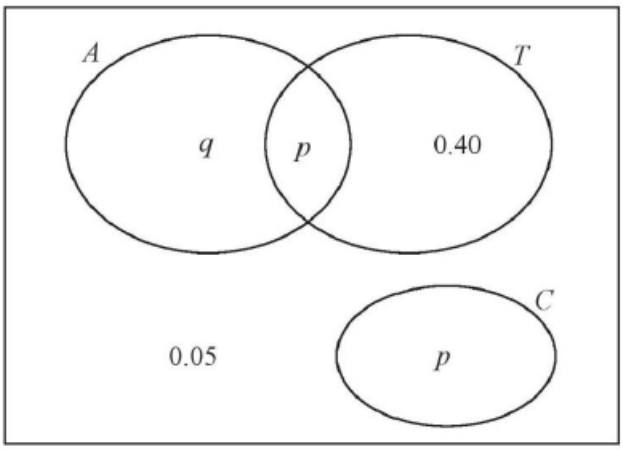
Questions

Q1.

The Venn diagram shows the probabilities for students at a college taking part in various sports.

A represents the event that a student takes part in Athletics.
T represents the event that a student takes part in Tennis.
C represents the event that a student takes part in Cricket.
p and *q* are probabilities.



The probability that a student selected at random takes part in Athletics or Tennis is 0.75

(a) Find the value of *p*.

(1)

(b) State, giving a reason, whether or not the events *A* and *T* are statistically independent. Show your working clearly.

(3)

(c) Find the probability that a student selected at random does not take part in Athletics or Cricket.

(1)

(Total for question = 5 marks)

Q2.

A factory buys 10% of its components from supplier *A*, 30% from supplier *B* and the rest from supplier *C*. It is known that 6% of the components it buys are faulty.

Of the components bought from supplier *A*, 9% are faulty and of the components bought from supplier *B*, 3% are faulty.

(a) Find the percentage of components bought from supplier *C* that are faulty.

(3)

A component is selected at random.

(b) Explain why the event "the component was bought from supplier *B* " is not statistically independent from the event "the component is faulty".

(1)

(Total for question = 4 marks)

Q3.

A biased spinner can only land on one of the numbers 1, 2, 3 or 4. The random variable X represents the number that the spinner lands on after a single spin and $P(X = r) = P(X = r + 2)$ for $r = 1, 2$

Given that $P(X = 2) = 0.35$

(a) find the complete probability distribution of X.

(2)

Ambroh spins the spinner 60 times.

(b) Find the probability that more than half of the spins land on the number 4

Give your answer to 3 significant figures.

(3)

The random variable $Y = \frac{12}{X}$

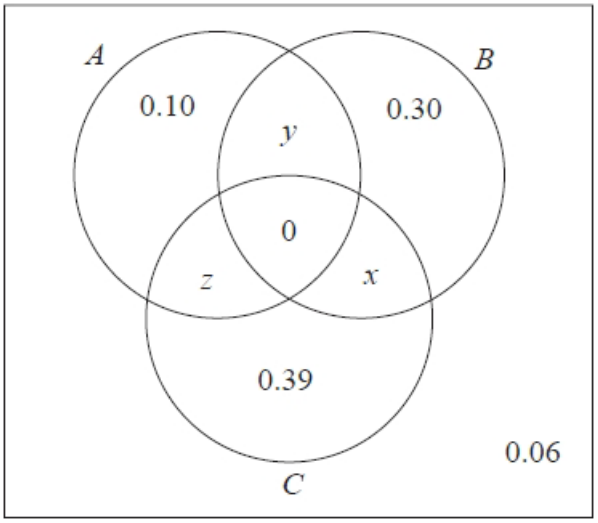
(c) Find $P(Y - X \leq 4)$

(3)

(Total for question = 8 marks)

Q4.

The Venn diagram shows three events, A, B and C, and their associated probabilities.



Events B and C are mutually exclusive.
Events A and C are independent.

Showing your working, find the value of x, the value of y and the value of z.

(Total for question = 5 marks)

Q5.

A fair 5-sided spinner has sides numbered 1, 2, 3, 4 and 5

The spinner is spun once and the score of the side it lands on is recorded.

(a) Write down the name of the distribution that can be used to model the score of the side it lands on.

(1)

The spinner is spun 28 times.

The random variable X represents the number of times the spinner lands on 2

(b) (i) Find the probability that the spinner lands on 2 at least 7 times.

(ii) Find $P(4 \leq X < 8)$

(5)

(Total for question = 6 marks)

Q6.

In a game, a player can score 0, 1, 2, 3 or 4 points each time the game is played.

The random variable S, representing the player's score, has the following probability distribution where a, b and c are constants.

s	0	1	2	3	4
$P(S = s)$	a	b	c	0.1	0.15

The probability of scoring less than 2 points is twice the probability of scoring at least 2 points.

Each game played is independent of previous games played.

John plays the game twice and adds the two scores together to get a total.

Calculate the probability that the total is 6 points.

(Total for question = 6 marks)

Q7.

Afrika works in a call centre.

She assumes that calls are independent and knows, from past experience, that on each sales call

that she makes there is a probability of $\frac{1}{6}$ that it is successful.

Afrika makes 9 sales calls.

(a) Calculate the probability that at least 3 of these sales calls will be successful.

(2)

The probability of Afrika making a successful sales call is the same each day.

Afrika makes 9 sales calls on each of 5 different days.

(b) Calculate the probability that at least 3 of the sales calls will be successful on exactly 1 of these days.

(2)

Rowan works in the same call centre as Afrika and believes he is a more successful salesperson.

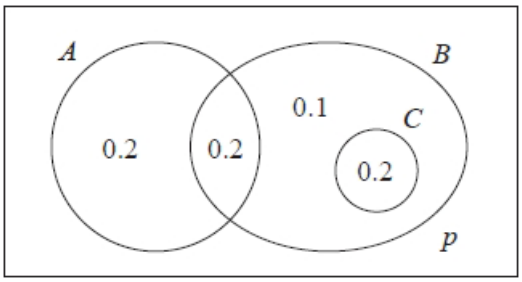
To check Rowan's belief, Afrika monitors the next 35 sales calls Rowan makes and finds that 11 of the sales calls are successful.

(c) Stating your hypotheses clearly test, at the 5% level of significance, whether or not there is evidence to support Rowan's belief.

(4)

(Total for question = 8 marks)

Q8.



The Venn diagram, where p is a probability, shows the 3 events A, B and C with their associated probabilities.

(a) Find the value of p.

(1)

(b) Write down a pair of mutually exclusive events from A, B and C.

(1)

(Total for question = 2 marks)

Q9.

Two bags, **A** and **B**, each contain balls which are either red or yellow or green.

Bag **A** contains 4 red, 3 yellow and n green balls.
Bag **B** contains 5 red, 3 yellow and 1 green ball.

A ball is selected at random from bag **A** and placed into bag **B**.
A ball is then selected at random from bag **B** and placed into bag **A**.

The probability that bag **A** now contains an equal number of red, yellow and green balls is p.

Given that $p > 0$, find the possible values of n and p.

(Total for question = 5 marks)

Q10.

Helen believes that the random variable C, representing cloud cover from the large data set, can be modelled by a discrete uniform distribution.

(a) Write down the probability distribution for C.

(2)

(b) Using this model, find the probability that cloud cover is less than 50%

(1)

Helen used all the data from the large data set for Hurn in 2015 and found that the proportion of days with cloud cover of less than 50% was 0.315

(c) Comment on the suitability of Helen's model in the light of this information.

(1)

(d) Suggest an appropriate refinement to Helen's model.

(1)

(Total for question = 5 marks)

Q11.

Magali is studying the mean total cloud cover, in oktas, for Leuchars in 1987 using data from the large data set. The daily mean total cloud cover for all 184 days from the large data set is summarised in the table below.

Daily mean total cloud cover (oktas)	0	1	2	3	4	5	6	7	8
Frequency (number of days)	0	1	4	7	10	30	52	52	28

One of the 184 days is selected at random.

(a) Find the probability that it has a daily mean total cloud cover of 6 or greater.

(1)

Magali is investigating whether the daily mean total cloud cover can be modelled using a binomial distribution.

She uses the random variable X to denote the daily mean total cloud cover and believes that $X \sim B(8, 0.76)$

Using Magali's model,

(b) (i) find $P(X \geq 6)$

(2)

(ii) find, to 1 decimal place, the expected number of days in a sample of 184 days with a daily mean total cloud cover of 7

(2)

(c) Explain whether or not your answers to part (b) support the use of Magali's model.

(1)

There were 28 days that had a daily mean total cloud cover of 8
For these 28 days the daily mean total cloud cover for the **following** day is shown in the table below.

Daily mean total cloud cover (oktas)	0	1	2	3	4	5	6	7	8
Frequency (number of days)	0	0	1	1	2	1	5	9	9

(d) Find the proportion of these days when the daily mean total cloud cover was 6 or greater.

(1)

(e) Comment on Magali's model in light of your answer to part (d).

(2)

(Total for question = 9 marks)

Q12.

The discrete random variable D has the following probability distribution

d	10	20	30	40	50
$P(D = d)$	$\frac{k}{10}$	$\frac{k}{20}$	$\frac{k}{30}$	$\frac{k}{40}$	$\frac{k}{50}$

where k is a constant.

(a) Show that the value of k is $\frac{600}{137}$

(2)

The random variables D_1 and D_2 are independent and each have the same distribution as D.

(b) Find P ($D_1 + D_2 = 80$)

Give your answer to 3 significant figures.

(3)

A single observation of D is made.

The value obtained, d, is the common difference of an arithmetic sequence.

The first 4 terms of this arithmetic sequence are the angles, measured in degrees, of quadrilateral Q

(c) Find the exact probability that the smallest angle of Q is more than 50°

(5)

(Total for question = 10 marks)

Q13.

The discrete random variable X has the following probability distribution

x	a	b	c
$P(X = x)$	$\log_{36} a$	$\log_{36} b$	$\log_{36} c$

where

- a, b and c are distinct integers ($a < b < c$)
- all the probabilities are greater than zero

(a) Find

(i) the value of a
(ii) the value of b
(iii) the value of c
Show your working clearly.

(5)

The independent random variables X_1 and X_2 each have the same distribution as X

(b) Find $P(X_1 = X_2)$

(2)

(Total for question = 7 marks)

Q14.

(a) State one disadvantage of using quota sampling compared with simple random sampling.

(1)

In a university 8% of students are members of the university dance club.

A random sample of 36 students is taken from the university.

The random variable X represents the number of these students who are members of the dance club.

(b) Using a suitable model for X, find

(i) $P(X = 4)$
(ii) $P(X \geq 7)$

(3)

Only 40% of the university dance club members can dance the tango.

(c) Find the probability that a student is a member of the university dance club and can dance the tango.

(1)

A random sample of 50 students is taken from the university.

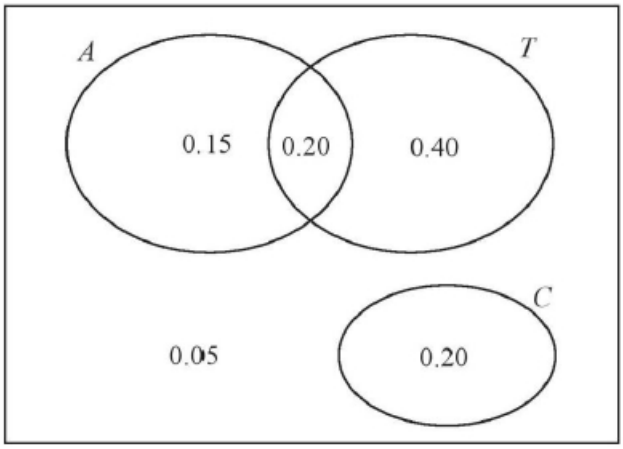
(d) Find the probability that fewer than 3 of these students are members of the university dance club and can dance the tango.

(2)

(Total for question = 7 marks)

Mark Scheme

Q1.

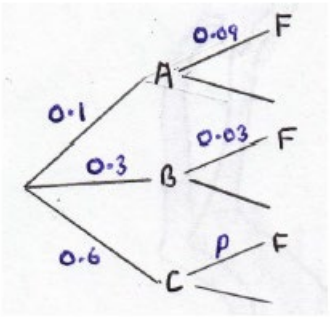
Question	Scheme	Marks	AOs
(a)	$p = [1 - 0.75 - 0.05 =]$ **0.20**	B1	1.1b
		(1)	
(b)	$q =$ **0.15**	B1ft	1.1b
	P(A) = 0.35 P(T) = 0.6 P(A and T) = 0.20 P(A)×P(T) = 0.21	M1	2.1
	Since 0.20 ≠ 0.21 therefore A and T are **not** independent	A1	2.4
		(3)	
			
(c)	P(not [A or C]) = **0.45**	B1	1.1b
		(1)	
			(5 marks)

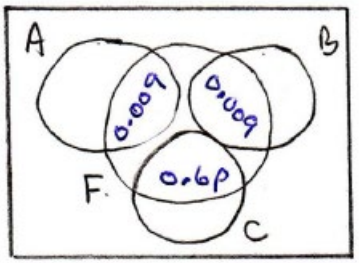
Part	Notes
(a)	B1cao for $p = 0.20$
(b)	B1ft for use of their p and P(A or T) to find q i.e. 0.75 – "p" – 0.40 or q = 0.15
	M1 for the statement of all probabilities required for a suitable test and sight of any appropriate calculations required.
	A1 All probabilities correct, correct comparison and suitable comment.
(c)	B1cao for 0.45

Q2.

Qu	Scheme	Marks	AO
(a)	[Let $p = P(F \mid C)$]		
	Tree diagram or some other method to find an equation for p	M1	2.1
	$0.1\times0.09+0.3\times0.03+0.6\times p=0.06$	A1	1.1b
	$p = 0.07$ i.e. 7%	A1	1.1b
		(3)	
(b)	e.g. $P(B \text{ and } F) = 0.3\times0.03 = 0.009$ but		
	$P(B)\times P(F) = 0.3\times0.06 = 0.018$	B1	2.4
	These are not equal so not independent		
		(1)	
		(4 marks)	

	Notes
(a)	M1 for selecting a suitable method to find the missing probability e.g. sight of tree diagram with 0.1, 0.3, 0.6 and 0.09, 0.03, p suitably placed e.g. sight of VD with 0.009 for $A\cap F$ and $B\cap F$ and $0.6p$ suitably placed or attempt an equation with at least one correct numerical and one "p" product (not necessarily correct) on LHS or for sight of $0.06 - (0.009 + 0.009)$ (o.e. e.g. $6 - 1.8 = 4.2\%$) 1st A1 for a correct equation for p (May be implied by a correct answer) or for the expression $\frac{0.06-(0.009+0.009)}{0.6}$ (o.e.) 2nd A1 for 7% (accept 0.07) **Correct Ans:** Provided there is no incorrect working seen award 3/3 e.g. may just see tree diagram with 0.07 for p (probably from trial and improv')
(b)	B1 for a suitable explanation…may talk about 2nd branches on tree diagram and point out that $0.03 \neq 0.06$ but need some supporting calculation/words Can condone incorrect use of set notation (it is not on AS spec) provided the rest of the calculations and words are correct.





Q3.

Qu	Scheme	Marks	AO
(a)	$P(X=4)=P(X=2)$ so $P(X=4)=0.35$ $P(X=1)=P(X=3)$ and $P(X=1)+P(X=3)=1-0.7$ So x | 1 | 2 | 3 | 4 $P(X=x)$ | 0.15 | 0.35 | 0.15 | [0.35]	M1 A1 (2)	2.1 1.1b
(b)	Let A = number of spins that land on 4 $A \sim B(60, "0.35")$	B1ft	3.3
	$[P(A>30)=]\ 1-P(A\leqslant 30)$ $=1-0.99411\ldots$ = **awrt 0.00589**	M1 A1 (3)	3.4 1.1b
(c)	$Y-X\leqslant 4 \Rightarrow \frac{12}{X}-X\leqslant 4$ or $12-X^2\leqslant 4X$ (since $X>0$) o.e.	M1	3.1a
	i.e. $0\leqslant X^2+4X-12 \Rightarrow 0\leqslant (X+6)(X-2)$ so $X\geqslant 2$	M1	1.1b
	$P(Y-X\leqslant 4)=P(X\geqslant 2)=0.35+0.15+0.35$ = **0.85**	A1 (3)	3.2a
		(8 marks)	

Notes

(a)
M1 for using the given information to obtain $P(X=4)$
Award for statement $P(X=4)=P(X=2)$ or writing $P(X=4)=0.35$
A1 for getting fully correct distribution (any form that clearly identifies probs)
e.g. can be list $P(X=1)=0.15$, $P(X=3)=\ldots$etc
or as a probability function $P(X=x)=\begin{cases}0.15 & x=1,3\\0.35 & x=2,4\end{cases}$
[Condone missing $P(X=2)$ as this is given in QP]

(b)
B1 for selecting a suitable model, sight of B(60, their 0.35) o.e. in words
f.t. their $P(X=4)$ from part (a).
Can be implied by $P(A\leqslant 30)$ = awrt 0.9941 or final answer = awrt 0.00589
M1 for using their model and interpreting "more than half"
Need to see $1-P(A\leqslant 30)$. Can be implied by awrt 0.00589
Can ignore incorrect LHS such as $P(A\geqslant 30)$
A1 for awrt 0.00589

(c)
1st M1 for translating the prob. problem into a correct mathematical inequality
Just an inequality in 1 variable. May be inside a probability statement.
ALT **Table of values:**

X	1	2	3	4
Y	12	6	4	3

or values of $Y-X=11, 4, 1, -1$

2nd M1 for solving the inequality leading to a range of values, allow 1 or 2 slips
May be a quadratic or cubic but must lead to a set of values of X or $Y-X$
ALT **Table or values:** They must state clearly which values are required
Both Ms can be implied by a correct answer (or correct ft of their distb'n)
A1 for interpreting the inequality and solving the problem i.e. 0.85 cao

Q4.

Question	Scheme	Marks	AOs
	$x = 0$	B1	2.2a
	$\mathrm{P}(A) = 0.1 + z + y \quad \mathrm{P}(C) = 0.39 + z[+x] \quad \mathrm{P}(A \text{ and } C) = z$	M1	2.1
	$\mathrm{P}(A \text{ and } C) = \mathrm{P}(A) \times \mathrm{P}(C) \rightarrow z = (0.1 + z + y) \times (0.39 + z[+x])$	M1	1.1b
	$\left[\sum p = 1\right]$ $0.06 + 0.3 + 0.39 + 0.1 + z + y[+x] = 1 \rightarrow \quad [z + y[+x] = 0.15]$	M1	1.1b
	Solving (simultaneously) leading to $\underline{z = 0.13} \quad \underline{y = 0.02}$	A1	1.1b
			(5 marks)

Notes	
	B1: for $x = 0$, may be seen on Venn diagram
	M1: Identifying the probabilities required for independence and at least 2 correct These must be labelled If there are no labels, then this may be implied by $z = (0.1 + z + y)(0.39 + z\ [+x])$, allow one numerical slip Allow e.g. $\mathrm{P}(A') = 0.39 + 0.30 + 0.06[+x] \quad \mathrm{P}(C) = 0.39 + z[+x] \quad \mathrm{P}(A' \text{ and } C) = 0.39$ [Not on spec. but you may see use of conditional probabilities]
	M1: Use of independence equation with their labelled probabilities in terms y, z [and x] All their probabilities must be substituted into a correct formula Sight of a correct equation e.g. $z = (0.1 + z + y)(0.39 + z\ [+x])$ scores M1M1
	M1: Using $\Sigma p = 1$ Implied by $[x +] y + z = 0.15$ or their $x + y + z = 0.15$ where x, y, and z are all probabilities or e.g. $\mathrm{P}(A) = 0.25$
	A1: both $y = 0.02$ and $z = 0.13$

Q5.

Question	Scheme	Marks	AOs
(a)	(Discrete) uniform (distribution)	B1	1.2
		(1)	
(b)	B(28, 0.2)	B1	3.3
(i)	$P(X \geq 7) = 1 - P(X \leq 6)$ [= 1 – 0.6784…]	M1	3.4
	awrt **0.322**	A1	1.1b
(ii)	$P(4 \leq X < 8) = P(X \leq 7) - P(X \leq 3)$ [= 0.818… – 0.160…]	M1	3.1b
	awrt **0.658**	A1	1.1b
		(5)	
			(6 marks)

Notes	
(a)	Continuous uniform is B0
(b)	**B1:** for identifying correct model, B(28, 0.2) allow B, bin or binomial may be implied by one correct answer or sight one correct probability i.e. awrt 0.678, awrt 0.818 or awrt 0.160 B(0.2, 28) is B0 unless it is used correctly
(i)	**M1:** Writing or using $1 - P(X \leq 6)$ or $1 - P(X < 7)$ **A1:** awrt 0.322 (correct answer only scores M1A1)
(ii)	**M1:** Writing or using $P(X \leq 7) - P(X \leq 3)$ or $P(X < 8) - P(X < 4)$ or $P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7)$ Condone P(4) as $P(X = 4)$, etc. **A1:** awrt 0.658 (correct answer only scores M1A1)

Q6.

Question	Scheme	Marks	AOs
	Overall method	M1	2.1
	$a+b=2c+0.5$ oe or $a+b=2(1-a-b)$	B1	2.2a
	$a+b+c=0.75$ oe	B1	1.1b
	$3c=0.25$ $\left[c=0.0833...\text{ or }\frac{1}{12}\right]$	M1	1.1b
	P(scoring 2,4 or 4,2 or 3,3) = $2\times"\frac{1}{12}"\times0.15+0.1^2$	M1	3.1b
	= 0.035 oe	A1cso	1.1b
		(6)	
			(6 marks)

Notes		
	M1:	A fully correct method with all the required steps. For gaining 2 correct equations with at least one correct(allow if unsimplified). Attempting to solve to find a value of c followed by **correct method** to find the probability
	B1:	Forming a correct equation from the information given in the question
	B1:	A correct equation using the sum of the probabilities equals 1
	M1:	Correct method for solving 2 equations to find c Implied by $c=\frac{1}{12}$
	M1:	Recognising the ways to get a total of 6. Condone missing arrangments or repeats. Do not ignore extras written unless ignored in the calculation. May be implied by $m\times"\frac{1}{12}"\times0.15+n\times0.1^2$ where m and n are positive integers
	A1cso:	Cao 0.035, $\frac{7}{200}$ oe

Q7.

Question	Scheme	Marks	AOs
(a)	Let C = the number of successful calls. $C \sim B\left(9, \frac{1}{6}\right)$	M1	3.3
	$P(C \geq 3) = 1 - P(C \leq 2) = 0.1782\ldots$ **awrt 0.178**	A1	1.1b
		(2)	
(b)	Let X = the number of occasions when at least 3 calls are successful. $P(X = 1) = 5 \times ("0.1782\ldots") \times ("0.8217\ldots")^4$	M1	1.1b
	$= 0.4061\ldots$ **awrt 0.406**	A1	1.1b
		(2)	
(c)	$H_0 : p = \frac{1}{6}$ $H_1 : p > \frac{1}{6}$	B1	2.5
	Let R = the number of successful calls $R \sim B\left(35, \frac{1}{6}\right)$	M1	3.3
	$P(R \geq 11) = 1 - P(R \leq 10) = 0.02\ldots$	A1	3.4
	There is sufficient evidence to support that **Rowan** has more successful sales calls than Afrika.	A1	2.2b
		(4)	
			(8 marks)

Notes		
(a)	**M1:**	For selecting the right model
	A1:	awrt 0.178
(b)	**M1:**	For $5 \times ("\text{their}(a)") \times ("1 - \text{their}(a)")^4$
	A1:	awrt 0.406
(c)	**B1:**	for correctly stating both hypotheses in terms of p or π Accept $p = 0.1\dot{6}$
	M1:	For selecting a suitable model. May be implied by a correct probability or CR
	A1:	Correct probability statement and answer of 0.02 or better (0.02318…) (CR $R \geq 11$ and either $P(R \leq 9) = 0.9450$ or $P(R \leq 10) = 0.9768$ or $1 - P(R \leq 10) = 0.0232$)
	A1:	Dependent on M1A1 but can ignore hypotheses. For conclusion in context supporting **Rowan's** belief / **Rowan** is a better sales person
		Do not accept Rowan can reject H_0

Q8.

Qu	Scheme	Marks	AO
(a)	$[p = 1 - (0.2 + 0.2 + 0.1 + 0.2)] = \underline{0.3}$	B1 (1)	1.1b
(b)	A and C are mutually exclusive. [NOT P(A) and P(C)]	B1 (1)	1.2
		(2 marks)	

	Notes
(a)	B1 for
(b)	B1 for A and C [NB $A \cap C$ or $A \cap C = \varnothing$ is B0] If more than one case given they must <u>all</u> be correct e.g. $A \cap B$ and C

Q9.

Qu	Scheme	Marks	AO
	Must end up with 3 of each colour or 4 of each colour	M1	3.1b
	$\underline{n = 2}$ requires 1st red and 2nd green <u>or</u> red from **A** and green from **B**	M1	2.2a
	P(1st red and 2nd green) $= \frac{4}{9} \times \frac{1}{10} = \frac{4}{90}$ or $\frac{2}{45}$ $p = \underline{\underline{\frac{2}{45}}}$	A1	1.1b
	$\underline{n = 5}$ requires 1st green and 2nd yellow <u>or</u> green from **A** and yellow from **B**	M1	2.2a
	P(1st green and 2nd yellow) $= \frac{5}{12} \times \frac{3}{10} = \frac{15}{120}$ or $\frac{1}{8}$ $p = \underline{\underline{\frac{1}{8}}}$	A1	1.1b
		(5)	
		(5 marks)	

	Notes
	1st M1 for an overall strategy realising there are 2 options. Award when evidence of both cases (3 of each colour or 4 of each colour) seen. 2nd M1 for $n = 2$ <u>and</u> attempt at 1st red and 2nd green May be implied by e.g. $\frac{4}{9} \times \frac{1}{9}$ 1st A1 for $p = \underline{\underline{\frac{2}{45}}}$ or exact equivalent 3rd M1 for $n = 5$ <u>and</u> attempt at 1st green and 2nd yellow May be implied by e.g. $\frac{5}{12} \times \frac{3}{9}$ 2nd A1 for $p = \underline{\underline{\frac{1}{8}}}$ or exact equivalent
NB	If both correct values of p are found and then added (get $\frac{61}{360}$) , deduct final A1 only (i.e. 4/5)

Q10.

Qu	Scheme	Marks	AO
(a)	c: 0 1 2 3 4 5 6 7 8 $P(C = c)$: $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$	B1 B1ft (2)	1.2 1.2
(b)	$P(C < 4) = \frac{4}{9}$ (accept 0.444 or better)	B1 (1)	3.4
(c)	Probability lower than expected suggests model is <u>not</u> good	B1ft (1)	3.5a
(d)	e.g. Cloud cover will vary from month to month and place to place So e.g. use a non-uniform distribution	B1 (1)	3.5c
		(5 marks)	

Notes

(a) 1st B1 for a correct set of values for c. Allow $\{\frac{1}{8}, \frac{2}{8}, \ldots \frac{8}{8}\}$

2nd B1ft for correct probs from their values for c, consistent with discrete uniform distrib'n Maybe as a prob. function. Allow $P(X = x) = \frac{1}{9}$ for $0 \leqslant x \leqslant 8$ provided $x = \{0, 1, 2, \ldots, 8\}$ is clearly defined somewhere.

(b) B1 for using correct model to get $\frac{4}{9}$ (o.e.)

SC **Sample space {1, …, 8}** If scored B0B1 in (a) for this allow $P(C < 4) = \frac{3}{8}$ to score B1 in (b)

(c) B1ft for comment that states that the model proposed is or is not a good one based on their model in part (a) and their probability in (b)

|(b) – 0.315| > 0.05 Allow e.g. "it is not suitable"; "it is not accurate" etc

|(b) – 0.315| ⩽ 0.05 Allow a comment that suggests it <u>is</u> suitable

No prob in (b) Allow a comparison that mentions 50% or 0.5 and rejects the model

No prob in (b) and no 50% or 0.5 or (b) > 1 scores B0

Ignore any comments about location or weather patterns.

(d) B1 for a sensible refinement considering variations in month or location

Just saying "not uniform" is B0

Context & "non-uniform" Allow mention of different locations, months <u>and</u> non-uniform <u>or</u> use more locations to form a new distribution with probabilities based on frequencies

Context & "binomial" Allow mention of different locations, months <u>and</u> binomial

Just refined model Model must be outlined and discrete and non-uniform

e.g. higher probabilities for more cloud cover <u>or</u> lower probabilities for less cloud cover

Continuous model Any model that is based on a continuous distribution. e.g. normal is B0

Q11.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{523+52+28}{184} = \frac{132}{184} = 0.717$	B1	This mark is given for a correct value for the probability for the cloud cover
(b)(i)	$P(X \geq 6) = 1 - P(X \leq 5)$	M1	This mark is given for using $1 - P(X \leq 5)$ with B(8, 0.76)
	$= 1 - 0.2967$ $= 0.703$	A1	This mark is given for finding as correct value for the probability
(b)(ii)	$184 \times P(X = 7)$ $= 184 \times 0.2811$	M1	This mark is given for using $184 \times P(X = 7)$ with B(8, 0.76)
	$= 51.7$	A1	This mark is given for finding as correct value for the probability
(c)	The answer to part (b)(i) of 0.703 is similar to 0.7127 in part (a) The answer to part (b)(ii) of 51.7 is very close to 52 found in the data set	B1	This mark is given for a correct evaluation of the outcomes from part (b) to determine the appropriateness of Magali's model
(d)	$\frac{5+9+9}{28} = \frac{23}{28} = 0.821$	B1	This mark is given for a correct value for the probability for the cloud cover
(e)	The answer to part (d) of 0.821 is greater than that in part (a) of 0.717 This shows that there is a higher chance of having high cloud cover if the previous day had high cloud cover	B1	This mark is given for a correct comparison for the answer to part (d) with the data set
	Thus independence does not hold so a binomial model might not be suitable	B1	This mark is given for a correct conclusion stated
			(Total 9 marks)

Q12.

	Scheme	Marks	AO
(a)	$\frac{k}{10}+\frac{k}{20}+\frac{k}{30}+\frac{k}{40}+\frac{k}{50}=1$ or $\frac{1}{600}(60k+30k+20k+15k+12k)=1$	M1	1.1b
	So $k=\frac{600}{137}$ (*)	A1cso	1.1b
		(2)	
(b)	(Cases are:) $D_1=30, D_2=50$ and $D_1=50, D_2=30$ and $D_1=40, D_2=40$	M1	2.1
	$\mathrm{P}(D_1+D_2=80)=\frac{k}{50}\times\frac{k}{30}\times 2+\left(\frac{k}{40}\right)^2$	M1	3.4
	$=0.0375619\ldots$ awrt **0.0376**	A1	1.1b
		(3)	
(c)	Angles are: $a,\ a+d,\ a+2d,\ a+3d$	M1	3.1a
	$S_4=a+(a+d)+(a+2d)+(a+3d)=360$	M1	2.1
	$2a+3d=180$ (o.e.)	A1	2.2a
	Smallest angle is $a>50$ consider cases: $d=10$ so $a=75$ or $d=20$ so $a=60$ [$d=30$ gives $a=45$ no good]	M1	3.1b
	$\mathrm{P}(D=10 \text{ or } 20)=\frac{3k}{20}=\frac{90}{137}$	A1	1.1b
		(5)	
		(10 marks)	

	Notes
(a)	M1 for clear use of sum of probabilities = 1 (all terms seen)
	A1 cso (*) M1 scored and no incorrect working seen.
Verify	**(Assume $k=\frac{600}{137}$)** to score the final A1 they must have a final comment "$\therefore k=\frac{600}{137}$"
(b)	1st M1 for selecting at least 2 of the relevant cases (may be implied by their correct probs) e.g. allow 30, 50 and 50,30 i.e. D_1 and D_2 labels not required
	2nd M1 for using the model to obtain a correct expression for two different probabilities. May use letter k or their value for k.
	Allow for $\frac{k}{50}\times\frac{k}{30}+\left(\frac{k}{40}\right)^2$ or $2\times\left(\frac{k}{50}\times\frac{k}{30}+\left(\frac{k}{40}\right)^2\right)$
	A1 for awrt 0.0376 (exact fraction is $\frac{705}{18769}$)
(c)	1st M1 for recognising the 4 angles and finding expressions in terms of d and their a
	2nd M1 for using property of quad with these 4 angles (equation can be un-simplified) Allow these two marks for use of a (possible) value of d
	e.g. $a+a+10+a+20+a+30=360$ (If at least 3 cases seen allow A1 for e.g. $4a=300$)
	or allow M1M1 for a set of 4 angles with sum 360 and possible value of d (3 cases for A1)
	e.g. (for $d=20$) 60, 80, 100, 120
	1st A1 for $2a+3d=180$ condition (o.e.) [Must be in the form $pa+qd=N$]
	3rd M1 for examining cases and getting $d=10$ and $d=20$ only
	2nd A1 for $\frac{90}{137}$ or exact equivalent
	The correct answer and no obviously incorrect working will score 5/5
	A final answer of awrt 0.657 (0.65693…) with no obviously incorrect working scores 4/5

Q13.

	Scheme	Marks	AO
(a)	[Sum of probs = 1 implies] $\log_{36} a+\log_{36} b+\log_{36} c=1$	M1	3.1a
	$\Rightarrow \log_{36}(abc)=1$ so $abc=36$	A1	3.4
	All probabilities greater than 0 implies each of a, b and c >1	B1	2.2a
	$36=2^2\times3^2$ (or 3 numbers that multiply to give 36 e.g. 2, 2, 9 etc)	dM1	2.1
	Since a, b and c are distinct must be **2, 3, 6** **($a=2$, $b=3$, $c=6$)**	A1 (5)	3.2a
(b)	$(\log_{36} a)^2+(\log_{36} b)^2+(\log_{36} c)^2$	M1	3.4
	[= 0.0374137…+ 0.09398737…+0.25] = 0.38140… awrt **0.381**	A1 (2)	1.1b
		(7 marks)	

	Notes
(a)	1st M1 for a start to the problem using sum of probabilities leading to eq'n in a, b and c
	1st A1 for reducing to the equation $abc = 36$ [Must follow from their equation.]
NB	Can go straight from $abc = 36$ to the answer for full marks for part (a).
	B1 for deducing that each value > 1 (may be implied by 3 integers all > 1 in the next line)
	2nd dM1 (dep on M1A1) for writing 36 as a product of prime factors or 3 values with product = 36 and none = 1
	2nd A1 for 2, 3 and 6 as a list or $a = 2$, $b = 3$ and $c = 6$
SC Ans only	M0M0 If no method marks scored but a correct answer given score: M0A0B1M0A1 (2/5) This gets the SC score of 2/5 [Question says show your working clearly]
(b)	M1 for a correct expression in terms of a, b and c or values; ft their integers a, b and c Condone invisible brackets if the answer implies they are used.
	A1 for awrt 0.381

Q14.

	Scheme	Marks	AO
(a)	**Disadvantage:** e.g. Not random; cannot use (reliably) for inferences	B1 (1)	1.1b
(b)	[Sight or correct use of] $X \sim B(36, 0.08)$	M1	3.3
(i)	$P(X = 4) = 0.167387\ldots$ awrt **0.167**	A1	1.1b
(ii)	$[P(X \ldots 7) = 1 - P(X \text{„} 6) =]$ 0.022233… awrt **0.0222**	A1 (3)	1.1b
(c)	P(In dance club and dance tango) = 0.4×0.08 = **0.032** or $\frac{4}{125}$ or **3.2%**	B1 (1)	1.1b
(d)	[Let T = those who can dance the Tango. Sight or use of] $T \sim B(50, \text{“}0.032\text{”})$	M1	3.3
	$[P(T < 3) = P(T \text{„} 2) =]$ 0.7850815… awrt **0.785**	A1 (2)	1.1b
		(7 marks)	

Notes

(a) B1 for a suitable disadvantage:

Allow (B1)	Do NOT allow (B0)
Not random or less random (o.e.)	Not representative
Cannot use (reliably) for inferences	Less accurate
(More likely to be) biased	Any comment based on time or cost
	Any mention of skew
	Any mention of non-response

(b) M1 for sight of B(36, 0.08) Allow in words: binomial with $n = 36$ and $p = 0.08$
may be implied by one correct answer to 2sf or sight of $P(X \text{„} 6) = 0.97776\ldots$ i.e. awrt 0.98
Allow for $36C4 \times 0.08^4 \times 0.92^{32}$ as this is "correct use"

(i) 1st A1 for awrt 0.167 NB An answer of just awrt 0.167 scores M1(⇒)1st A1

(ii) 2nd A1 for awrt 0.0222

(c) B1 for 0.032 o.e. (Can allow for sight of 0.4×0.08)

(d) M1 for sight of B(50, “0.032”) ft their answer to (c) provided it is a probability ≠ 0.08
may be implied by correct answer
or sight of $[P(T \text{„} 3)] = 0.924348\ldots$ i.e. awrt 0.924 or $P(T \text{„} 2)$ as part of $1 - P(T \text{„} 2)$ calc.
A1 for awrt 0.785

MR Allow MR of 50 (e.g. 30) provided clearly attempting $P(T \text{„} 2)$ and score M1A0