1. There are four human blood groups; these are called O, A, B and AB. Each person has one of these blood groups. The table below shows the distribution of blood groups in a large country.

Blood group	Proportion of population
O	49%
A	38%
B	10%
AB	3%

Two people are selected at random from this country.

(a) Find the probability that at least one of these two people has blood group O. [2]

(b) Find the probability that each of these two people has a different blood group. [3]

2. Sakura and Emily are playing a table tennis match. The winner of the match is the first player to win three games. The probability that Sakura wins a game is 0.55, independently of all other games. Games cannot be drawn.

(i) Find the probability that Sakura wins the match in three games. [2]

(ii) Find the probability that Emily wins the match. [5]

3. Tom is carrying out a survey into the way in which students travel to school. He selects 50 students and asks each of them 'How did you get to school this morning?' The results are given in the table below.

Walk	Cycle	Bus	Car
17	9	13	11

Tom then randomly selects 4 of these students to interview in more detail.

(i) Find the number of ways in which Tom can select the 4 students. [2]

(ii) Find the probability that all 4 of these students walked to school. [2]

(iii) Find the probability that at least 2 of the 4 students used the same method to get to school. [4]

4. A normal pack of 52 playing cards contains 4 aces. A card is drawn at random from the pack. It is then replaced and the pack is shuffled, after which another card is drawn at random.

i. Find the probability that neither card is an ace.

[2]

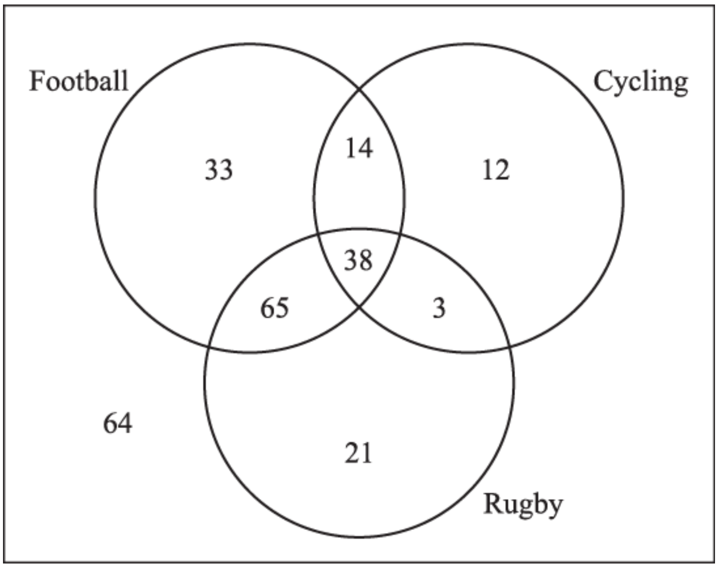
ii. This process is repeated 10 times. Find the expected number of times for which neither card is an ace.

[1]

5. A survey is being carried out into the sports viewing habits of people in a particular area. As part of the survey, 250 people are asked which of the following sports they have watched on television in the past month.

- Football
- Cycling
- Rugby

The numbers of people who have watched these sports are shown in the Venn diagram.



One of the people is selected at random.

i. Find the probability that this person has in the past month

A. watched cycling but not football,

[1]

B. watched either one or two of the three sports.

[2]

ii. Given that this person has watched cycling, find the probability that this person has not watched football.

[2]

6. i. There are 5 runners in a race. How many different finishing orders are possible? [You should assume that there are no 'dead heats', where two runners are given the same position.]

[1]

For the remainder of this question you should assume that all finishing orders are equally likely.

ii. The runners are denoted by A, B, C, D, E. Find the probability that they either finish in the order ABCDE or in the order EDCBA.

[2]

iii. Find the probability that the first 3 runners to finish are A, B and C, in that order.

[1]

iv. Find the probability that the first 3 runners to finish are A, B and C, in any order.

[2]

7. In a hockey league, each team plays every other team 3 times. The probabilities that Team A wins, draws and loses to Team B are given below.

- P(Wins) = 0.5
- P(Draws) = 0.3
- P(Loses) = 0.2

The outcomes of the 3 matches are independent.

i. Find the probability that Team A does not lose in any of the 3 matches.

[1]

ii. Find the probability that Team A either wins all 3 matches or draws all 3 matches or loses all 3 matches.

[2]

iii. Find the probability that, in the 3 matches, exactly two of the outcomes, 'Wins', 'Draws' and 'Loses'occur for Team A.

[4]

8. In a game show, each contestant is asked a number of questions. Each question has a choice of three possible answers.

The probability that John knows the answer to any given question is $\frac{2}{5}$.

If John doesn't know the answer he guesses. The probability that he guesses correctly is $\frac{1}{3}$.

Calculate the probability that John answers two consecutive random questions correctly. **[4]**

9. The probability that Chipping FC win a league football match is $P(W) = 0.4$.

(a) Calculate the probability that Chipping FC fail to win each of their next two league football matches. [1]

The probability that Chipping FC lose a league football match is $P(L) = 0.3$.

(b) Explain why $P(W) + P(L) \neq 1$. [1]

10. **You must show detailed reasoning in this question.**

In the summer of 2017 in England a large number of candidates sat GCSE examinations in **both** mathematics **and** English. 56% of these candidates achieved at least level 4 in mathematics and 80% of these candidates achieved at least level 4 in English. 14% of these candidates did not achieve at least level 4 in either mathematics or English.

Determine whether achieving level 4 or above in English and achieving level 4 or above in mathematics were independent events. [5]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		a	$1 - 0.51^2$ $= 0.7399$	M1(AO3.1b) A1(AO1.1) [2]	Accept 0.74 or 0.740	
		b	$1 - 0.49^2 - 0.38^2 - 0.1^2 - 0.03^2$ $= 0.6046$	M1(AO3.1b) M1(AO1.1) A1(AO1.1)	For squaring probabilities OR products of pairs For complementary event OR doubling products of pairs $2 \times (0.1862 + 0.049 + 0.0147 + 0.038 + 0.0114 + 0.003)$	

				[3]		
			Total	5		
2		i	$0.55^3 = 0.166 \quad (0.166375) \left(= \dfrac{1331}{8000}\right)$	M1 A1 [2]	For 0.55^3 Accept 0.17 with working Condone answer of 0.166375 (over-specified) **Examiner's Comments** Around 95% of candidates answered this correctly. Most gave a decimal answer of 0.166 or 0.1664.	
		ii	P(Wins in 3 games) = 0.45^3 = 0.091125 P(Wins in 4 games) = 3 × 0.45^2 × 0.55 × 0.45 = 0.150356 P(Wins in 5 games) = 6 × 0.45^2 × 0.55^2 × 0.45 = 0.165392 NB Answer if no coefficients used is 0.168809… = 0.091125 + 3 x 0.05011875 + 6 x 0.0275653125 P(Emily wins) = 0.407 (0.406873..) 1 – P(Sakura wins) can get all marks (use similar scheme) so eg 1 - 0.55^3 gets M1M0M0M0A0	M1 M1 M1 M1 A1 [5]	For P(Wins in 3 games) P(Wins in 4 games) with any or no coefficient P(Wins in 5 games) with any or no coefficient For either coefficient correct CAO SC2 for P(Sakura wins) = 0.593 (0.593126…) **Examiner's Comments** This caused numerous problems for many candidates – time was spent drawing tree diagrams and trying to list all combinations. Most candidates found the probability of Emily winning in 3 games, 0.45^3, and thus gained a mark. A large number also found the probability of her winning in 4 games (in some order) and in 5 games (in some order). The number of valid combinations was frequently wrong – sometimes failing to find them all systematically and often using ${}^4C_3$ and ${}^5C_3$. Some candidates used 1-P(Sakura wins) and these candidates achieved mostly between 1 and 3 marks with mistakes with the coefficients resulting in not gaining full credit. An elegant solution seen was from a candidate who stated that Emily must win the last game and so worked on the possible ways of Emily winning two of the previous matches.	

			Total	7		
3		i	$\binom{50}{4} = \frac{50!}{4!46!} = 230300$	M1 A1 [2]	**Examiner's Comments** This part was answered well by the majority of the candidates. A small minority of the candidates used permutations instead of combinations and a very few simply gave an answer of 4! = 24.	
		ii	$\frac{17}{50}\times\frac{16}{49}\times\frac{15}{48}\times\frac{14}{47} = \frac{17}{1645} = 0.0103$ Or: $\binom{17}{4} \div \binom{50}{4} = \frac{2380}{230300} = 0.0103$	M1 A1 [2] M1 A1	$17/50 \times$ or $0.34\times$ <u>NB</u> $\left(\frac{17}{50}\right)^4$ or 0.34^4 scores M1A0 But M0 if part of a binomial expression CAO Uncancelled fraction gets M1A0 Uncancelled fraction gets M1A0 Allow 0.010 with working but not 0.01 **Examiner's Comments** This was answered well by many candidates. However a common misconception was to assume 'with replacement' probability giving an answer of $0.34^4 = 0.0134$. A few candidates gave their answer in fractional form but failed to cancel, hence losing the accuracy mark.	
		iii	$1-4! \times \frac{17}{50}\times\frac{9}{49}\times\frac{13}{48}\times\frac{11}{47}$ = 1 – 24 x 0.0003958.. = 1 – 0.09500217 = 0.905 (0.904997…)	M1 M1 M1 A1 [4]	For correct product For x4! For 1– with product of four fractions but with or without a coefficient CAO If denominators all 50 then max M0M1M1A0 Allow 0.90 with working For product of four correct nCr terms	

			Or: $$1-\left[\binom{17}{1}\times\binom{9}{1}\times\binom{13}{1}\times\binom{11}{1}\div\binom{50}{4}\right]=1-0.09500=0.905$$	M1 M1 M1 A1	For division of product of four nCr terms by 50C4 For 1 - product of four nCr terms divided by 50C4 **Examiner's Comments** This question was found to be rather difficult. Many candidates wrote more than one page as they attempted to find all the probabilities of two or more using the same method, almost always without success. In fact around half of the candidature scored zero on this question part. A fair number of candidates did find the correct product and took their answer away from 1, but few found the correct multiplier of 4!, and thus only gained 2 marks.	
			Total	8		
4		i	$$\text{P(Neither is an ace)} = \left(1-\frac{4}{52}\right)^2$$	M1	For 48/52 oe seen	
		i	$$=\frac{2304}{2704}=\frac{144}{169}=0.852\ \ (0.8572071\ldots)$$	A1	CAO **Examiner's Comments** This was answered very well. However a few ignored the question and assumed 54 or 50 cards in a pack, or that the card had not been replaced. Another common wrong method was to find 1 - P(both aces) = 1 – $(4/52)^2$ = 168/169.	Allow 0.85 with working
		ii	Expected number = 10 × 0.852 = 8.52	B1	FT their (i) if seen **Examiner's Comments** Surprisingly, only two thirds of candidates scored this easy mark. Most realised that they had to multiply their answer to part (i) by 10 but some then rounded their answer to a whole number, thus losing the mark. A smaller number incorrectly raised their answer to part (i) to the power of 10.	Do not allow whole number final answer even if 8.52 seen first. Allow fractional answer
			Total	3		

5		i	(*A*) P(Watched cyc but not fb) = $\frac{15}{250}=\frac{3}{50}$ = 0.06	B1	CAO (aef) **Examiner's Comments** This was answered very well, although a number of candidates gave the number that watched cycling and not football rather than the probability. A few had the wrong divisor, usually 186 or 100.	
		i	(*B*) P(Watched one or two) = $\frac{33+12+21+14+3+65}{250}$	M1	OR: $\frac{250-(64+38)}{250}$ =	For M1 terms must be added with no extra terms (added or subtracted)
		i	$=\frac{148}{250}=\frac{74}{125}$ = 0.592	A1	CAO (aef) **Examiner's Comments** Again this was very well-answered with only a small minority of candidates making errors. The most common errors were to include those people who watched all three sports or to miss out one of the six who watched 1 or 2 sports..	
		ii	P(Not watched fb|watched cyc) = $\frac{15}{67}$ = 1567= 0.224 (0.223880597…)	M1		For denominator of either 67 or 67/250 or 0.268
		ii		A1	CAO (aef) **Examiner's Comments** Approximately two thirds of candidates answered this correctly. Of the rest, some were able to get a method mark for the correct denominator but then failed to get the correct numerator, often thinking that it was 12 rather than 15. Some candidates	Allow 0.22 with working

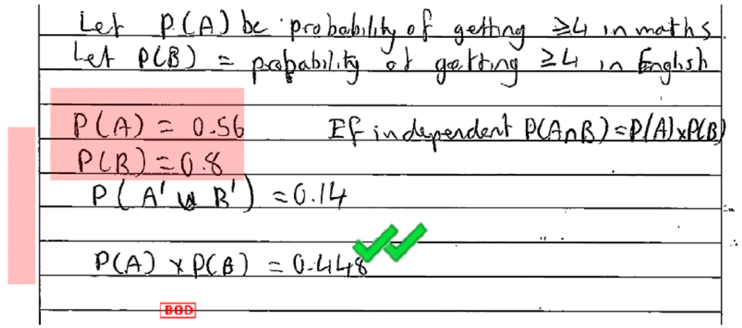
					either did not recognize this as a conditional probability question, or did not know about conditional probability.	
			Total	5		
6		i	Number of ways = 5! = 120	B1	**Examiner's Comments** This was generally well answered.	
		ii	Probability = 2/120	M1	For division by their 120	M1 for $^k/_{120}$
		ii	= 1/60 or 0.0167 or $0.01\dot{6}$	A1	CAO **Examiner's Comments** Again this was generally well answered, although some candidates truncated their decimal rather than correctly rounding. The use of fractions was preferable here.	Condone final answer of 2/120 Do not allow 0.016
		iii	$\frac{1}{5}\times\frac{1}{4}\times\frac{1}{3}=\frac{1}{60}$ or 0.0167 or $0.01\dot{6}$	B1	Condone 2/120 for B1 **Examiner's Comments** Only around two thirds of candidates scored the mark here. The most common error was to use combinations.	Do not allow 0.016
		iv	$\frac{3}{5}\times\frac{2}{4}\times\frac{1}{3}=\frac{1}{10}$	M1	For 3/5 ×	Listing options gives 12/120
		iv		A1	CAO	Or (3! / 120) × 2 Or ($^3P_3$ × $^2P_2$)/120)
		iv	or $1/^5C_3$ = 1/10	M1	For division by $^5C_3$ **Examiner's Comments**	SC2 for 3! × their part (iii)

		iv		A1	Again about two thirds of candidates scored both marks, with many scoring the marks for their answer to part (iii), correct or not, multiplied by 6, rather than for the fairly simple $^3/_5 \times {}^2/_4 \times {}^1/_3$.	or 6 × their part (iii)
			Total	**6**		
7		i	P(Does not lose any match) = 0.83 = 0.512 = $^{64}/_{125}$	B1	**Examiner's Comments** Many candidates thought that not losing meant winning, and hence gave the common wrong answer of $0.5^3 = 0.125$. Others tried to consider combinations of wins and draws, often without success. The fact that the question part was only worth 1 mark should have been a clue to the fact that there was an easier approach.	
		ii	P(Wins all 3 or draws all 3 or loses all 3) = $0.5^3 + 0.3^3 + 0.2^3$	M1	Including addition **Examiner's Comments** This was generally well answered although a few candidates interpreted this as 'find three separate probabilities which they did, and listed them but with no addition thus scoring zero.	
		ii	= 0.16 = $^4/_{25}$	A1		
		iii	P(all three outcomes occur) = 3! × 0.5 × 0.3 × 0.2	M1*		Allow M1 for k × 0.5 × 0.3 × 0.2 even if k = 1 Even if cubed
		iii	= 0.18	A1		
		iii	Required probability = 1 – '0.18' – 0.16	*M1 dep		Not if cubed
		iii	= 0.66 = $^{33}/_{50}$	A1	CAO	

		iii	OR:			
		iii	P(WWW') + P(DDD') + P(LLL')	M1	For any one product (no need for '3 ×')	Even if cubed
		iii	$3 \times 0.5^2 \times 0.5 + 3 \times 0.3^2 \times 0.7 + 3 \times 0.2^2 \times 0.8$	M1	For '3 ×'	Dep on at least 1 correct term
		iii	0.375 + 0.189 + 0.096	M1	For sum of three correct terms (no need for '3 ×') And no incorrect terms	NB common wrong answer of 0.22 from omitting '3 ×' or 0.44 from '2×' scores M1M0M1A0 Not if cubed
		iii	0.66	A1	CAO	
		iii	OR:			
		iii	P(WWD) + P(WWL) + P(DDW) + P(DDL) + P(LLW) + P(LLD)	M1	For any one product (no need for '3 ×')	Even if cubed
		iii	$3\times0.5^2 \times 0.3 + 3 \times 0.5^2 \times 0.2 + 3 \times 0.3^2 \times 0.5 + 3 \times 0.3^2 \times 0.2 + 3 \times 0.2^2 \times 0.5 + 3 \times 0.2^2 \times 0.3$	M1	For '3 ×'	Dep on at least 1 correct term
		iii	0.225 + 0.15 + 0.135 + 0.054 + 0.06 + 0.036	M1	For sum of six correct terms (no need for '3×') And no incorrect terms	Not if cubed
		iii	0.66	A1	CAO **Examiner's Comments**	

					Only a small proportion of candidates used the most elegant approach (the first method in the mark scheme) and of those who did, many forgot to multiply by 6. Most candidates gave lists or tree diagrams to show P(WWL) etc., but many then did not multiply by 3, so the most common answer was 0.22, rather than the correct 0.66.	
			Total	7		
8			P(correct) $= \frac{2}{5} + \frac{3}{5} \times \frac{1}{3}$ $= 0.6 \ (\frac{3}{5})$ *their* 0.6^2 $0.36 \ (\frac{9}{25})$	M1(AO3.1b) A1(AO1.1b) M1(AO1.1b) A1(AO1.1b) [4]		
			Total	4		
9		a	0.36	B1 (AO 1.1) [1]	<u>Examiner's Comments</u> A small minority of candidates found the probability to win the next two league football matches.	
		b	P(draw) ≠ 0 oe	B1 (AO 2.4)	allow any comment which identifies that eg winning and losing are not exhaustive	

				[1]	<table><tr><td>other outcomes are possible</td><td></td></tr></table> **Examiner's Comments** Generally answered well, although some mini essays seen.	
			Total	2		
10			use of contingency table or Venn diagram or P(A or B) = P(A) + P(B) − P(A and B) P(A and B) = 0.5 P(A) × P(B) = 0.56 × 0.80 = 0.448 seen 0.448 ≠ 0.5 or 0.56 × 0.80 ≠ 0.5 so not independent	M1 (AO 3.1b) A1 (AO 2.1) M1 (AO 1.1) A1 (AO 1.1) A1 (AO 3.2a) [5]	<table><tr><td>0.56, 0.8 and 0.14 must be correctly placed; eg 1 − 0.14 = 0.56 + 0.8 − P(A and B) or $P(A/B) = \frac{0.5}{0.80}$</td><td>where A denotes "passing" maths and B denotes "passing" English</td></tr><tr><td></td><td>the first M1A1 may be awarded for working with percentages</td></tr><tr><td>= 0.625 or ⅝ seen 0.625 or ⅝ $\neq 0.56$</td><td>allow equivalent argument based on showing A' and B' not independent</td></tr></table> **Examiner's Comments** Candidates who did well in this question defined appropriate events and set out a clear, reasoned argument. They usually worked with a Venn diagram and clearly showed that p(A and B) ≠ p(A) × p(B) in this case. Candidates who did less well wrote down relevant ideas but were unable to draw	

					them together successfully. They sometimes stated results instead of giving details of the calculation. Exemplar 1  Let P(A) be probability of getting ≥4 in maths Let P(B) = probability of getting ≥4 in English P(A) = 0.56 If independent P(A∩B) = P(A)×P(B) P(B) = 0.8 P(A′ ∪ B′) = 0.14 P(A) × P(B) = 0.448 BOD This candidate has been given BOD M1A1 for the calculation of p(*A*)×p(*B*) being correctly done, as highlighted by the statements on the LHS. The events are clearly defined. No attempt was made to find the actual value of p(*A* and *B*), so no further progress was made.	
			Total	5		