Conditional Probability Cheat Sheet

Previously, you have looked at how to calculate probabilities relating to independent events. We will now look at how to solve problems involving two or more events that are **not** independent.

The meaning of independency

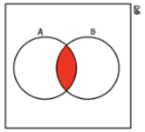
If we have two events that are independent, this means that the outcome of one event does not affect whether the other event occurs. For example, rolling a 3 on the first roll of a six-sided die and then rolling a 2 on the second roll are independent, since the outcome of the first roll does not have an impact on the outcome of the second.

- If two events A and B are independent, we have that $P(A) \times P(B) = P(A \cap B)$

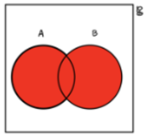
Set notation

You need to be comfortable with using set notation to describe events within a sample space. With the help of Venn diagrams, we will go through examples highlighting what each piece of notation means:

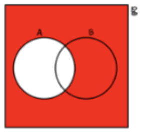
- The event that A and B both occur can be written as $A \cap B$ (A intersection B)



- The event that A or B occur can be written as $A \cup B$. (A union B)



- The event that A does not occur can be written as A'. (the complement of A)



- $n(A)$ is sometimes used to indicate the number of outcomes in each event. For example, if A is the event that a card selected at random from a pack of 52 playing cards is an ace, then $n(A) = 4$ since there are 4 aces in a pack.

- The empty set is denoted as $\emptyset$. If two events A and B are mutually exclusive, then we can write $A \cap B = \emptyset$

Probability formulae

When two events are not independent, this means that the outcome of one event is affected by the outcome of the other. For example, the probability that you receive an A on a statistics test may change depending on whether you completed all of your homework or not. For such scenarios, we use conditional probability.

- The probability that event A occurs given that event B has already occurred is denoted as $P(A|B)$.

- If A and B are independent, then $P(A|B) = P(A)$ since the occurrence of B does not affect A.

To solve conditional probability problems, you can use the formula:

- $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

The following formula relating the intersection to the union is also helpful for some problems:

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

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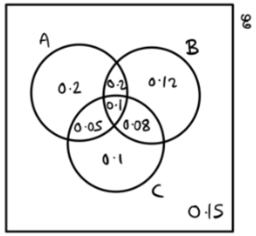
Example 1: The Venn diagram shows the probabilities of three events, A, B and C.
Find: a) $P(A|B)$ b) $P(C|A')$ c) $P(C \cup A')$



a) Using the first formula above:

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0.1+0.2}{0.2+0.1+0.08+0.12} = 0.6$$

everything inside the C circle but outside of A

$$\text{b) } P(C|A') = \frac{P(C\cap A')}{P(A')} = \frac{0.08+0.1}{0.1+0.08+0.12+0.15} = 0.4$$

everything outside of A

c) Using the union and intersection formula:

$$P(C \cup A') = P(C) + P(A') - P(C \cap A')$$

$$= \underbrace{(0.1 + 0.05 + 0.08 + 0.1)}_{P(C)} + \underbrace{(0.12 + 0.08 + 0.1 + 0.15)}_{P(A')} - \underbrace{(0.1 + 0.08)}_{P(C\cap A')}$$

$= 0.41$

Tree diagrams

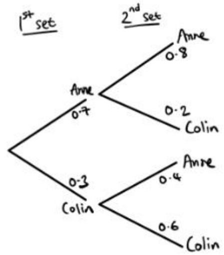
You need to be able to draw and use tree diagrams to calculate conditional probabilities.

The second set of branches in a tree diagram represent the conditional probabilities.

Example 2: In a tennis match, the probability that Anne wins the first set against Colin is 0.7. If Anne wins the first set, the probability she wins the second set is 0.8. If Anne loses the first set, the probability she wins the second set is 0.4. A match is won when one player wins two sets.

A) draw the tree diagram to illustrate this situation
b) find the probability that the game is over after two sets.
c) find the probability that Anne wins given that the game is over after two sets.

a) We draw the tree diagram and fill in the probabilities:



b) If the game is over after two sets then either Anne wins twice, or Colin wins twice. We must find each of these probabilities and sum them up.

$\Rightarrow P(Anne\ wins\ twice) = 0.7 \times 0.8 = 0.56$
$\Rightarrow P(Colin\ wins\ twice) = 0.3 \times 0.6 = 0.18$
$\therefore P(Required) = 0.56 + 0.18 = 0.74$

c) Using the conditional probability formula:

$P(Anne\ wins\ |\ game\ is\ over\ after\ 2\ sets)$

$= P(Anne\ wins\ twice)$

$$= \frac{P(Anne\ wins\ AND\ game\ is\ over\ in\ two\ sets)}{P(game\ is\ over\ in\ two\ sets)}$$

= answer to part b

using part b: $P(required) = \frac{0.56}{0.74} = \frac{28}{37}$