) [2]	FT their 0.6 **BC** FT their 0.6	or 0.0728 or 0.07283
			Total	5		
7		a	0.4 × 0.6 or 0.6 × 0.1 p(in, out) + p(out, out) 0.3	B1(AO 3.1b) M1(AO 3.1a) A1(AO 1.1) [3]	If **B0M0** allow **SC1** for correct tree diagram with all possible outcomes shown.	
		b	1 – p(out every night) $1 - 0.6 \times 0.1^6$ $= 0.9999994 \approx 1$ so extremely likely	M1(AO 3.1b) A1(AO 1.1) A1(AO 3.2a) [3]		
			Total	6		
8		a	use of $B \sim (20,0.7)$ soi 0.191638982753…rounded to 2 or more dp isw BC	M1 (AO 3.1b) A1 (AO 1.1)		

				[2]	NB 0.1916 or 0.192 or 0.19 Examiner's Comments This was answered well when clear working was shown. Mistakes may have been due to going straight to the calculator and mistakenly finding at least 14 rather than exactly 14.	
		b	P($X \le 13$) found soi 0.608009811813…rounded to 2 or more dp isw BC	M1 (AO 3.1b) A1 (AO 1.1) [2]	NB 0.391990188187 NB 0.6080 or 0.608 or 0.61 **M0** if P($X =$ 13) used NB 0.1643… if **M0** allow **SC1** for 1 – P($X \le$ 14) = 1 – 0.58362… = 0.41637083 rounded to 2 or more dp Examiner's Comments Candidates who did well in this question made efficient use of the binomial distribution function in their calculator. Candidates who did less well mistook "at least" for	

					"greater than" in this part or made arithmetic slips when using traditional methods to calculate the probabilities.	
			Total	4		
9		a	$k(1 \times 6 + 2 \times 5 + 3 \times 4 + 4 \times 3 + 5 \times 2 + 6 \times 1) = 1$ oe $[k =]\frac{1}{56}$ sw	M1 (AO 3.1a) A1 (AO 1.1) [2]	allow one slip in arithmetic B2 if unsupported Examiner's Comments Candidates who did well in this question found k successfully and used the result in fractional form in part (b) to calculate the requested probability.	
		b	$(6 \times k)^3 \times (12 \times k)^2$ oe seen $\frac{243}{4302592}$ or 0.000056477584 rounded to 2 or more sf	M1 (AO 2.1) A1 (AO 1.1) [2]	FT their k Examiner's Comments The most common error was for candidates to add the two probabilities instead of multiplying.	
		c	$40 \times 6k$ 4.286 or 4.29 or 4.3	M1 (AO 3.1b) A1 (AO 3.2b) [2]	FT their k mark the final answer Examiner's Comments There was a significant number of candidates that	

					spoiled their answer to this part by rounding to the nearest whole number.	
			Total	6		
10		a	$\frac{1}{24}+\frac{23}{24}\times\frac{1}{23}+\frac{23}{24}\times\frac{22}{23}\times\frac{1}{22}$ $=\frac{1}{8}$	M1 (AO 2.1) M1(AO 2.4) M1(AO 1.1) [3]	$\frac{1}{24}$ seen addition of three terms	
		b	Use of $B(24, \frac{1}{8})$ 0.139(009338…) to 3, 4 or 5 sf	M1 (AO 3.3) M1(AO 1.1) [2]		
		c	$\left(\frac{1}{8}\right)\left(\frac{7}{8}\right)^{23}$ $(0.0057955….)^2$ = 0.000033588489… to 3, 4 or 5 sf	M1 (AO 3.1a) A1(AO 1.1) [2]	soi NB 0.0057955….	
		d	[6000 ÷ 120] = 50 $[1 - 0.13900…]^{49} \times 0.0057955…$ 0.00000376653… so Mark is wrong – it's more like 4 in a million	B1 (AO 3.1b) M1(AO 3.1a) A1(AO 1.1) A1(AO 3.2a) [4]	FT their part **(b)**	
			Total	11		