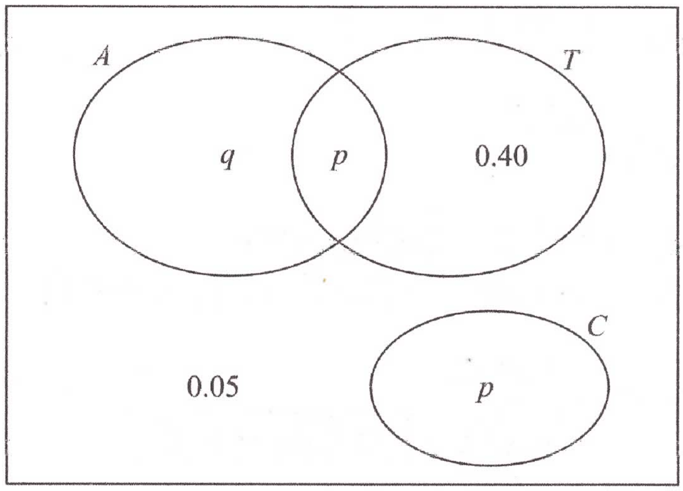
1. The Venn diagram shows the probabilities for students at a college taking part in various sports.

A represents the event that a student takes part in Athletics.
T represents the event that a student takes part in Tennis.
C represents the event that a student takes part in Cricket.
p and q are probabilities.



The probability that a student selected at random takes part in Athletics or Tennis is 0.75

(a) Find the value of p.
(1)

(b) State, giving a reason, whether or not the events A and T are statistically independent. Show your working clearly.
(3)

(c) Find the probability that a student selected at random does not take part in Athletics or Cricket.
(1)

(a) $P(A \cup T) = 0.75$

$P(C) = 1-(0.75+0.05)$

$= 1 - 0.8$

$= 0.2$

$\therefore \boxed{p = 0.2}$

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(b) $P(A) = 0.35 \quad P(T) = 0.6$

$\underline{P(A \cap T) = 0.2}$

$P(A) \times P(T) = 0.35 \times 0.6$

$\underline{= 0.21}$

$0.2 \neq 0.21$, so events A and T are <u>NOT statistically independent.</u>

(c) $P(A' \cup C') = 0.4 + 0.05$

$= \boxed{0.45}$

2. A factory buys 10% of its components from supplier A, 30% from supplier B and the rest from supplier C. It is known that 6% of the components it buys are faulty.

Of the components bought from supplier A, 9% are faulty and of the components bought from supplier B, 3% are faulty.

(a) Find the percentage of components bought from supplier C that are faulty. **(3)**

A component is selected at random.

(b) Explain why the event "the component was bought from supplier B" is not statistically independent from the event "the component is faulty". **(1)**

2a) $0.1(0.09) + 0.3(0.03) + 0.6p = 0.06$

$0.6p = 0.042$

$p = 0.07$

$= 7\%$

b) $P(B) = 0.3$

$P(F) = 0.06$

$P(B \cap F) = 0.03$

$P(B) \times P(F) = 0.3(0.06)$

$= 0.018$

$P(B) \times P(F) \neq P(B \cap F)$

$\therefore$ They are not independent

3. A biased spinner can only land on one of the numbers 1, 2, 3 or 4. The random variable X represents the number that the spinner lands on after a single spin and $P(X = r) = P(X = r + 2)$ for $r = 1, 2$

Given that $P(X = 2) = 0.35$

(a) find the complete probability distribution of X.

(2)

Ambroh spins the spinner 60 times.

(b) Find the probability that more than half of the spins land on the number 4
Give your answer to 3 significant figures.

(3)

The random variable $Y = \dfrac{12}{X}$

(c) Find $P(Y - X \leqslant 4)$

(3)

a) $r = 1, \quad P(X=1) = P(X = 1+2)$
$= P(X=3)$
$r = 2, \quad P(X=2) = P(X=2+2)$
$= P(X=4)$
$2(0.35) + 2P(X=1) = 1$
$P(X=1) = 0.15$

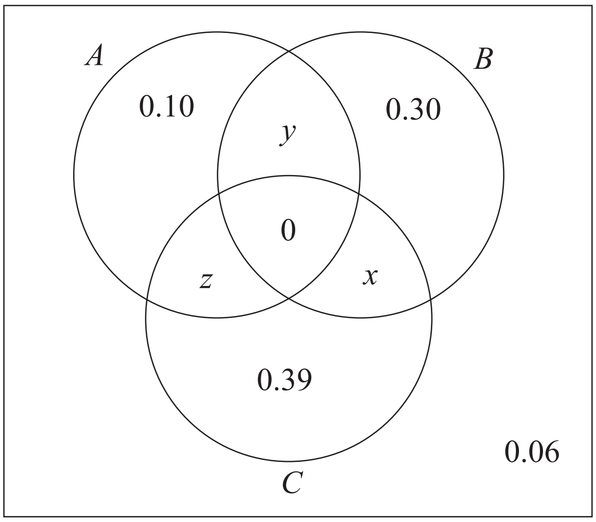
x	1	2	3	4
$P(X=x)$	0.15	0.35	0.15	0.35

b) Let Y = no. of spins which land on 4
$Y \sim B(60, 0.35)$
$P(Y > 30) = 1 - P(Y \leq 30)$
$= 1 - 0.99411$
$= 0.00589$

c) $Y - X \leq 4$
$\frac{12}{X} - X \leq 4$
$12 - X^2 \leq 4X$
$X^2 + 4X - 12 \geq 0$
$(X-2)(X+6) \geq 0$
$X - 2 \geq 0 \qquad X + 6 \geq 0$
$X \geq 2 \qquad X \geq -6$ n/a

$P(X \geqslant 2) = 1 - 0.15$
$= 0.85$

4. The Venn diagram shows three events, A, B and C, and their associated probabilities.



Events B and C are mutually exclusive.
Events A and C are independent.

Showing your working, find the value of x, the value of y and the value of z.

(5)

B and C mutually exclusive $\Rightarrow x = 0$

$P(A) = 0.1 + y + z$
$P(C) = 0.39 + z + (x) \rightarrow = 0$

$P(A \text{ and } C) = z$

A and C independant $\Rightarrow P(A \text{ and } C) = P(A) \times P(C)$

$\Rightarrow z = (0.1 + y + z)(0.39 + z)$

Σ probability $= 1$. $1 = 0.06 + 0.1 + 0.39 + 0.3 + y + z$

$y + z = 0.15$

$\Rightarrow z = (0.1 + 0.15)(0.39 + z)$

$0.75z = 0.0975$

↓

$z = \frac{0.0975}{0.75} = 0.13$

$y = 0.15 - z = 0.15 - 0.13 = 0.02$

$x = 0$
$y = 0.02$
$z = 0.13$

5. A fair 5-sided spinner has sides numbered 1, 2, 3, 4 and 5

The spinner is spun once and the score of the side it lands on is recorded.

(a) Write down the name of the distribution that can be used to model the score of the side it lands on.

(1)

The spinner is spun 28 times.

The random variable X represents the number of times the spinner lands on 2

(b) (i) Find the probability that the spinner lands on 2 at least 7 times.

(ii) Find $P(4 \leqslant X < 8)$

(5)

a. Discrete uniform distribution.

b. i. At least 7 times: $P(X \geq 7) = 1 - P(X \leq 6)$

$= 1 - 0.6784$

$= 0.322$ (3dp)

ii. $P(4 \leq X < 8) = P(X \leq 7) - P(X \leq 3)$

$= 0.818 - 0.160$

$= 0.658$ (3dp)

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6. In a game, a player can score 0, 1, 2, 3 or 4 points each time the game is played.

The random variable S, representing the player's score, has the following probability distribution where a, b and c are constants.

s	0	1	2	3	4
$P(S = s)$	a	b	c	0.1	0.15

The probability of scoring less than 2 points is twice the probability of scoring at least 2 points.

Each game played is independent of previous games played.

John plays the game twice and adds the two scores together to get a total.

Calculate the probability that the total is 6 points.

(6)

$P(S<2) = 2 \times P(S \geq 2) \Rightarrow a+b = 2(c+0.1+0.15)$

$a+b = 2c+0.5$ ①

total probability $= 1 \Rightarrow a+b+c+0.1+0.15 = 1$

$a+b+c = 0.75$ ②

sub in ①: $2c+0.5+c = 0.75$

$3c = 0.25$

$c = \frac{1}{12}$

ways to score 6: 2+4, 4+2, 3+3

↳ so we don't need a or b

$P(T.S.=6) = \underbrace{\frac{1}{12} \times 0.15}_{P(2,4)} + \underbrace{0.15 \times \frac{1}{12}}_{P(4,2)} + \underbrace{0.1^2}_{P(3,3)}$

$= 0.035$

7. Afrika works in a call centre.

She assumes that calls are independent and knows, from past experience, that on each sales call that she makes there is a probability of $\frac{1}{6}$ that it is successful.

Afrika makes 9 sales calls.

(a) Calculate the probability that at least 3 of these sales calls will be successful.
(2)

The probability of Afrika making a successful sales call is the same each day.

Afrika makes 9 sales calls on each of 5 different days.

(b) Calculate the probability that at least 3 of the sales calls will be successful on exactly 1 of these days.
(2)

Rowan works in the same call centre as Afrika and believes he is a more successful salesperson.

To check Rowan's belief, Afrika monitors the next 35 sales calls Rowan makes and finds that 11 of the sales calls are successful.

(c) Stating your hypotheses clearly test, at the 5% level of significance, whether or not there is evidence to support Rowan's belief.
(4)

a) independent, constant probability of success ⇒ prompts us to use the binomial distribution.

let C = number of successful calls.

$C \sim B(9, \frac{1}{6})$

no. attempts

probability

$P(C \geq 3) = 1 - P(C \leq 2) = 0.1782...$ by calculator

b) we use the value of $P(C \geq 3)$ from (a) ← imagine we 'trial' the 9 calls on 5 days, with a new p

let X = the number of days when at least 3 calls succeed.

$X \sim (5, P(C \geq 3))$

$P(X=1) = 5 \times (0.1782) \times (1-0.1782)^4$

$= 0.4061\ldots$

c) 1. state your hypotheses

$H_0: p = \frac{1}{6}$ $H_1: p > \frac{1}{6}$

2. define your variables & calculate test statistic

let R = number of successful calls

$R \sim B(35, \frac{1}{6})$

$P(R \geq 11) = 1 - P(R \leq 10) = 0.02\ldots$

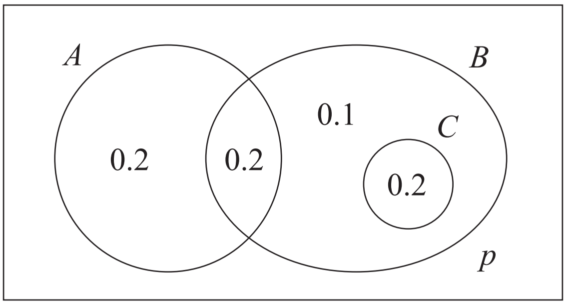
3. form conclusion

$0.02 < 0.05 \Rightarrow$ there is sufficient evidence to support that

Rowan has more successful calls than Afrika. reject H_0.

always link to context of test

8.



The Venn diagram, where p is a probability, shows the 3 events A, B and C with their associated probabilities.

(a) Find the value of p.

(1)

cannot happen at the same time

(b) Write down a pair of mutually exclusive events from A, B and C.

(1)

(a) Sum of probabilities = 1

$0.2 + 0.2 + 0.2 + 0.1 + p = 1$

$0.7 + p = 1$

$p = 0.3$ 1

(b) Events A and C are mutually exclusive 1

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9. Two bags, **A** and **B**, each contain balls which are either red or yellow or green.

Bag **A** contains 4 red, 3 yellow and n green balls.
Bag **B** contains 5 red, 3 yellow and 1 green ball.

A ball is selected at random from bag **A** and placed into bag **B**.
A ball is then selected at random from bag **B** and placed into bag **A**.

The probability that bag **A** now contains an equal number of red, yellow and green balls is p.

Given that $p > 0$, find the possible values of n and p.

(5)

p is the probability of having an equal number of red, yellow and green balls.

to get p, we can have either 2 of these scenarios

1) 3 of each colour (in bag A) = when a red ball is taken from A and put into B and a green ball is added into A from B.

1

For this scenario = $n+1 = 3$

$n = 2$ 1

green B → A

$\therefore p(\text{red from A and green from B}) = \frac{4}{9} \times \frac{1}{10} = \frac{4}{90} = \frac{2}{45}$ 1

red A → B

extra 1 because ball from A is added

2) 4 of each colour (in bag A) = when a green ball is taken from A and put into B and a yellow ball is taken from B and put into A.

For this scenario = $n-1 = 4$

$n = 5$ 1

$\therefore p(\text{green from A and yellow from B}) = \frac{5}{12} \times \frac{3}{10} = \frac{15}{120} = \frac{1}{8}$ 1

green A → B   yellow B → A

$\therefore$ when $n = 2$, $p = \dfrac{2}{5}$ ✱

$\therefore$ when $n = 5$, $p = \dfrac{1}{8}$ ✱

10. Helen believes that the random variable C, representing cloud cover from the large data set, can be modelled by a discrete uniform distribution. ↳oktas

(a) Write down the probability distribution for C.

(2)

(b) Using this model, find the probability that cloud cover is less than 50%

(1)

Helen used all the data from the large data set for Hurn in 2015 and found that the proportion of days with cloud cover of less than 50% was 0.315

(c) Comment on the suitability of Helen's model in the light of this information.

(1)

(d) Suggest an appropriate refinement to Helen's model.

(1)

a)

c	0	1	2	3	4	5	6	7	8
$P(C=c)$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

-① -①

The random variable C can only be between 0 and 8 as cloud coverage is measured on a scale of 0-8 (oktas)

b) $P(C<4) = P(C=0) + P(C=1) + P(C=2) + P(C=3)$

$= 4 \times \frac{1}{9}$

$= \frac{4}{9}$ -①

c) $\frac{4}{9} = 0.\dot{4}$

$0.\dot{4} > 0.315$

Probability is lower than expected, suggesting Helen's model isn't suitable. -①

d) As cloud coverage varies over different months and places, Helen could use a non-uniform distribution instead. -①

11. Magali is studying the mean total cloud cover, in oktas, for Leuchars in 1987 using data from the large data set. The daily mean total cloud cover for all 184 days from the large data set is summarised in the table below.

Daily mean total cloud cover (oktas)	0	1	2	3	4	5	6	7	8
Frequency (number of days)	0	1	4	7	10	30	52	52	28

One of the 184 days is selected at random.

(a) Find the probability that it has a daily mean total cloud cover of 6 or greater. **(1)**

$52+52+28=132$ $\quad \frac{132}{184}=\frac{33}{46}$ 1

Magali is investigating whether the daily mean total cloud cover can be modelled using a binomial distribution.

She uses the random variable X to denote the daily mean total cloud cover and believes that $X \sim \mathrm{B}(8, 0.76)$

n p

Using Magali's model, $P(X \leqslant x)$ $\quad$ 4 5 | 6 7 8

← Form needed for calculator 1

(b) (i) find $\mathrm{P}(X \geqslant 6)$ 1 **(2)**

$P(X \geqslant 6) = 1 - P(X \leqslant 5) = 1 - 0.296722\ldots = 0.703$ (3dp)

(ii) find, to 1 decimal place, the expected number of days in a sample of 184 days with a daily mean total cloud cover of 7 **(2)**

1 1

$P(X=7) \times 184 = 0.2811\ldots \times 184 = 51.7385\ldots = 51.7$ (1dp)

(c) Explain whether or not your answers to part (b) support the use of Magali's model. **(1)**

Part (a) and part (b)(i) are similar and the expected number of 7s (51.7) matches number of 7s in the data set (52) so Magali's model is supported 1

There were 28 days that had a daily mean total cloud cover of 8
For these 28 days the daily mean total cloud cover for the **following** day is shown in the table below.

Daily mean total cloud cover (oktas)	0	1	2	3	4	5	6	7	8
Frequency (number of days)	0	0	1	1	2	1	5	9	9

(d) Find the proportion of these days when the daily mean total cloud cover was 6 or greater. **(1)**

$5+9+9=23$ $\quad \frac{23}{28} = 0.82142\ldots = 0.821$ (3dp) 1

(e) Comment on Magali's model in light of your answer to part (d). **(2)**

Part d (0.821) differs from part (a) and (b)(i) (≈ 0.7) 1
therefore Magali's model may not be suitable 1
(since this means independence does not hold)

12. The discrete random variable D has the following probability distribution

d	10	20	30	40	50
$P(D=d)$	$\frac{k}{10}$	$\frac{k}{20}$	$\frac{k}{30}$	$\frac{k}{40}$	$\frac{k}{50}$

where k is a constant.

(a) Show that the value of k is $\frac{600}{137}$

(2)

The random variables D_1 and D_2 are independent and each have the same distribution as D.

(b) Find $P(D_1 + D_2 = 80)$
Give your answer to 3 significant figures.

(3)

The value obtained, d, is the common difference of an arithmetic sequence.

The first 4 terms of this arithmetic sequence are the angles, measured in degrees, of quadrilateral Q

(c) Find the exact probability that the smallest angle of Q is more than 50°

(5)

a) $\frac{k}{10}+\frac{k}{20}+\frac{k}{30}+\frac{k}{40}+\frac{k}{50}=1$ (1) $\quad \frac{137k}{600}=1 \quad k=\frac{600}{137}$ as needed (1)

b) $40+40=80$
or $50+30=80$

multiplication

$D_1=40, D_2=40$ For 'And' (X) ✓
$D_1=50, D_2=30$ For 'Or' (+) ✓ addition
$D_1=30, D_2=50$ (1)

$P(D_1+D_2=80)=P(D_1=40 \text{ and } D_2=50)+P(D_1=30 \text{ and } D_2=50)+P(D_1=50+D_2=30)$

$=\left(\frac{k}{40}\times\frac{k}{40}\right)+\left(\frac{k}{50}\times\frac{k}{30}\right)+\left(\frac{k}{30}\times\frac{k}{50}\right)=\frac{k^2}{1600}+\frac{k^2}{1500}+\frac{k^2}{1500}$

(1) $=\frac{47k^2}{24000}$ since $k=\frac{600}{137}$

$=\frac{47\left(\frac{600}{137}\right)^2}{24000}=0.0375619\ldots=0.0376$ (3sf) (1)

c) let a = first term in sequence

Angles: $a, a+d, a+2d, a+3d$ (1)

Interior angles in a quadrilateral add to 360°

$a + a + d + a + 2d + a + 3d = 360$ (1)

$4a + 6d = 360$

÷2 ↓

$2a + 3d = 180$ ÷2 (1)

For $a > 50$ only possible cases (see working)

$d = 10, a = 75$ or $d = 20, a = 60$ (1)

let $d = D$

Sub $k = \frac{600}{137}$

$P(D = 10 \text{ or } D = 20) = \left(\frac{k}{10} + \frac{k}{20}\right) = \left(\frac{3k}{20}\right) = \frac{90}{137}$ (1)

Possibe cases:

Since $2a + 3d = 180$

when $d = 10$

$2a + 3(10) = 180$

$2a = 180 - 30$

$2a = 150$

$a = 75$

Meets condition $a > 50$

when $d = 20$

$2a + 3(20) = 180$

$2a = 180 - 60$

$2a = 120$

$a = 60$

Meets condition $a > 50$

when $d = 30$

$2a + 3(30) = 180$

$2a = 180 - 90$

$2a = 90$

$a = 45$

doesn't meet condition $a > 50$ and no point trying any more possibilities for d since as d gets bigger a gets smaller

13. The discrete random variable X has the following probability distribution

x	a	b	c
$P(X = x)$	$\log_{36} a$	$\log_{36} b$	$\log_{36} c$

where

- a, b and c are distinct integers $(a < b < c)$
- all the probabilities are greater than zero

(a) Find

(i) the value of a

(ii) the value of b

(iii) the value of c

Show your working clearly.

(5)

The independent random variables X_1 and X_2 each have the same distribution as X

(b) Find $P(X_1 = X_2)$

(2)

a) $\log_{36} a + \log_{36} b + \log_{36} c = 1$ ① (because sum of probabilities = 1)

$\log_{36} (abc) = 1$

$1 \times abc = 36$

$abc = 36$ ①

All probabilities are greater than 0 so $a, b, c > 1$ ①

As $a \times b \times c = 36$, $2 \times 3 \times 6 = 36$ so: ①

i) $a = 2$

ii) $b = 3$

iii) $c = 6$ ①

as a, b, c must be distinct (different).

b) $(\log_{36} a)^2 + (\log_{36} b)^2 + (\log_{36} c)^2$ ①

$= (\log_{36} 2)^2 + (\log_{36} 3)^2 + (\log_{36} 6)^2$

$= 0.0374... + 0.0939... + 0.25$

$= 0.381$ (3.s.f) ①

14. (a) State one disadvantage of using quota sampling compared with simple random sampling.

(1)

In a university 8% of students are members of the university dance club. so $p = 0.08$

A random sample of 36 students is taken from the university. and $n = 36$

The random variable X represents the number of these students who are members of the dance club.

(b) Using a suitable model for X, find

(i) $P(X = 4)$

(ii) $P(X \geqslant 7)$

(3)

Only 40% of the university dance club members can dance the tango.

(c) Find the probability that a student is a member of the university dance club and can dance the tango.

use multiplication rule for AND

(1)

A random sample of 50 students is taken from the university.

$P(X < 3)$

(d) Find the probability that fewer than 3 of these students are members of the university dance club and can dance the tango.

(2)

a) One disadvantage is that quota sampling is not random, so it cannot be used reliably for inferences. 1

OR

More likely to be biased.

OR

Not random / less random

b) i) $X \sim B(36, 0.08)$ 1 where $n = 36$, $p = 0.08$

$P(X = 4) = 0.16738... = 0.167$ (3.s.f) 1

ii) $P(X \geqslant 7) = 1 - P(X \leqslant 6)$

$= 1 - 0.97776..$

$= 0.022233..$

$= 0.0222$ (3 s.f) 1

c) $P(\text{dance club AND tango}) = 0.08 \times 0.4$

$= 0.032$ 1

or 3.2% or $\frac{4}{125}$

d) T = people who can dance the tango

$T \sim B(50, 0.032)$ ① 0.032 from c)

$P(T < 3) = P(T \leq 2) = 0.785$ (3.s.f) ①