1. i. Kathryn is allowed three attempts at a high jump. If she succeeds on any attempt, she does not jump again. The probability that she succeeds on her first attempt is $\frac{3}{4}$. If she fails on her first attempt, the probability that she succeeds on her second attempt is $\frac{3}{8}$. If she fails on her first two attempts, the probability that she succeeds on her third attempt is $\frac{3}{16}$. Find the probability that she succeeds.

[3]

ii. Khaled is allowed two attempts to pass an examination. If he succeeds on his first attempt, he does not make a second attempt. The probability that he passes at the first attempt is 0.4 and the probability that he passes on either the first or second attempt is 0.58. Find the probability that he passes on the second attempt, given that he failed on the first attempt.

[3]

2. The table shows the numbers of members of a swimming club in certain categories.

	Male	Female
Adults	78	45
Children	52	*n*

It is given that $\frac{5}{8}$ of the female members are children.

i. Find the value of *n*.

[2]

ii. Find the probability that a member chosen at random is either female or a child (or both).

[2]

The table below shows the corresponding numbers for an athletics club.

	Male	Female
Adults	6	4
Children	5	10

iii. Two members of the athletics club are chosen at random for a photograph.

a. Find the probability that one of these members is a female child and the other is an adult male.

[2]

b. Find the probability that exactly one of these members is female and exactly one is a child.

[2]

3. Each question on a multiple-choice examination paper has n possible responses, only one of which is correct. Joni takes the paper and has probability p, where $0 < p < 1$, of knowing the correct response to any question, independently of any other. If she knows the correct response she will choose it, otherwise she will choose randomly from the n possibilities. The events K and A are 'Joni knows the correct response' and 'Joni answers correctly' respectively.

i. Show that $\mathrm{P}(A) = \dfrac{q + np}{n}$ where $q = 1 - p$.

[3]

ii. Find $P(K|A)$.

[3]

A paper with 100 questions has $n = 4$ and $p = 0.5$. Each correct response scores 1 and each incorrect response scores – 1.

iii.

a. Joni answers all the questions on the paper and scores 40. How many questions did she answer correctly?

[1]

b. By finding the distribution of the number of correct answers, or otherwise, find the probability that Joni scores at least 40 on the paper using her strategy.

[6]

4. During an outbreak of a disease, it is known that 68% of people do not have the disease. Of people with the disease, 96% react positively to a test for diagnosing it, as do m% of people who do not have the disease.

i. In the case $m = 8$, find the probability that a randomly chosen person has the disease, given that the person reacts positively to the test.

[5]

ii. What value of m would be required for the answer to part **(i)** to be 0.95?

[4]

5. For the events A and B it is given that

$$P(A) = 0.6,\ P(B) = 0.3 \text{ and } P(A \text{ or } B \text{ but not both}) = 0.4.$$

i. Find $P(A \cap B)$.

[3]

ii. Find $P(A' \cap B)$.

[1]

iii. State, giving a reason, whether A and B are independent.

[1]

6. Events A and B are such that $P(A) = 0.5$, $P(B) = 0.6$ and $P(A|B') = 0.75$.

i. Find $P(A \cap B)$ and $P(A \cap B)$.

[6]

ii. Determine, giving a reason in each case,

a. whether A and B are mutually exclusive,

b. whether A and B are independent.

[2]

iii. A further event C is such that $P(A \cup B \cup C) = 1$ and $P(A \cap B \cap C) = 0.05$. It is also given that $P(A \cap B' \cap C) = P(A' \cap B \cap C) = x$ and $P(A \cap B' \cap C') = 2x$.
Find $P(C)$.

[3]

7. Each of the 30 students in a class plays at least one of squash, hockey and tennis.

- 18 students play squash
- 19 students play hockey
- 17 students play tennis
- 8 students play squash and hockey
- 9 students play hockey and tennis
- 11 students play squash and tennis

(a) Find the number of students who play all three sports. [3]

A student is picked at random from the class.

(b) Given that this student plays squash, find the probability that this student does not play hockey. [1]

Two different students are picked at random from the class, one after the other, without replacement.

(c) Given that the first student plays squash, find the probability that the second student plays hockey. [4]

8. (a) Events *A* and *B* are independent, and $P(A \cap B) = \frac{1}{24}$ and $P(A \cup B) = \frac{3}{8}$.

Find P(*A*) and P(*B*). [5]

(b) Events *C* and *D* are such that P(*C*) = 0.6, P(*D*) = 0.3 and P (*C* ∪ D) = 0.8. Find P (*D*|*C*'). [4]

9. For events *A*, *B* and *C* it is given that P(*A*) = 0.6, P(*B*) = 0.5, P(*C*) = 0.4 and P (*A* ∩ *B* ∩ *C*) = 0.1. It is also given that events *A* and *B* are independent and that events *A* and *C* are independent.

(i) Find P (*B* | *A*) [1]

(ii) Given also that events *B* and *C* are independent, find P (*A*'∩*B*'∩*C*'). [4]

(iii) Given instead that events *B* and *C* are **not** independent, find the greatest and least possible values of P (*A*'∩*B*'∩*C*'). [5]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$\frac{3}{4} + \frac{1}{4} \times \frac{3}{8}$	M1	$\frac{1}{4} \times \frac{5}{8} \times \frac{13}{16}$ $\left(= \frac{65}{512} \text{ or } 0.127\right)$	
		i	$+ \frac{1}{4} \times \frac{5}{8} \times \frac{3}{16}$	M1	$1 - \frac{1}{4} \times \frac{5}{8} \times \frac{13}{16}$	
		i	$= \frac{447}{512}$ or 0.873 (3 sf)	A1	Examiner's Comments Most candidates answered this part correctly. A few omitted the probability of success at either the first attempt or the third attempt. Others thought that the probability of success at the third attempt was $\frac{1}{4} \times \frac{5}{8} \times \frac{13}{16}$ instead of $\frac{1}{4} \times \frac{5}{8} \times \frac{3}{16}$. Only a few chose the more elegant method using the complement.	
		ii	$0.6p$ or equiv seen	B1	Tree diag alone insufficient for mark.	NB 0.6 × 0.3 = 0.18 seen at the end is
		ii	$0.4 + 0.6p = 0.58$	M1	Or $0.6p = 0.18$. "0.18" alone insufficient	probably a check, not an answer.
		ii	$p = 0.3$	A1	Examiner's Comments Many good answers were seen. Some candidates appeared not to understand the difference between P(he passes on the 2nd attempt) and P(he passes on the 2nd attempt, given that he failed on the first), giving an answer of 0.58 – 0.4 = 0.18. A few formed an equation, but with the term "0.4p" instead of "0.6p". Some found the correct answer of 0.3 but then unnecessarily continued by using the formula $\text{P(A\|B)} = \frac{\text{P}(A \cap B)}{\text{P(B)}} = \frac{0.6 \times 0.3}{0.6} = 0.3$ Others, having found the correct value of 0.3, continued with 0.6 × 0.3 = 0.18.	But if 0.3 seen and 0.18 is very clearly indicated as the ans then B1M1A0

					If candidates did not make clear that this last line was only a check, rather than an answer, they were likely to lose the final mark. Many candidates gave incorrect attempts based on misunderstandings of conditional probability, such as 0.4 ÷ 0.58 = 0.690.	Probability
			Total	6		
2		i	$\frac{n}{n+45} = \frac{5}{8}$ or $n : 45 = 5 : 3$ or $\frac{3}{8} : 45 =$ $\frac{5}{8} : n$	M1	$\frac{3F}{8} = 45$ & $n = \frac{5}{8} \times F$; $45 \times \frac{8}{3} - 45$; $45 \times \frac{8}{3} \times \frac{5}{8}$	correct first step involving n or complete correct method for finding n
		i	$n = 75$	A1	**Examiner's Comments** Although many candidates had no problem with this part, a disappointing number appeared to have trouble coping with simple ratio and proportion. A common incorrect response was $\frac{5}{8} \times 45 = 28.125$, rounded to 28. Another common response was $n = 45 \times \frac{8}{5} = 72$.	
		ii	$\frac{45+"75"+52}{45+"75"+52+78}$ alone oe	M1	$1 - \frac{78}{45+"75"+52+78}$ oe or $\frac{"250"-78}{"250"}$ oe Completely correct method ft their integer answer to (i)	$\frac{45+"75"}{"250"} + \frac{52+"75"}{"250"} - \frac{"75"}{"250"}$ or 0.48 + 0.508 – 0.48×0.508
		ii	$= \frac{86}{125}$ or $\frac{172}{250}$ or 0.688 (3 sf) oe	A1 ft	eg if their (i) is 28, ans 0.616 or M1A1ft **Examiner's Comments** This is a simple question requiring the addition of three numbers (or	

					subtraction of one number from the total), followed by division by the total. However, presumably because of unfamiliarity with two-way tables, only a minority of candidates used this approach. Some candidates tried to use multiplication of probabilities, i.e. P(female)×P(adult) + P(female)×P(child) + P(male)×P(child). These candidates generally failed to realise that the second probability in each product must be a conditional probability. A correct version of this method is possible, but this method is unnecessarily long and was rarely carried out correctly. Many candidates added the frequencies (or the probabilities) for females plus children, thus including the female children twice. Others added females plus children plus female children, thus including the female children three times. Perhaps these mistakes were prompted by a misunderstanding of the phrase “or both” in the question. A few correctly added females plus children and subtracted female children.	Probability
		iii	(a) $\frac{10}{25}\times\frac{6}{24}$ or $\frac{6}{25}\times\frac{10}{24}$ seen (or $\frac{2}{5}\times\frac{1}{4}$ or $\frac{6}{25}\times\frac{5}{12}$)	M1	or $\frac{10}{25}\times\frac{6}{25} + \frac{6}{25}\times\frac{10}{25}$ or $\frac{10}{25}\times\frac{6}{25}\times 2$ oe $\frac{{}^{10}C_1\times{}^{6}C_1}{{}^{25}C_2}$ oe or $\frac{10\times 6}{300}$ oe	ie allow M1 if ‘2 ×’ is omitted **OR** if 25 instead of 24, but not both errors allow M1 for correct num or denom
		iii	($\frac{10}{25}\times\frac{6}{24} + \frac{6}{25}\times\frac{10}{24}$ or $\frac{10}{25}\times\frac{6}{24}\times 2$) $= \frac{1}{5}$	A1	**Examiner's Comments** Many candidates attempted P(female child) × P(adult male). Most of these made one or both of two errors: omission of “2 ×” and/or failure to reduce the second denominator to 24. Thus $\frac{4}{25}\times\frac{6}{25}$ and $\frac{4}{25}\times\frac{6}{24}$ and $2\times\frac{4}{25}\times\frac{6}{25}$ were frequently seen. A correct method using combinations was seen, but some candidates added ${}^{10}C_1$ and ${}^{6}C_1$ in the numerator instead of multiplying. Others had a denominator of 25 instead of ${}^{25}C_2$.	NB long methods may be correct, eg $(\frac{14}{25}\times\frac{10}{14})\times(\frac{11}{24}\times\frac{6}{11})$ same as $\frac{10}{25}\times\frac{6}{24}$

		iii	(b) FA + MC or FC + MA $\underline{\text{Either}}\ \frac{4}{25}\times\frac{5}{24}\ \times 2$ $\underline{\text{or}}\ \frac{10}{25}\times\frac{6}{24}\ \times 2$ NB ft their (iiia)	M1	Allow $\frac{10}{25}\times\frac{6}{25}\times 2$ or $\frac{4}{25}\times\frac{5}{25}\times 2$ or $\frac{10}{25}\times\frac{6}{24}+\frac{4}{25}\times\frac{5}{24}$ or $\frac{10}{25}\times\frac{6}{25}+\frac{4}{25}\times\frac{5}{25}$ NB ft their (iii)(a) $\frac{{}^{10}C_1\times{}^{6}C_1}{{}^{25}C_2}+\frac{{}^{4}C_1\times{}^{5}C_1}{{}^{25}C_2}$ oe or $\frac{60+20}{300}$ oe	Probability ie allow 25 instead of 24 AND allow one case with × 2 or both cases without × 2 ie allow 25 and one of these two errors cf scheme for (iii)(a)
		iii	$\left(\frac{4}{25}\times\frac{5}{24}\ \times 2+\frac{10}{25}\times\frac{6}{24}\ \times 2=\frac{1}{5}+\frac{1}{15}\right)$ $=\frac{4}{15}$ or 0.267 (3 sf)	A1	cao **Examiner's Comments** Mistakes made in part (i) were carried over into this part. Some candidates found P(female child) × P(adult male), but not P(adult female) × P(male child). Some very common errors involved such working as P(female) × P(child) = $\frac{14}{25}\times\frac{15}{25}$.	allow M1 if one of these fracts correct NB ${}^{25}C_2$ in denom NOT M1, cf (iii)(a) NB see note on long methods in 7(iiia)
			Total	8		
3		i	$P(A) = P(K)\times 1 + P(K')\times {}^{1}/_{n}$	M1		
		i	$= p + (1 - p)/n$	A1	Allow $p+\frac{q}{n}$	
		i	$=\frac{q+np}{n}$ AG	B1	**Examiner's Comments** Almost all the candidates scored full marks.	
		ii	$P(K \cap A) = p$	B1		

		ii	$$P(K\|A) = \frac{p}{\frac{q+np}{n}} = \frac{np}{q+np}$$	M1		Probability
		ii		A1	AEF **Examiner's Comments** Almost all the candidates scored full marks.	
		iii	If X answers are correct $100 - X$ are incorrect so score = $2X - 100 = 40$ giving $X = 70$	B1	70 seen	
		iii	P(A) = 5/8 (α) $E(X) = 100 \times {}^5/_8 = 62.5$	B1		
		iii	$\mathrm{Var}(X) = s^2 = 100 \times {}^5/_8 \times {}^3/_8$ $(= 23.4375)$ $(= \frac{375}{16})$	M1A1	Allow M1 from wrong p	
		iii	$P(X \geq 70) = 1 - \Phi\,[(69.5 - 62.5)/s]$	M1A1	Normal approximation. Allow M1 from 40/70 or wrong p	
		iii	= 0.0741	A1	Standardise M1 only if no or wrong cc, A1 for 0.0607	
		iii	(β) $E(2X - 100) = 25$	B1		
		iii	$\mathrm{Var}(2X - 100) = 93.75$	M1A1		
		iii	$P(2X - 100 \geq 40) = 1 - \Phi\,[(39 - 25)/\sqrt{(93.75)}]$	M1A1	Standardise, M1 only for no or wrong cc, A1 for 0.0671	
		iii	= 0.0741	B1		
		iii	(γ) Score per question = S $E(S) = 1 \times {}^5/_8 - 1 \times {}^3/_8 = ¼$	B1		

		iii	$\text{Var}(S) = 1^2 \times {}^5/_8 + 1^2 \times {}^3/_8 - ({}^1/_4)^2$	M1A1		Probability
		iii	Total, $T \sim N(25, 93.75)$ $P(T \geq 40) = 1 - \Phi\,[39 - 25]/\sqrt{(93.75)}]$	M1A1	As for β	
		iii	= 0.0741	B1	**Examiner's Comments** Over half the candidates gained full marks, but there were some very confused attempts, muddling the various methods given on the mark scheme. Many obtained an incorrect value of p, using the answer to (ii) instead of (i).	
			Total	13		
4		i	0.32 × 0.96 or 0.68 × 0.08	M1		Allow M marks for 0.8 instead of 0.08 or incorrect 1 – 0.68.
		i	Both, added.	M1		
		i	= 0.3616	A1	May be implied.	
		i	0.32 × 0.96 ÷ “0.3616”	M1		
		i	0.850	A1	Allow 0.85 or $\frac{96}{113}$ **Examiner's Comments** Almost all candidates scored full marks.	
		ii	$\dfrac{0.32 \times 0.96}{0.32 \times 0.96 + 0.68 \times p} = 0.95$	M1,A1	Allow 0.3072	
		ii	Solve	M1	Allow failure to multiply brackets correctly, but NOT divide instead of subtract or vv.	

		ii	$p = 0.0238$, so $m = 2.38$	A1	$\frac{192}{1075}$ **Examiner's Comments** Most candidates scored full marks. A few lost the final mark by saying $m = 0.0238$. Some weaker candidates could not solve the equation.	Probability
			Total	9		
5		i	Let $P(A \cap B) = x$, $0.6 - x + 0.3 - x = 0.4$	M1A1	M1 for attempt to set up equation in x. **Examiner's Comments** Almost all gained full marks. Those who did not usually obtained the answer 0.5.	x must appear more than once.
		i	$x = 0.25$	A1		
		ii	0.05	B1 ft	0.3-(i). Ans must be ≤ 0. **Examiner's Comments** Those who were correct in (i) were also correct here.	
		iii	No, $0.6 \times 0.3 \neq 0.25$	B1 ft	Must have an answer to (i) **Examiner's Comments** Almost all the candidates earned this mark. Those who were incorrect in (i) usually scored the mark on follow through.	$P(B\|A') = 0.05 \div 0.4 = 0.125 \neq P(B)$
			Total	5		
6		i	$P(A \cap B') = 0.75 \times 0.4 = 0.3$	M1A1		
		i	$P(A \cap B) = 0.5 - \text{“}0.3\text{”} = 0.2$	M1A1		

		i	P($A \cup B$) = 0.5 + 0.6 – "0.2" = 0.9	M1A1	Examiner's Comments Four-fifths of the candidates obtained full marks.	Probability
		ii	(a) No, P($A \cap B$) ≠ 0 oe	B1		
		ii	(b) No, 0.5 × 0.6 ≠ 0.2 oe	B1	Examiner's Comments Three-quarters of the candidates obtained full marks. Some candidates did not fully understand the concept of mutual exclusiveness.	
		iii	P($A' \cap B' \cap C$) = 0.1 soi	B1ft	1 – (i)	
		iii	x = 0.1	B1		
		iii	P(C) = 2x + 0.05 + 0.1 = 0.35	B1	Examiner's Comments This was by far the most difficult question on the paper. Half the candidates gained no marks. Many did not realise that P(A ∪ B)=0.9 ⇒ P(A′ ∩ B′ ∩ C)=0.1. A few found the value of x correctly, but could make no further progress. Candidates who drew Venn diagrams did better than those who wrote down many equations but did not know what to do with them.	
			Total	11		
7		a	Attempt to represent information e.g. by Venn diagram with x in centre and 3 other correct values in terms of x Attempt total (in terms of x) = 30 x = 4 so n(S ⋂ H ⋂ T) = 4	B1(AO3.3) M1(AO3.4) E1(AO1.1) [3]	Any equivalent method — OR B1 $\frac{18}{30}+\frac{19}{30}+\frac{17}{30}-\left(\frac{8}{30}+\frac{9}{30}+\frac{11}{30}\right)\left(=\frac{26}{30}\right)$ Or the number doing all — M1 $1-"\frac{26}{30}"\left(=\frac{4}{30}\right)$	

					three is 4. **E0** for just $x = 4$	Probability
	b	$\frac{5}{9}$ oe	B1FT(AO2.2a) [1]	FT their **(a)**	
	c	$\frac{5}{9} \times \frac{19}{29}$ $\frac{4}{9} \times \frac{18}{29}$ $\frac{5}{9} \times \frac{19}{29} + \frac{4}{9} \times \frac{18}{29}$ $= \frac{167}{261}$ oe or 0.640 (3 s.f.)	B1(AO2.2a) B1(AO2.2a) M1(AO2.2a) A1(AO1.1) [4]	All correct	
		Total	8		
8	a	$P(A) \times P(B) = \frac{1}{24}$ $P(A) + P(B) = \frac{1}{24} + \frac{3}{8}$ $P(A) + \frac{1}{24P(A)} = \frac{5}{12}$ $24(P(A))^2 - 10P(A) + 1 = 0$ $((6P(A) - 1)(4P(A) - 1) = 0)$	M1(AO1.1a) M1(AO1.1) M1(AO3.1a) A1(AO1.1) A1(AO1.1) [5]	Attempt equation in one P Correct quadratic	

			$P(A) = \frac{1}{6}$ and $P(B) = \frac{1}{4}$ or vice versa		equation in one P Allow without "vice versa"	Probability
		b	$P(C') = 1 - P(C)$ $(= 0.4)$ $P(D \cap C') = P(C \cup D) - P(C)$ $(= 0.2)$ $P(D \mid C') = \frac{P(D \cap C')}{P(C')}$ $= \frac{0.2}{0.4} = 0.5$	M1(AO 1.2) M1(AO 1.1) M1(AO 1.1) M1(AO 1.2) [4]	Attempted	
			Total	9		
9		i	0.5	B1 [1]	**Examiner's Comments** Almost all candidates answered this question correctly.	
		ii	0.16 + 0.2 + 0.1 + 0.14 + 0.1 + 0.1 + 0.06	M2	M1 for at least 4 correct. | 0.6 + 0.5 + 0.4 – 0.3 – 0.24 – 0.2 + 0.1 = 0.86 M2A1. M1 if incorrect	

			= 0.86 1 – “0.86” 0.14	A1 A1 [5]	coefficient of P($A \cap B \cap C$) used in otherwise correct formula. **Examiner’s Comments** Most candidates gained full marks. A few made errors in the formula for P($AUBUC$).	Probability
		iii	greatest : P($A' \cap B \cap C'$) = 0.04, P($A' \cap B \cap C$) = 0.16 P($A' \cap B' \cap C$) = 0 least: P($A' \cap B \cap C'$) = 0.2, P($A' \cap B \cap C$) = 0 P($A' \cap B' \cap$C) = 0.16 greatest 1 – (0.16 + 0.2 + 0.04 + 0.14 + 0.1 + 0.16) = 0.2 least 1 – (0.16 + 0.2 + 0.2 + 0.14 + 0.1 + 0.16) = 0.04	M1 M1 M1A1 A1 [5]	for any of these soi eg P(B∩C) = 0.26 — Greatest: 1 – (0.6 + 0.5 + 0.4 – 0.3 – 0.24 – 0.26 + 0.1) = 0.2 for any of these soi eg P(B∩C) = 0.1 — Least 1 – (0.6 + 0.5 + 0.4 – 0.3 – 0.24 – 0.1 + 0.1) = 0.04 M1 for fully correct method for either. **Examiner’s Comments**	

					This was the most difficult question on the paper. Those who drew Venn diagrams and put the correct probabilities in the correct places did better than those who tried to repeat the method in part (ii). There were few fully correct solutions, but many found one of the correct limits.	Probability
			Total	10		