1. A random variable has the distribution B(n, p). It is required to test $H_0: p = \frac{2}{3}$ against $H_1: p < \frac{2}{3}$ at a significance level as close to 1% as possible, using a sample of size n = 8, 9 or 10. Use tables to find which value of n gives such a test, stating the critical region for the test and the corresponding significance level.

[4]

2. In a city the proportion of inhabitants from ethnic group Z is known to be 0.4. A sample of 12 employees of a large company in this city is obtained and it is found that 2 of them are from ethnic group Z. A test is carried out, at the 5% significance level, of whether the proportion of employees in this company from ethnic group Z is less than in the city as a whole.

   i. State an assumption that must be made about the sample for a significance test to be valid.

[1]

   ii. Describe briefly an appropriate way of obtaining the sample.

[2]

   iii. Carry out the test.

[7]

   iv. A manager believes that the company discriminates against ethnic group Z. Explain whether carrying out the test at the 10% significance level would be more supportive or less supportive of the manager's belief.

[2]

3. i. State an advantage of using random numbers in selecting samples.

[1]

   ii. It is known that in analysing the digits in large sets of financial records, the probability that the leading digit is 1 is 0.25. A random sample of 18 leading digits from a certain large set of financial records is obtained and it is found that 8 of the leading digits are 1s. Test, at the 5% significance level, whether the probability that the leading digit is 1 in this set of records is greater than 0.25.

[7]

4. 55% of the pupils in a large school are girls. A member of the student council claims that the probability that a girl rather than a boy becomes Head Student is greater than 0.55. As evidence for his claim he says that 6 of the last 8 Head Students have been girls.

i. Use an exact binomial distribution to test the claim at the 10% significance level.

[7]

ii. A statistics teacher says that considering only the last 8 Head Students may not be satisfactory. Explain what needs to be assumed about the data for the test to be valid.

[1]

5. It is known that under the standard treatment for a certain disease, 9.7% of patients with the disease experience side effects within one year. In a trial of a new treatment, 450 patients with this disease were selected and the number, X, that experienced side effects within one year was noted.
It was found that 51 of the 450 patients experienced side effects within one year.

(a) Test, at the 10% significance level, whether the proportion of patients experiencing side effects within [7]
one year is greater under the new treatment than under the standard treatment.

(b) It was later discovered that all 450 patients selected for the trial were treated in the same hospital. [1]
Comment on the validity of the model used in part **(a)**.

6. **(a)** André throws a fair six-sided dice 30 times. The number of throws on which the score is six is denoted by X.

(i) State a suitable model for X, including the values of any parameters. [1]

(ii) Find $P(X = 9)$. [1]

(iii) Find $P(X \geq 9)$. [2]

(b) André has another six-sided dice. He suspects that this dice is biased so that it is more likely to show a six than if it were fair. He throws the dice 30 times and it shows a six on 9 throws. Test at the 5% significance level whether André's suspicion is justified. [6]

7. The acidity of paper is measured on the numerical pH scale. It is known that the writing paper generally used by a certain author has a mean pH of 6.3. The pH, X units, of a random sample of 36 pieces of paper thought to have been used by this author was measured, and the results are summarised as follows.

$$n = 36 \qquad \sum x = 222.48 \qquad \sum x^2 = 1380.5264$$

(i) Test at the 5% significance level whether the pH of the paper from which this sample is drawn differs from 6.3. [11]

(ii) State where the Central Limit Theorem was used in your test in part (i). [1]

8. A random variable X has the distribution B(60, p). A hypothesis test is to be carried out, at the 5% significance level, of the null hypothesis H_0: $p = 0.95$ against the alternative hypothesis H_1: $p > 0.95$.

(i) Explain why a normal approximation cannot be used. [1]

(ii) Verify that the critical region for the test is $X = 60$. [4]

(iii) State the value(s) of p for which a Type I error could occur, and give the corresponding probability or probabilities of a Type I error. [2]

(iv) Find the range of values of p for which the probability that a Type II error occurs is less than 0.6. [4]

9. It is known that 20% of plants of a certain type suffer from a fungal disease, when grown under normal conditions. Some plants of this type are grown using a new method. A random sample of 250 of these plants is chosen, and it is found that 36 suffer from the disease. Test, at the 2% significance level, whether there is evidence that the new method reduces the proportion of plants which suffer from the disease. [7]

10. Briony suspects that a particular 6-sided dice is biased in favour of 2. She plans to throw the dice 35 times and note the number of times that it shows a 2. She will then carry out a test at the 4% significance level. Find the rejection region for the test. [7]

11. Research has shown that drug A is effective in 32% of patients with a certain disease.

In a trial, drug B is given to a random sample of 1000 patients with the disease, and it is found that the drug is effective in 290 of these patients.

Test at the 2.5% significance level whether there is evidence that drug B is effective in a lower proportion of patients than drug A. [7]

END OF QUESTION paper

Question			Answer/Indicative content	Marks	Part marks and guidance	
1			$n = 9$	B1	Stated explicitly	
			CR is ≤ 2	M1A1	2 seen but not ≤: M1A0. Allow "P(≤ 2)"	CR must be stated explicitly for A1
			0.0083	A1	Or more SF. "n = 9, CR ≥ 3", 0.0083 seen: B1M1A0A1 Examiner's Comments Most knew what to do, though many lost marks by failure to spell out the answers for the critical region and the significance level. Some wrongly attempted a right-hand tail.	SR: ≤ 3 with 0.0424: (B1)M1A0 SR: If 0, give B1 for at least 3 of 0.0083, 0.0113, 0.0026, 0.0197, 0.0034 seen
			Total	4		
2		i	Sample is random	B1	Indicate random sample. Allow "unbiased sample" or "randomly selected" or "all equally likely". Allow "representative" provided it's clearly "of company" (not city) Not just "independent". Withhold if extra wrong bits. Examiner's Comments Most realised that the sample had to be a random one. Those who used the word "representative" sometimes indicated, wrongly, that it had to be representative of the city rather than of the company. Some even negated the point of the test by stating that the proportion from group Z had to be the same in the company as in the city.	
		ii	List population, number sequentially	B1	List can be implied; must imply employees or people. "Sequential" can be assumed.	
		ii	Select using random numbers	B1	Not "select numbers randomly", Don't need "ignore outside range" etc. Number randomly *and* select randomly, B1, but "assign random nos &	

					arrange", B2 SC: Put names into hat / lottery machine and take them out: B2 SC: Systematic: B1 for list, can get second B1 if starting-point random **Examiner's Comments** Many candidates do not appreciate the difference between "select numbers randomly" and "select using random numbers". The word "randomly" in the former context is not specific and gives no indication of how the selection is to be done, whereas "random numbers" represents a specific mathematical concept, knowledge of which is required by the specification.	
		iii	H_0: $p = 0.4$; H_1: $p < 0.4$	B2	Both correct, B2. Allow π. One error, e.g, μ or no symbol, B1, but $\bar{x}$, z etc: B0.	
		iii	B(12, 0.4)	M1	B(12, 0.4) stated or implied. Can be implied by N(4.8, 2.88) but no further marks. 0.1673, 0.0398, 0.1513, 0.0421: M1A0(A1M1A1)	
		iii	P(≤ 2) = **0.0834**	A1	P(≤ 2) = 0.0834, or P(≥ 2) = 0.9166.	
		iii	> 0.05	A1	Compare numerical P(≤ 2) with 0.05, or P(> 2) with 0.95	
		iii	CR is ≤ 1	A1	CR is ≤ 1 stated.	
		iii	0.0196 seen and compare 2 with ≤ 1	A1	Explicitly compare 2 with CR, probability 0.0196 must be seen	
		iii	Do not reject H_0.	M1	Correct first conclusion, needs P(≤ 2 | p = 0.4) or fully consistent equivalent	
		iii	Insufficient evidence that proportion of employees from group Z is less.	A1ft	In context (mention "employees", "city" etc), acknowledge uncertainty ("evidence") *Not* "there is evidence that the proportion of employees is 0.4" FT on wrong p-value or wrong critical value if previous mark gained SC: Normal: B2 M1 max SC: P(= 2) or P(≥ 2) or P(< 2): B2 M1 max SC: two–tailed: can get B1B0 M1A1A0 M1A1 (don't give second A1 for 0.05)	

					Examiner's Comments This question was often well done, perhaps because finding the relevant probability for a left-hand tail is easier than for a right-hand tail. Few made the error of finding P(< 2) or P(= 2) as opposed to the correct P(≤ 2). Those who used the critical region method generally did so correctly but this method always needs validation by displaying the relevant probability (here 0.0196). Too many stated the acceptance of H_0 meant that there is significant evidence that the proportion from group Z is 40%. This is wrong. The correct statement is that there is not sufficient evidence that the proportion from group Z is less than 40%.	
		iv	Yes as H_0 is rejected	M1	Realise this changes conclusion (FT!), or "more likely to reject H0", "larger CR" More supportive [just "more supportive" without evidence is M0A0]	
		iv		A1	**Examiner's Comments** This question could be successfully answered either in hypothetical terms ("it is more likely that the null hypothesis is rejected") or in terms of the actual sample ("the null hypothesis is now rejected"). Some thought that the issue was whether causality could be proved – but once again that is answering last year's question and not this one.	
			Total	12		
3		i	Avoids (reduces) bias, or "representative" or "allows calculations to be done" or "allows reliable estimates"	B1	– unbiased (allow "fair") – representative (allow "reliable") – allows use of distribution Both right and wrong: B1 **Examiner's Comments**	*Not:* – all equally likely to be selected – selections independent – quick / easy / cheap – random sample

					This unfamiliar request was in fact answered well. About half the candidates wrote that it avoids (or reduces) bias, or that it ensures a representative sample (this latter statement is not really true but it was given credit). Sophisticated answers seen included 'use of random numbers allows distributions such as the binomial to be used'.	
		ii	B(18, 0.25)	M1	B(18, 0.25) stated or used	
		ii	H_0: p = 0.25, H_1: p > 0.25	B2	One error, B1; x or $\overline{x}$ B0; π: B2	*Any* symbol can get B2 if explicitly defined
		ii	α: P(≥ 8) = 1 − P(≤ 7) = **0.0569**	A1	0.0569 seen	Allow 0.9431 only if "> 0.95" and vice versa.
		ii	> 0.05	A1	Explicit comparison with 0.05	"> 8" (0.0193), "≤ 8" (0.9807) or "= 8" (.0376): max M1B2 [A0A0M0A0], 3/7
		ii	β: CR is ≥ 9, and 8 < 9	A1dep*	Correct CR and explicit comparison	
		ii	probability 0.0193	*A1	0.0193 explicitly seen If more than one probability seen, assume method is β. Note that this requires explicit comparison for either A1; but can get final M1A1	
		ii	Do not reject H_0.	M1	Correct first conclusion, e.g. "reject H_1"	M1 needs correct method, comparison, like-with-like, ≥ 8 (or ≤ 7 but only if used consistently)
		ii	Insufficient evidence that proportion of 1's is greater than 25%.	A1ft	Interpreted, in context, consistent with p, acknowledge uncertainty. FT on wrong CR/p *Not:* "significant evidence that proportion of 1s is 25%"	Allow "change" instead of "increase" **SR**: 2-tail, max M1B1B0A1A0M1A1
		ii			**Examiner's Comments** This was a standard binomial hypothesis test and many scored full marks, although some poor conclusions were seen. As usual, weaker candidates using the probability method attempted to use P(> 8) or P(= 8) as opposed to the correct P(≥ 8). It is not, however, sufficient to write down two probabilities, tell us that one is > 0.05 and one is < 0.05, and then say 'do not reject H_0'; it is not clear whether this is using the critical value method or the probability method. Using the probability method,	

					only one probability can be given; using the critical region, an explicit statement and comparison such as CR is ≥ 9, and 8 < 9 is essential. The final conclusion was usually well stated, though it is incorrect to say that 'there is evidence that the proportion of 1's *is* 25%'. A double negative is required: 'there is insufficient evidence that the proportion of 1's is not 25%'.	
			Total	8		
4		i	H_0: $p = 0.55$, H_1: $p > 0.55$	B2	All correct, B2. One error (e.g. ≠, wrong or no letter) B1, but *r*, *x* etc: B0	
		i	R ~ B(8, 0.55) where *R* is the number of girls	M1	B(8, 0.55) stated or implied, e.g. N(4.4, 1.98)	
		i	α: P($R \geqslant 6$) = 1 − 0.7799 = 0.2201	A1	P(⩾ 6) = 0.2201, or P(< 6) = 0.7799	
		i	> 0.1	B1	Compare P(⩾ 6) with 0.1 or P(< 6) with 0.9	
		i	β: CR is ⩾ 7 and 6 < 7	B1	Correct CR stated and explicit comparison with 6	
		i	$p = 0.0632$	A1	This probability seen, a.r.t. 0.0632. Award if 0.9368 seen and CR is correct. If CR not clearly stated, cannot get last M1A1	
		i	Do not reject H_0. There is insufficient evidence that the girls are proportionately more likely to become	M1	Correct first conclusion, requires B(8, 0.55), not P(> 6) [= 0.0632] or P(⩽ 6) [= 0.9368] or P(= 6) [= 0.1569]. Allow 0.7799 if compared with 0.9 Interpreted, in context, acknowledge uncertainty, double negative. SC: Normal: max B2 M1 SC: Two different attempts: max B2 M1 unless both correct	
		i	Head Student.	A1	**Examiner's Comments** A standard hypothesis test for a binomial parameter. The proportion of candidates who considered the wrong tail, or no tail at all, seemed lower than in the past, which is pleasing. To make the point clearly: with a sample value of 6 (and an expected value of 4.4) the probability that	

					has to be found is P(≥ 6), and not P(≥ 6) or P(= 6). The use of P(< 6), although not wrong, should be discouraged, as comparison with large probabilities is not in the spirit of hypothesis testing. Often those who used the critical value method did not make it clear what the actual critical region was. "Critical value is 7" is not enough; it has to be "critical region is ⩾ 7", and then "6 is not in the CR", or "6 < 7" has to be clearly stated. Candidates who did not state the critical region unambiguously risked losing the last two marks as well as earlier ones. $p = 0.2201 \geq 0.1$, do not reject.	
		ii	Assume that the last 8 years are a random sample of years when Head Student has been chosen	B1	Refer to random sample, allow implied by any method described Must be choosing *years*, not *students* Not quote conditions for random sample unless explicitly "years" Extras: ignore unless clearly wrong, in which case B0 **Examiner's Comments** This verbal question revealed a lot of muddled thinking. The issue is whether the Head Students from the last 8 years can be taken as a representative of all Head Students from the period under discussion, and so the focus has to be on selecting the *years*. Many candidates instead attempted to apply standard binomial conditions to the way in which the Head Student was chosen (elected?) each year ("each Head Student must be chosen independently of the previous Head Student").	
			Total	8		
5		a	$H_0 : p = 0.097$ $H_1 : p > 0.097$ where p is the proportion of patients experiencing side effects within a year $X \sim B(450, 0.097)$ and $X = 50$ $P(X \geq 51) = 1 - 0.862 = 0.138$(3 s.f.)	B1 (AO1.1) B1 (AO2.5) M1 (AO3.3) A1 (AO3.4) A1 (AO1.1) M1 (AO1.1) A1 (AO2.2b)	Must be stated in terms of parameters Undefined p **B1B0** Stated or implied Only 0.138 seen without parameters/	

			Comparison with 0.1 Do not reject H_0 No evidence (at 10% level) that proportion under new treatment greater than under standard treatment	[7]	BC In context, not definite, e.g. Proportion not greater **A0**	distribution **M1AO** FT their 0.138, but not comparison with 0.1
		b	E.g. The patients could be treated together so they are not independent, so the binomial model is not valid. E.g. The 450 patients are not a random sample from the population, so the binomial model is not valid. E.g. It is not known whether the proportion of patients experiencing side effects under the standard treatment is 9.7%, so the binomial model used may not be valid.	B1 (AO3.5a) [1]	In context, referring to independence or random sampling. Must include a comment on appropriateness.	
			Total	8		
6		a	i) $\mathrm{B}(30, \frac{1}{6})$	B1(AO3.3) [1]		
		a	ii) 0.0309	B1(AO1.1) [1]	BC	

		a	iii) $1 - P(X \le 8)$ $= 0.0506$	M1(AO3.4) A1(AO1.1) [2]		
		b	$H_0: p = \frac{1}{6}$ where $p = P(\text{score is } 6)$ $H_1: p > \frac{1}{6}$ $P(X \ge 9) =$ '0.0506' or their **(a)(ii)** comp 0.05 Not reject H_0 No evidence that dice biased towards 6	B1(AO1.1) B1(AO2.5) B1(AO3.4) A1(AO1.1) M1(AO2.2b) A1f(AO3.5a) [6]	Undefined *p*: B0B1 BC ft their **(a)(ii)** ft their **(a)(ii)** In context, not definite	dice is unbiased B0 dice is biased towards 6 B1
			Total	10		
7		i	$\hat{\mu} = \bar{x} = 6.18$ $\hat{\sigma}^2 = \frac{36}{35}\left(\frac{1380.5264}{36} - 6.18^2\right)$ = 0.16 $H_0: \mu = 6.3$, $H_1: \mu \ne 6.3$	B1 M1 M1 A1 B2 M1	6.18 seen somewhere Correct formula for biased estimate Multiply by 36/(36 – 1) 0.1556: M1M0A0. Allow e.g. 5.6/35 One error, B1, but	Single formula: M2 if right, M1 if wrong but with 35 divisor *somewhere*

			α: $z = \dfrac{6.18-6.3}{\sqrt{0.16/36}} = -1.8, p = 0.0359$	A1	x, $\bar{x}$, t: B0 Standardise, 36 needed (if omitted, no more marks in (i)	H_0: $\lambda = 6.3$, H_1: $\lambda \neq 6.3$: B1
				A1	1.8 or –1.8 or a.r.t. 0.0359	u rather than μ: B1B0 if unquestionably u, else BOD
			−1.8 > −1.96 or 0.0359 > 0.025		Compare $-z$ with −1.96 or z with 1.96 or p with 0.025, like-with-like	Allow 0.9641 *only* if compared with 0.95 or 0.975
				M1		Wrong or no notation (e.g. "cdfnorm"): full marks if right, M0A0A0 (M1A1) if numbers wrong in any way
			β: CV $6.3-1.96\sqrt{\dfrac{0.16}{36}} = 6.1693$	A1	$6.3 - z\sqrt{(\sigma^2/36)}$, allow √ errors, cc, ±	
				A1	$z = 1.96$	6.18 + $z\sqrt{(\sigma^2/36)}$: M1 and no further marks in (i)
			6.18 > 6.1693	M1	Compare $\bar{x}$ with 6.17 (or with 6.19 from 1-tail)	6.17 (and no working) can imply mark for 1.96
			Do not reject H_0.		Requires essentially correct method, 36 divisor, like-with-like, hypotheses involving 6.3	SC 1-tail: 6.18 > 6.19, reject H_0, etc: M1A0A1, M1A1
			Insufficient evidence that pH of paper is not 6.3	A1ft 11	Contextualised, acknowledge uncertainty. Allow "insufficient	

					<table><tr><td>evidence that pH hasn't changed"</td><td>Withhold A1 if no context or too assertive, e.g. "evidence that pH of paper is 6.3"</td></tr></table> **Examiner's Comments** A standard normal hypothesis test question. Common mistakes included: not finding an unbiased variance estimate, or the omission of a factor of √36 in standardising. Wrong hypotheses were rarer this year. As usual with tests where the outcome is not to reject H_0, the final conclusion needed to be stated as a double negative, for example "there is insufficient evidence that the mean pH is not 6.3"; it is wrong to say "there is significant evidence that the mean pH is 6.3".	
		ii	In comparing z with z_{crit}	B1 1	<table><tr><td>Or in using 1.96 for CV, etc, or "in assuming that the sample (mean) is normally distributed" (must answer "where?", mustn't leave it vague as to whether it's X or $\bar{X}$)</td><td>No extra answers. "Calculating variance" or "dividing by $\sqrt{n}$" : B0. Not "because the population is not known to be normally distributed". But allow if OK *and also* explained why it can be used.</td></tr></table>	

					Examiner's Comments This question was poorly answered. For a start, many candidates didn't answer this question but their own variation of the question; the question did *not* ask "do you need to use the CLT?", or "can you use the CLT?" but "where is the CLT used?" Answers such as "Yes because we are not told the parent distribution" were disappointingly common. The central limit theorem appeared to be a widely misunderstood topic. It is *not* a statement about dividing the variance by n; that result is true for any distribution. Many candidates seem to believe that the CLT turns any distribution into an approximately normal one; "we are not told that the distribution is normal but the CLT allows us to assume that it is" was a common wrong response. It is important for candidates to realise that the CLT refers to *two different* distributions: the "parent" distribution, and the distribution of the sample mean (not "the sample", either). The statement is: "regardless of the parent distribution, the distribution of the sample mean is approximately normal for a large enough sample". The key words are "sample mean". A correct answer would, therefore, have been, "in assuming that the sample mean is approximately normally distributed".	
			Total	12		
8		i	$nq = 3 < 5$, or p not close to ½ and n not large enough	B1 1	www. 3 and 5 must be seen if inequality used (No need to mention Poisson – ignore any mention) — Withhold if extra wrong statements seen but ignore irrelevant statements (e.g. np). Do *not* allow $npq = 2.85$ Examiner's Comments	

					It was necessary to focus on *nq* and quote its value as 3, which is less than 5 (or to make an equivalent statement about both *n* and *p*: "*p*□ not close to ½" is not enough as it depends also in the value of *n*). The wrong condition *npq* < 5 was quite often used and this did not score the mark.	
		ii	$P(X = 60) = 0.95^{60} = 0.046$ | | < 0.05 | $P(X \geq 59) = 0.046 + 0.1455$ [= 0.19155] | | > 0.05 |	B1dep* dep*B1 B1dep† dep†B1 4	In range [0.0460, 0.0461], *or* [0.9539, 0.954] Correct tail explicitly compared In range [0.191, 0.192], *or* [0.808, 0.809] Correct tail explicitly compared (no final conclusion needed) 0.145 or 0.855 qualifies for these two marks SC: Po(3): $P(X = 60) = 0.0498$ — B1dep* < 0.05, — dep*B1 $P(X \geq 59) = 0.1991$ — B1dep† > 0.05: — dep†B1 **Examiner's Comments** Solutions to this question were often too casual. Many were content to find P(= 60) and state that this was less than 0.05, but it was also necessary to show that is P(≥ 59) is greater than 0.05. Use of the Poisson distribution in this and subsequent parts of the question was possible, though unnecessary.	

		iii	$p = 0.95$ 0.046	B1 B1ft 2	Question requires this to be stated FT on their 0.046 from (ii) *NOT* 0.05. NB: if Po(3) used in (ii), 0.0498 gets B1 **Examiner's Comments** This was found surprisingly hard. Few realised that in order to make a Type I error the value of p had to equal 0.95 and nothing else. Answers such as $p < 0.95$ or $p > 0.95$ were common, but so were answers such as $X = 60$. Candidates seemed to be confused about what p represented. A more understandable common wrong answer to the probability of a Type I error was 0.05.	
		iv	P(≤ 59) < 0.6 $1 - p^{60} < 0.6$ $p^{60} > 0.4$ $0.985 < p \le 1$	M1 A1 A1 A1 4	Can be implied Range required, allow any combination of </≤ Withhold if more than 5 sf seen $p^{60} > 0.6$ or $p^{60} < 0.6$: can get M1 if P(≤ 59) < 0.6 stated explicitly, otherwise 0 SC: T&I or tables: $p > 0.985$ or better, B4, else 0 Allow $p > 0.958$ SC: 60 – R~Po(λ): 1 – P(≤ 0) < 0.6 — M1 $e^{-\lambda} > 0.4$ — A1 ($\lambda < 0.916$)	

					<table><tr><td></td><td><table><tr><td>$60(1 - p) < 0.916$</td><td></td><td>M1, needs M1A1</td></tr><tr><td>$p > 0.985$</td><td>A1</td></tr></table></td></tr></table> **Examiner's Comments** There was a clear-cut division between those candidates who could answer this question confidently and accurately, and those that struggled to make any progress. Some tried to use a normal distribution, and a common wrong solution involved comparing p^{60} with 0.6 instead of 0.4.	
			Total	11		
9			H_0: p = 0.2 where p = P(A plant gets disease) H_1: $p < 0.2$ (not $p \leq 0.2$) $X \sim B(250, 0.2)$ and $X = 36$ (allow 35) $P(X \leq 36) = 0.0139$ or 0.014 $0.0139 < 0.02$ Reject H_0 (Allow Accept H_1)	B1 (AO1.1) B1 (AO2.5) M1 (AO3.3) A1 (AO3.4) A1f (AO1.1) M1	<table><tr><td>Allow "possibility" or "proportion". Not p = %age having disease Undefined p: B1B0 Stated or implied eg by 0.0139 (or 0.00884) cao **BC** NB dep attempt P(X ≤ 36) ft their P(X ≤ 36) (< 0.02)</td><td>**If 2-tail test:** H_0: $p = 0.2$ (defined p) B1 H_1: $p \neq 0.2$ B0 M1 A1 $0.0139 > 0.01$ A1 No more marks</td></tr></table>	

			There is evidence that new method reduces prop of diseased plants	(AO2.2b) A1f (AO3.5a) [7]	Must see this statement NB dep attempt P(X ≤ 36) or P(X < 36) and dep comp 0.02, ft their P(X ≤ 36), possibly not reject H_0 In context, not definite ft only their P(X ≤ 36) or P(X < 36) possibly "no evidence.." Ignore all else P(X < 36): max B1B1M1A0A0M1A1 **<u>Examiner's Comments</u>** Many candidates had clearly been well prepared for a hypothesis test question. However, even these often made errors. Examples of such errors were as follows. • Failure to define "p" in the hypotheses • H1: p ≠ 0.02 • Finding P(X < 36) instead of P(X ≤ 36) • Finding P(X = 36) instead of P(X ≤ 36) • Writing P(X ≤ 0.2) instead of P(X ≤ 36) • Comparing the probability with 0.2 instead of 0.02 • Omitting to state "Reject H0" in the conclusion

					• Giving a definite conclusion such as "Reject Ho. The new method reduces the proportion . . . ." • Stating that because 0.0139 < 0.02, we do not reject H_0 There was a significant minority that could not access this question on Hypothesis Testing.	
			Total	7		
10			$H_0: p = \frac{1}{6}$ $H_0: p > \frac{1}{6}$ where p = P(2 on one throw) B(35, $\frac{1}{6}$) P($X \geq 10$) = 1 – P($X \leq 9$) or P($X \geq 11$) = 1 – P($X \leq 10$) P($X \geq 10$) = 0.055 P($X \geq 11$) = 0.023 (0.04 lies between these hence) rejection region is $X \geq 11$ Allow eg $a \geq 11$ <u>Special case, using N~Bin; Method A</u> $H_0: \mu = \frac{35}{6}$ $H_0: \mu > \frac{35}{6}$ where μ = pop mean no. of 2's	B1 (AO 1.1) B1 (AO 2.5) M1 (AO 3.3) M1 (AO 1.1a) A1 (AO 2.1) A1 (AO 3.4) A1 (AO 2.2a) B1 (AO 1.1) B1 (AO 2.5)	**B1B0** one error eg undefined p or two-tail stated or implied unless clearly using N( ) ≥ 1 of these probabilities stated **BC** **BC** dep ≥ one of above probs seen & correct || or P($X \leq 9$), P($X \leq$ 10) P($X \leq 9$) = 0.945 P($X \leq 10$) = 0.977 (0.96 between these) rej'n region is $X \geq$ 11	

Answer	Marks	Guidance	
$N(\frac{35}{6}, \frac{175}{36})$ or N(5.833, 4.861) soi	M1 (AO 3.3)	**B1B0** one error eg undefined μ or two-tail	
$P(X \geq 10) = 1 - P(X < 9.5)$ or $P(X \geq 11) = 1 - P(X < 10.5)$	M1 (AO 1.1a)	Allow incorrect variance	
$P(X \geq 10) = 0.048$	A0 (AO 2.1)	≥ 1 of these probabilities attempted	$P(X < 9.5)$ or $P(X < 10.5)$
$P(X \geq 11) = 0.017$	A1 (AO 3.4)	**BC**	$P(X < 9.5) = 0.952$
(0.04 lies between these hence)		**BC**	$P(X < 10.5) = 0.983$ (0.96 between these)
rejection region is $X \geq 11$	A1 (AO 2.2a)	dep ≥ one of above probs seen & correct	rej'n region is $X \geq 11$
<u>Special case, using N~Bin; Method B</u>			
$H_0\text{: } \mu = \frac{35}{6}$	B1 (AO 1.1)		
$H_0\text{: } \mu > \frac{35}{6}$ where μ = pop mean no. of 2's	B1 (AO 2.5)		
$N(\frac{35}{6}, \frac{175}{36})$ or N(5.833, 4.861) soi	M1 (AO 3.3)	**B1B0** one error eg undefined μ or two-tail	
$P(X < a) = 0.04$ soi	M1 (AO 1.1a)	Allow incorrect variance	
$\frac{35}{6} + 1.751 \times \sqrt{\frac{175}{36}}$	A1 (AO 2.1)	$z = \Phi^{-1}(0.96)$ (= 1.751) dep $\Phi^{-1}(0.96)$	
= 9.69 or 9.7	A0 (AO 3.4) A1 (AO 2.2a)		

			rejection region is $X \geq 11$	[7]	attempt. May be implied **BC** Examiner's Comments This question was well answered by many candidates. A common error was the omission of a definition of p in the hypotheses. To gain full marks candidates had to use an exact method, using the binomial distribution. This involved finding $P(X \geq 10)$ and $P(X \geq 9)$ or a similar method. Many candidates stated that they were finding, for example, $P(X < 10)$ but their result was actually the value for $P(X \geq 10)$. (Perhaps this is due to insufficient familiarity with the use of the binomial function on the calculator). Some candidates found only one of $P(X \geq 10)$ and $P(X \geq 9)$. These lost a mark. Some gave completely correct probabilities, but gave an incorrect final answer of $X \geq 10$ instead of $X \geq 11$. Some candidates misread the question and attempted actually to carry out a test. Some candidates used the normal approximation to the binomial distribution. (These were able to gain a maximum of six marks out of the seven). Many of these candidates arrived correctly at $a = 9.69$, but then gave an answer of $X \geq 10$.	
			Total	7		
11			H_0: $p = 0.32$ where p = P(drug B is effective) H_1: $p < 0.32$ $X \sim B(1000, 0.32)$ and $X = 290$	B1 (AO 1.1) B1 (AO 2.5) M1 (AO 3.3) A1 (AO 3.4) A1 (AO 1.1) M1 (AO 1.1) A1FT (AO	B1 for one error, eg undefined p, or $\neq$ soi	

			$P(X \leq 290) = 0.0221$ Comp 0.025 Reject H_0 There is evidence that drug B is effective in a lower prop of patients than drug A	2.2b) [7]	BC In context. Not definite. Full statement	
			Total	7		