Statistical Distributions Cheat Sheet

Probability distributions

A variable can take any of a range of specific values. A variable is random if the outcome is not known until the experiment is carried out. Random variables are written as upper case letters, for example X or Y. The particular values the random variable can take are written as equivalent lower case letters, for example x or y. The probability that the random variable X takes a particular value x is written as:

$$\mathrm{P}(X = x)$$

A probability distribution fully describes the probability of any outcome in the sample space. The probability distribution of a discrete random variable can be describe using probability mass function, a table or a diagram.

When all probabilities are the same, the distribution is known as discrete uniform distribution. For example, the score when a fair dice is rolled.

Example 1: Three fair coins are tossed. A random variable, X is defined as the number of heads when the three coins are tossed. Shown the probability distribution as X as a a) table, b) probability mass function, c) diagram.

All possible outcomes when the coins are tossed:
HHH, HHT, HTH, THH, HTT, THT, TTH, TTT
Since the coins are fair, the probability of getting each outcome listed above is the same.

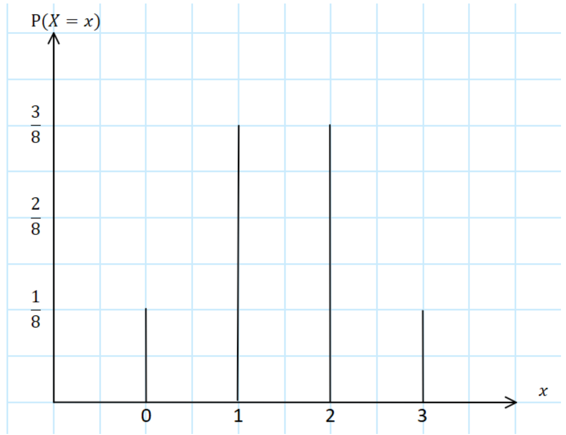
a. Table

No. of heads, x	0	1	2	3
$\mathrm{P}(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

b. Probability mass function:

$$\mathrm{P}(X = x)\begin{cases} \frac{1}{8} & x = 0,3 \\ \frac{3}{8} & x = 1,2 \\ 0 & \text{otherwise} \end{cases}$$

c. Diagram:



Binomial distribution

You can define a random variable X to represent the number of successful trials when you are carrying out a number of trials.

You can model X with a binomial distribution $\mathrm{B}(n, p)$ if:

- There are a fixed number of trials, n
- There are two possible outcomes (success and failure)
- There is a fixed probability of success, p
- The trials are independent of each other

If a random variable X has the binomial function $\mathrm{B}(n, p)$, then its probability mass function is given by:

$$\mathrm{P}(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$$

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = {}^{n}\mathrm{C}_{r}$$

n is also called the index and p is called the parameter.

You can also use the binomial probability distribution function in the calculator to work out the binomial probabilities.

Example 2: The probability that a randomly chosen member of a reading group is left-handed is 0.15. A random sample of 20 members of the group is taken.

a. Suggest a suitable model for the random variable X, the number of members in the sample who are left-handed. Justify your choice.

The random variable can only take two values, left-handed or right-handed. There are a fixed number of trials: 20, and a fixed probability of success: 0.15. Assuming each member in the sample is independent, a suitable model is $X \sim \mathrm{B}(20, 0.15)$.

b. Use your model to calculate the probability that:

i. Exactly 7 of the members in the sample are left-handed

$$\mathrm{P}(X = 7) = \binom{20}{7} \times (0.15)^7 (0.85)^{13}$$
$$= 0.01601 \ldots$$
$$= 0.0160 \text{ (3 s.f.)}$$

ii. Fewer than two of the members in the sample are left-handed

$$\mathrm{P}(X < 2) = \mathrm{P}(X = 0) + \mathrm{P}(X = 1)$$
$$= 0.03875 \ldots + 0.13679 \ldots$$
$$= 0.176 \text{ (3s.f.)}$$

Cumulative probabilities

$\mathrm{P}(X \leq x)$ gives the sum of all individual probabilities for values up to and including x.
$\mathrm{P}(X < x)$ gives the sum of all individual probabilities for values not greater than x.
$\mathrm{P}(X \geq x)$ gives the sum of all individual probabilities for x and values greater than x.
$\mathrm{P}(X > x)$ gives the sum of all individual probabilities for values greater than x.

You can use the tables in the formula book or the binomial cumulative probability function in the calculator to find cumulative probabilities for $X \sim \mathrm{B}(n, p)$.

Example 3: A spinner is designed so that the probability it lands on red is 0.3. Jane has 12 spins. Find the probability that Jane obtains:

a. No more than 2 reds
Let X = number of reds in 12 spins.
$X \sim \mathrm{B}(12, 0.3)$
$\mathrm{P}(X \leq 2) = 0.2528$

b. At least 5 reds
$$\mathrm{P}(X \geq 5) = 1 - \mathrm{P}(X \leq 4)$$
$$= 1 - 0.7237$$
$$= 0.2763$$

Jane decides to use this spinner for a class competition. She wants the probability of winning a prize to be <0.05. Each student will have 12 spins and the number of reds will be recorded. Find out how many reds are needed to win a prize.

Let r =the smaller number of reds needed to win a prize
We need to find the value of r so that $\mathrm{P}(X \geq r) < 0.05$

From the table:
$\mathrm{P}(X \leq 5) = 0.8822$
$\mathrm{P}(X \leq 6) = 0.9614$
$\mathrm{P}(X \leq 7) = 0.9905$

$x = 6$ is the first value which gives a probability greater than 0.95, so we use the probability of $\mathrm{P}(X \leq 6)$ to find r.
$\mathrm{P}(X \leq 6) = 0.9614$ implies that :
$$\mathrm{P}(X \geq 7) = 1 - 0.9614$$
$$= 0.0386 < 0.05$$

$\therefore r = 7$

7 or more reds are needed to win a prize.