Probability Cheat Sheet

Stats/Mech Year 1

Calculating probabilities

An experiment is a repeatable process that gives rise to a number of outcomes. An event is a collection of one or more outcomes. A sample space is the set of all possible outcomes.

Probabilities can be written as decimals or fractions and are in the range of 0(impossible) to 1(certain).

If each outcome has an equal likelihood of occurring,

$$\text{Probability of event} = \frac{\text{number of possible outcomes in the event}}{\text{total number of possible outcomes}}$$

Example 1: The table shows the time taken, in minutes, for a group of students to complete a number puzzle.

Time, t(min)	$5 \le t < 7$	$7 \le t < 9$	$9 \le t < 11$	$11 \le t < 13$	$13 \le t < 15$
Frequency	6	13	12	15	4

A student is chosen at random. Find the probability that they finished the number puzzle:

a. In under 9 minutes

Total number of students: $6 + 13 + 12 + 15 + 4 = 50$

Number of students who finished under 9 minutes: $6 + 13 = 19$

P (finished under 9 minutes) = $\frac{19}{50}$

b. In over 10.5 minutes

10.5 minutes is $\frac{3}{4}$ through the $9 \le t < 11$ class. Estimate using interpolation:

$\frac{1}{4} \times 12 = 3$

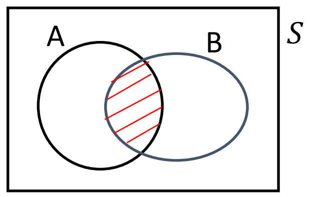
$3 + 15 + 4 = 22$

P (finished in over 10.5 minutes) = $\frac{22}{50}$

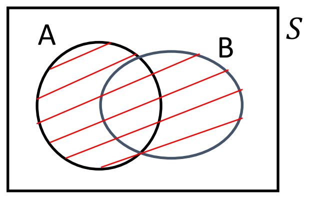
Venn Diagrams

A Venn diagram can be used to represent events graphically. Frequencies or probabilities can be placed in the regions of Venn diagrams.

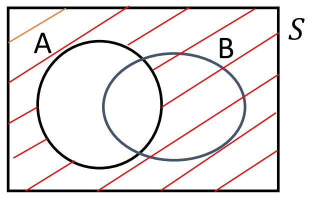
A rectangle represents the sample space, S. It contains closed curves which represent events.



Intersection of A and B shows the event in which both A and B occur



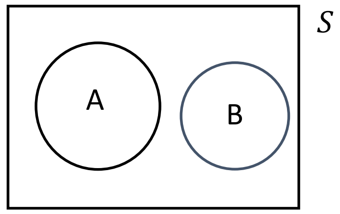
Union of A and B shows the event in which either A or B or both occur



The shaded area shows the event in which A does not occur

Mutually exclusive independent events

Events which have no outcomes in common are called mutually exclusive. The closed curves do not overlap in a Venn Diagram.



For mutually exclusive events,

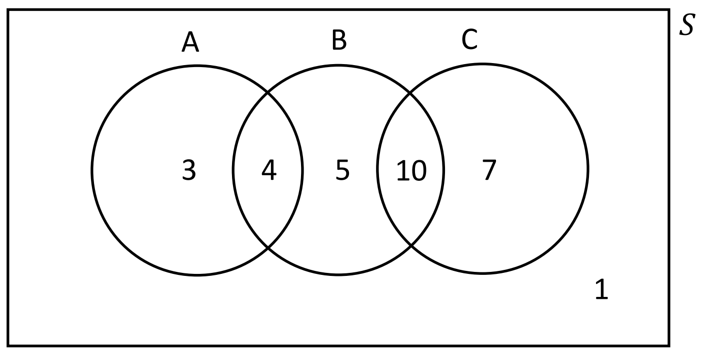
P(A or B) =P(A)+P(B)

When one event has no effect on another, they are independent. For independent events A and B, the probability of B happening is the same regardless of whether A happens. For independent events,

P(A and B)=P(A)×P(B)

You can also use this multiplication rule to check if events are independent.

Example 2: The Venn diagram shows the number of students in a particular class who watch any of three popular TV shows



a. Find the probability of a student chosen at random watches B or C or both.

$4 + 5 + 10 + 7 = 26$

P(watches B or C or both)= $\frac{26}{30} = \frac{13}{15}$

b. Determine whether watching A and watching B are statistically independent.

P(A)= $\frac{3+4}{30} = \frac{7}{30}$

P(B)= $\frac{4+5+10}{30} = \frac{19}{30}$

P(A and B)= $\frac{4}{30} = \frac{2}{15}$

P(A)×P(B)= $\frac{7}{30} \times \frac{19}{30} = \frac{133}{900}$

P(A and B)≠ P(A)×P(B)

Therefore, watching A and watching B are not statistically independent.

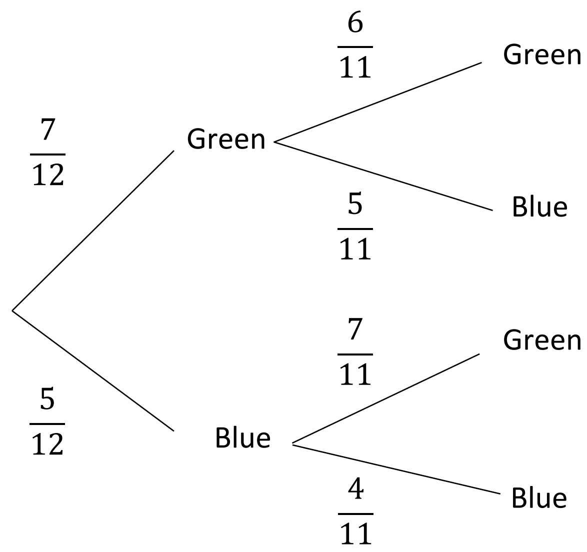
Tree diagrams

A tree diagram can be used to show the outcomes of two or more events happening in succession.

Example: A bag contains seven green beads and five blue beads. A bead is taken from the bag at random and not replaced. A second bead is then taken from the bag. Find the probability that:

a. Both beads are green

1. Draw a tree diagram to show the events.



2. Multiply along the branch of tree diagram:

P(green and green) $= \frac{7}{12} \times \frac{6}{11} = \frac{7}{22}$

b. The beads are different colours

P(different colours) = P(green then blue) + P(blue then green)

$= \frac{7}{12} \times \frac{5}{11} + \frac{5}{12} \times \frac{7}{11}$

$= \frac{35}{66}$