1. The scores, X, in Paper 1 of an English examination have an underlying Normal distribution with mean 76 and standard deviation 12. The scores are reported as integer marks. So, for example, a score for which $75.5 \leqslant X < 76.5$ is reported as 76 marks.

i. Find the probability that a candidate's reported mark is 76.

[4]

ii. Find the probability that a candidate's reported mark is at least 80.

[3]

iii. Three candidates are chosen at random. Find the probability that exactly one of these three candidates' reported marks is at least 80.

[2]

The proportion of candidates who receive an A* grade (the highest grade) must not exceed 10% but should be as close as possible to 10%.

iv. Find the lowest reported mark that should be awarded an A* grade.

[5]

The scores in Paper 2 of the examination have an underlying Normal distribution with mean μ and standard deviation 12.

v. Given that 20% of candidates receive a reported mark of 50 or less, find the value of μ.

[4]

2. The quality control department of a battery manufacturing company checks the lifetimes of the batteries produced by the company. The lifetimes, x minutes, for a random sample of 80 'Superstrength' batteries are shown in the table below.

Lifetime	$160 \le x < 165$	$165 \le x < 168$	$168 \le x < 170$	$170 \le x < 172$	$172 \le x < 175$	$175 \le x < 180$
Frequency	5	14	20	21	16	4

(a) Estimate the proportion of these batteries which have a lifetime of at least 174.0 minutes. [2]

(b) Use the data in the table to estimate

- the sample mean,
- the sample standard deviation.

[3]

The data in the table on the previous page are represented in the following histogram:

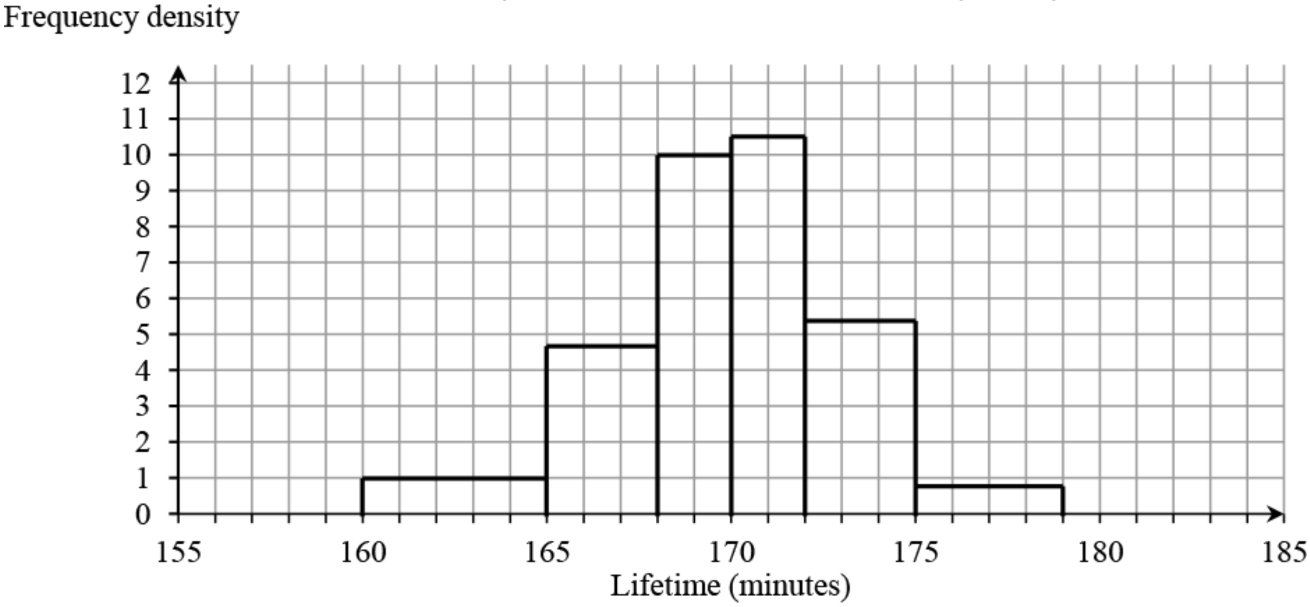

Fig. 15

A quality control manager models the data by a Normal distribution with the mean and standard deviation you calculated in part (b).

(c) Comment briefly on whether the histogram supports this choice of model. [2]

(d) (i) Use this model to estimate the probability that a randomly selected battery will have a lifetime of more than 174.0 minutes.

(ii) Compare your answer with your answer to part (a). [3]

The company also manufactures 'Ultrapower' batteries, which are stated to have a mean lifetime of 210 minutes.

(e) A random sample of 8 Ultrapower batteries is selected. The mean lifetime of these batteries is 207.3 minutes. Carry out a hypothesis test at the 5% level to investigate whether the mean lifetime is as high as stated. You should use the following hypotheses $H_0 : \mu = 210$, H1 : $\mu < 210$, where μ represents the population mean for Ultrapower batteries. You should assume that the population is Normally distributed with standard deviation 3.4. [5]

3. Between the islands of Tenerife and La Gomera there is a resident population of pilot whales. Studies in the past showed that 20% of the adult population were male. Due to a change in environmental conditions, it is thought that the proportion of males has decreased.

In order to investigate this, scientists caught and released a random sample of 43 different adult pilot whales. Exactly 3 of these whales were found to be male.

(a) Carry out a hypothesis test at the 5% level to investigate whether there is any evidence that the proportion of males has decreased. [6]

Previous studies also showed that the mean length of adult females was 2.98 metres. On another occasion the scientists caught a random sample of 39 different adult female pilot whales; the length, x metres, of each whale was measured before it was released. The data are summarized below.

$$\sum x = 120.7 \text{ and } \sum x^2 = 384.75$$

(b) Carry out a hypothesis test at the 5% level to investigate whether there is any evidence that the mean length of adult females has changed. [8]

4. The pre-release data set consists of a subset (see http:/www.ocr.org.uk/Images/308749-units-h630-and-h640-large-data-set-lds-sample-assessment-material.xls) of the data about countries from the CIA World Facebook.
Data for Croatia shows that the average number of mobile phone subscribers per 1000 population is 1111.72.

(a) Explain how this average can be greater than 1000. [1]

It is assumed that, once the data have been cleaned, the average number of phones per 1000 population in different countries can reasonably be approximated by a Normal distribution with mean 996 and standard deviation 407.

(b) State two ways in which the data may have been cleaned. [2]

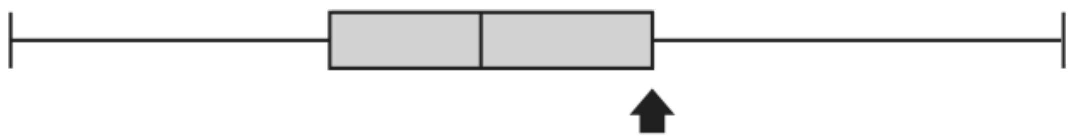

Fig. 11.1

Fig. 11.1 shows a box plot of the cleaned data.

(c) Give the name of the measure which the arrow is pointing to. [2]

(d) Use the approximate Normal distribution to estimate a value for this measure. [2]

Fatima thinks that the mean number of mobile phone subscribers per 1000 population for the countries of the world has increased since the data were collected. In order to test this he obtains some up-to-date data for a random sample of countries and uses software to conduct the appropriate hypothesis test at the 5% level of significance. The output from the software is shown in Fig. 11.2.

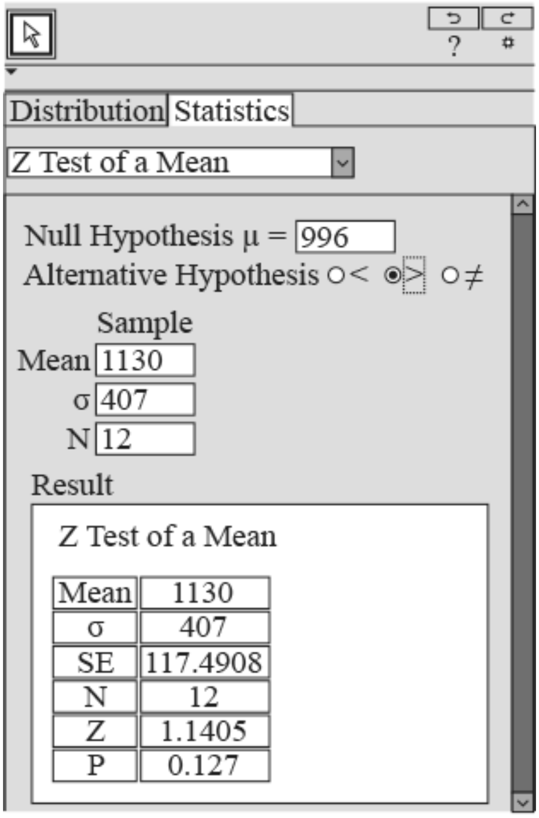

Fig. 11.2

(e) State the conclusion of Fatima’s test, explaining your reasoning. [4]

(f) With reference to the pre-release data set, comment on the sample size Fatima has chosen. [1]

5. A team of biologists are monitoring a subspecies of vole on a remote island. Previous studies have shown that the population mean mass of adult male voles is 26.2 g. It is believed that, due to changes in the environment, the mean mass may have altered. Three of the biologists conducted a new survey and each collected a random sample of adult male voles. The voles were weighed and tagged and then released unharmed.

Fig. 12 shows summary statistics for the 3 samples.

Statistics	Sample 1	Sample 2	Sample 3
Mean	22.6	22.8	23.7
Standard deviation	2.983287	3.258527	4.122196
Variance	8.9	10.618	16.9925
Minimum	19.2	18.6	18.4
Maximum	29.1	30.1	32
Q_1	20.3	20.1	19.8
Median	21.7	22.4	23.2
Q_3	23.95	24.3	26.1
Count	9	11	17

Fig. 12

(a) For each sample calculate Σx and Σx^2. [4]

(b) Use your answers to part **(a)** to calculate the mean and variance of the mass of all the voles caught in the survey. [2]

The biologists use the results of the survey to test, at the 5% level, the hypothesis H_0: $\mu = 26.1$ against the hypothesis H_1 : $\mu \neq 26.1$.

(c) Assuming that the distribution of the masses of adult male voles may be modelled by the Normal distribution with the value for the variance found in part **(b)**, calculate the critical region for this test. [3]

(d) State the conclusion reached by the biologists, explaining your answer. [1]

6. **In this question you must show detailed reasoning.**

Each evening Statto goes for a walk on the same circular route. Over a long period of time Statto noticed that the mean time taken to complete the walk was 56 minutes and the standard deviation was 4 minutes.

A few months ago Statto was ill and was unable to complete the walk for a month. Since then Statto's partner thinks that the mean time Statto takes to complete the evening walk has increased. Over a period of weeks Statto's partner collects a random sample of the times taken by Statto to complete the evening walk.

Statto's partner uses software to generate the summary statistics in Fig. 14.

Statistics ▼	
n	19
Mean	57.4737
σ	3.8848
s	3.9912
Σx	1092
Σx^2	63048
Min	52
Q1	54
Median	57
Q3	60
Max	65

Fig. 14

Use information from Fig. 14 to conduct a hypothesis test to determine whether there is any evidence at the 5% level to suggest that the mean time Statto takes to complete the evening walk has increased. [7]

END OF QUESTION paper

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$P(Y = 76) = P\left(\frac{75.5-76}{12} \le Z \le \frac{76.5-76}{12}\right)$ = P(−0.04166… < Z < 0.04166…) = Φ(0.04166…) − (1 − Φ(0.04166…)) = 2 × Φ(0.04166…) − 1 = 2 × 0.5167 − 1 = 0.0334	B1	For one correct continuity correction used	
		i		M1	For standardizing	
		i		M1	For correctly structured probability calculation. CAO inc use of diff tables. Allow 0.0330 – 0.0340 www. <u>Examiner's Comments</u>	
		i		A1	Most candidates obtained a correct answer. A small but significant number did not use one or both of the continuity corrections. Most used the difference column of the Standard Normal table correctly to provide suitably accurate answers. A relatively small number struggled with the structure of the calculation.	
		ii	$P(Y \ge 80) = P\left(Z \ge \frac{79.5-76}{12}\right)$ = P(Z> 0.2917) = 1 − Φ(0.2917) = 1 − 0.6148 = 0.3852 = 0.385 to 3 sig fig	B1	For correct cc used	
		ii		M1	For correct structure	

		ii		A1	CAO do not allow 0.386 **Examiner's Comments** Many candidates answered correctly, but a common mistake was to omit or to provide an incorrect continuity correction. A relatively small number did not use the difference column correctly.	
		iii	$3 \times 0.3852 \times 0.6148^2 = 0.4368$	M1	$3 \times$ their $p \times (1 - \text{their } p)^2$	
		iii		A1	FT their p. Allow 2sf if working seen. **Examiner's Comments** Most candidates knew and applied the method correctly but many were dependent on the FT to gain the 2 marks. A small number omitted the x3 from their binomial calculation.	
		iv	**EITHER:** P(Score $\geq k$) = 0.1			
		iv	$\Phi^{-1}(0.9) = 1.282$	B1	For 1.282	
		iv		M1	Allow $k - 0.5$ used for k. Positive z used.	
		iv	$k = 76 + (1.282 \times 12) = 91.38$ or $k = 76 + 0.5 + (1.282 \times 12) = 91.88$	A1	For 91.38 or 91.88	
		iv	$91.38 > 90.5$ or $91.88 > 91$	M1	Relevant comparison (e.g. diagram)	www
		iv	so lowest reported mark = 92	A1		
		iv	**OR** Trial and improvement method	M1	M1 for attempt to find P(Mark $\geq$ integer)	
		iv	P(Mark $\geq$ 91) = P(Score $\geq$ 90.5) = 0.1135	A1	A1 for 0.1135	
		iv	P(Mark $\geq$ 92) = P(Score $\geq$ 91.5) = 0.0982			
		iv	P(Mark $\geq$ 91) > 10% and P(Mark $\geq$ 92) < 10%	M1	M1 for comparisons	www

		iv	so lowest reported mark = 92	A1	**Examiner's Comments** 1.282 was identified by the majority of candidates who went on to set up a correct equation and arrive at 91.38 or 91.88. Many of these gave 92% as the final answer but many others gave 91%. Others rearranged incorrectly and arrived at 60.6 for the first calculation. Few candidates demonstrated a proper understanding of the requirement of this question.	
		v	$P(Y \leq 50) = 0.2$			
		v		B1	For 50.5 used	
		v		B1	For −0.8416. Condone – 0.842 Condone 0.8416 if numerator reversed.	
		v		M1	For structure.	
		v	$\mu = 50.5 + (12 \times 0.8416) = 60.6$	A1	CAO **Examiner's Comments** In this part, the continuity correction was omitted, or an incorrect value was used, by many candidates. Many used +0.8416 leading to 60.0992 which was a common answer. The issue of over-specification was most apparent in this part of the question.	
			Total	18		
2		a	Estimated number $= 4 + \frac{16}{3} = 9\frac{1}{3}$ $\frac{9\frac{1}{3}}{80} = 0.1166$ … so proportion is approximately 0.117	M1(AO3.1b) A1(AO1.1)	for attempt at interpolation	

				[2]		
		b	E.g. Midpoints Mean Standard deviation = 3.4	M1(AO1.1) A1(AO1.1) A1(AO1.1) [3]	evidence of valid method for estimation **BC** Mean in the range 169-171 **BC** SD in the range 3-3.5	
		c	The histogram e.g. seems to have a rough bell shape e.g. is symmetrical (around the estimated mean) e.g. appears to have all data within 3 s.d. of the mean so this does support the manager's belief	B1(AO3.5a) B1(AO3.5a) [2]	for one reason for at least two reasons and 'supports belief'	
		d	i P(Lifetime > 174) for N(170, 3.4^2) 0.1197	M1(AO3.4) A1(AO1.1)	oe **BC** FT their mean and standard deviation	
		d	ii Answer is very similar to estimate in part (a)	B1(AO3.5a)		

				[3]		
		e	Either Test statistic $= \dfrac{207.3 - 210}{3.4/\sqrt{8}} = -2.246$ Lower 5% level 1 tailed critical value of $z = -1.645$ $-2.246 < -1.645$ so significant or under $H_0,\ \overline{X} \sim N\left(210,\ \dfrac{3.4^2}{8}\right)$ $P\left(\overline{X} \le 207.3\right) = 0.01235$ $0.01235 < 0.05$ so significant There is sufficient evidence to reject H_0 There is sufficient evidence to conclude that the mean lifetime is less than 210 minutes.	M1(AO3.4) A1(AO1.1) B1(AO1.1) M1(AO3.4) A1(AO1.1) B1(AO1.1) A1(AO2.2b) E1(AO2.4) [5]	Must include $\sqrt{8}$ For comparison leading to correct conclusion BC	

			Total	15		
3		a	$H_0 p = 0.2$ $H_1 p < 0.2$ p is the proportion of male adult pilot whales $X \sim B(43,0.2)$ $P(X \leq 3) = 0.0178$ BC $0.0178 < 0.05$ so result is significant There is sufficient evidence to suggest at the 5% level that the proportion of males in the population of adult pilot whales is less than 20%	B1(AO 1.1) B1(AO 2.5) M1(AO 3.3) B1(AO 3.4) M1(AO 1.1) A1(AO 2.2b) [6]	For both hypotheses Definition of p soi Comparison of their p-value with 0.05	
		b	Sample mean is 3.09 Sample variance is 0.295 $H_0 : \mu = 2.98$ $H_1 : \mu \neq 2.98$ μ is the population mean length of female pilot whales $\bar{X} \sim N(2.98, \frac{0.295}{39})$	B1(AO 1.1) B1(AO 1.1) B1(AO 1.1) B1(AO 2.5) M1(AO 3.3) B1(AO 3.4) M1(AO 1.1) A1(AO	soi For both hypotheses soi Comparison of their	NB 3.09487179487 and 0.294709851552

			$p(\bar{X} \geq 3.09) = 0.1029$ 0.1029 > 0.025 so result not significant There is insufficient evidence at the 5% level to suggest that the mean length of female pilot whales has changed	2.2b) [8]	p-value with 0.025 BC	
			Total	14		
4		a	eg some people have more than one phone oe	E1(AO2.4) [1]	Some of the other figures in the LDS are greater than 1	
		b	E.g. data missing E.g. removal of outlier data items	B1(AO1.1b) B1(AO1.1b) [2]		
		c	Upper quartile	B1(AO1.2) [1]		
		d	X ~ N(996,407^2), $p = 0.75$ 1271	M1(AO3.4) A1(AO1.1b) [2]	BC allow awrt 1300	if **M0** allow **B2** for awrt 1300 unsupported
		e	p-value = 0.127	M1(AO1.1b) M1(AO1.1b)	from screenshot	

			> 0.05 not significant Insufficient evidence to suggest that the population mean of the number of mobile phones/1000 population has increased	A1(AO2.2b) E1(AO2.2b) [4]	comparison with 0.05 FT	
		f	Sample size of 12 is small compared to the number of countries in the population so inference may be unreliable	B1(AO2.3) [1]	Any sensible comment on the premise that 12 is a relatively small sample size	
			Total	**1**		
5		a	9×22.6 or 11×22.8 or 17×23.7 soi 203.4, 250.8 and 402.9 isw $8 \times 8.9 + 9 \times 22.6^2$ or $10 \times 10.618 + 11 \times 22.8^2$ or $16 \times 16.9925 + 17 \times 23.7^2$ soi 4668.04, 5824.42 and 9820.61 isw	M1(AO3.1a) A1(AO1.1b) M1(AO2.1) A1(AO1.1b) [4]	if **M0** allow **SC1** for use of n instead of $n - 1$ to find all three $\sum x^2$ accept answers to 4 sf or more	

		b	23.16486 to 3 sf or more 12.7(35…) to 3 sf or more	B1(AO1.1b) B1(AO1.1b) [2]	from $\frac{203.4+250.8+402.9}{37}$ from $\frac{4668.04+5824.42+9820.61-37\times 23.1648}{36}$	
		c	Use of $N(26.1, \frac{12.735}{37})$ $26.1 \pm 1.96 \times\sqrt{\frac{12.735}{37}}$ Critical region: mean < 24.95 and mean > 27.25	M1(AO3.3) M1(AO3.1a) A1(AO3.4) [3]	FT their variance of combined sample allow use of $z = 1.645$ for **M1**	
		d	The evidence **suggests** that the mean mass of adult voles has changed since the sample mean is inside the critical (rejection) region	E1(AO2.2b) [1]	or 'outside acceptance region'	
			Total	4		
6			H_0: $\mu = 56$ H_1: $\mu > 56$ μ is the population mean time taken by Statto to complete his walk Use of $N(56, \frac{4^2}{19})$ 0.9459 BC	B1 (AO1.1) B1 (AO2.5) M1 (AO3.3) A1 (AO1.1) M1 (AO3.4)	Both hypotheses	

			$1 - 0.9459 > 0.05$ No evidence to reject H_0 at 5% level oe There is no evidence to suggest at the 5% level that the mean time taken by Statto to complete his walk has increased	A1 (AO1.1) E1 (AO2.2b) [7]	FT their probability	
			Total	7		