## Questions

**Q1.**

(a) The discrete random variable $X \sim B(40, 0.27)$

Find $P(X \geq 16)$

**(2)**

Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager suspects that there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

(b) Write down the hypotheses that should be used to test the manager's suspicion.

**(1)**

(c) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager's suspicion. You should state the probability of rejection in each tail, which should be less than 0.05

**(3)**

(d) Find the actual significance level of a test based on your critical region from part (c).

**(1)**

One afternoon the manager observes that 12 of the 20 customers who bought baked beans, bought their beans in single tins.

(e) Comment on the manager's suspicion in the light of this observation.

**(1)**

Later it was discovered that the local scout group visited the supermarket that afternoon to buy food for their camping trip.

(f) Comment on the validity of the model used to obtain the answer to part (e), giving a reason for your answer.

**(1)**

**(Total for question = 9 marks)**

**Q2.**

Naasir is playing a game with two friends. The game is designed to be a game of chance so that the probability of Naasir winning each game is $\frac{1}{3}$

Naasir and his friends play the game 15 times.

(a) Find the probability that Naasir wins

(i) exactly 2 games,
(ii) more than 5 games.

**(3)**

Naasir claims he has a method to help him win more than $\frac{1}{3}$ of the games. To test this claim, the three of them played the game again 32 times and Naasir won 16 of these games.

(b) Stating your hypotheses clearly, test Naasir's claim at the 5% level of significance.

**(4)**

**(Total for question = 7 marks)**

**Q3.**

Past records show that 15% of customers at a shop buy chocolate. The shopkeeper believes that moving the chocolate closer to the till will increase the proportion of customers buying chocolate.

After moving the chocolate closer to the till, a random sample of 30 customers is taken and 8 of them are found to have bought chocolate.

Julie carries out a hypothesis test, at the 5% level of significance, to test the shopkeeper's belief.

Julie's hypothesis test is shown below.

$H_0 : p = 0.15$
$H_1 : p \geq 0.15$
Let $X$ = the number of customers who buy chocolate.
$X \sim B(30, 0.15)$
$P(X = 8) = 0.0420$
$0.0420 < 0.05$ so reject $H_0$
There is sufficient evidence to suggest that the proportion of customers buying chocolate has increased.

(a) Identify the first two errors that Julie has made in her hypothesis test.

**(2)**

(b) Explain whether or not these errors will affect the conclusion of her hypothesis test.

Give a reason for your answer.

**(1)**

(c) Find, using a 5% level of significance, the critical region for a one-tailed test of the shopkeeper's belief. The probability in the tail should be less than 0.05

**(2)**

(d) Find the actual level of significance of this test.

**(1)**

**(Total for question = 6 marks)**

**Q4.**

Afrika works in a call centre.

She assumes that calls are independent and knows, from past experience, that on each sales call

that she makes there is a probability of $\frac{1}{6}$ that it is successful.

Afrika makes 9 sales calls.

(a) Calculate the probability that at least 3 of these sales calls will be successful.

**(2)**

The probability of Afrika making a successful sales call is the same each day.

Afrika makes 9 sales calls on each of 5 different days.

(b) Calculate the probability that at least 3 of the sales calls will be successful on exactly 1 of these days.

**(2)**

Rowan works in the same call centre as Afrika and believes he is a more successful salesperson.

To check Rowan's belief, Afrika monitors the next 35 sales calls Rowan makes and finds that 11 of the sales calls are successful.

(c) Stating your hypotheses clearly test, at the 5% level of significance, whether or not there is evidence to support Rowan's belief.

**(4)**

**(Total for question = 8 marks)**

**Q5.**

A nursery has a sack containing a large number of coloured beads of which 14% are coloured red.

Aliya takes a random sample of 18 beads from the sack to make a bracelet.

(a) State a suitable binomial distribution to model the number of red beads in Aliya's bracelet.

**(1)**

(b) Use this binomial distribution to find the probability that

(i) Aliya has just 1 red bead in her bracelet,
(ii) there are at least 4 red beads in Aliya's bracelet.

**(3)**

(c) Comment on the suitability of a binomial distribution to model this situation.

**(1)**

After several children have used beads from the sack, the nursery teacher decides to test whether or not the proportion of red beads in the sack has changed.
She takes a random sample of 75 beads and finds 4 red beads.

(d) Stating your hypotheses clearly, use a 5% significance level to carry out a suitable test for the teacher.

**(4)**

(e) Find the *p*-value in this case.

**(1)**

**(Total for question = 10 marks)**

## Mark Scheme

**Q1.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | $P(X \geqslant 16) = 1 - P(X \leqslant 15)$ | M1 | 1.1b |
| | $= 1 - 0.949077\ldots$ = awrt **0.0509** | A1 | 1.1b |
| | | (2) | |
| (b) | $H_0 : p = 0.3$ $H_1 : p \neq 0.3$ (Both correct in terms of $p$ or $\pi$) | B1 | 2.5 |
| | | (1) | |
| (c) | $[Y \sim B(20, 0.3)]$ sight of $P(Y \leqslant 2) = 0.0355$ or $P(Y \leqslant 9) = 0.9520$ | M1 | 2.1 |
| | Critical region is $\{Y \leqslant 2\}$ or (o.e.) | A1 | 1.1b |
| | $\{Y \geqslant 10\}$ (o.e.) | A1 | 1.1b |
| | | (3) | |
| (d) | $[0.0355 + (1 - 0.9520)] = 0.0835$ or **8.35%** | B1ft | 1.1b |
| | | (1) | |
| (e) | (Assuming that the 20 customers represent a random sample then) 12 is in the CR so the manager's suspicion is supported | B1ft | 3.2a |
| | | (1) | |
| (f) | e.g. (e) requires the 20 customers to be a random sample or independent and the members of the scout group may invalidate this so binomial distribution would not be valid (and conclusion in (e) is probably not valid) | B1 | 3.5a |
| | | (1) | |
| | | | **(9 marks)** |

| Part | Notes |
|---|---|
| (a) | M1 for dealing with $P(X \geqslant 16)$ – they need to use cumulative prob. function on calc. |
| | A1 awrt 0.0509 (from calculator) |
| (b) | B1 for both hypotheses in terms of $p$ or $\pi$ and $H_1$ must be 2-tail |
| (c) | M1 for correct use of tables to find probability associated with critical value. |
| | 1st A1 for the correct lower limit of the CR. Do not award for $P(Y \leqslant 2)$ |
| | 2nd A1 for the correct upper limit. |
| (d) | B1ft ft on their 0.0355 and (1 – their 0.9520) provided each probability is less than 0.05 |
| (e) | B1ft for a comment that relates 12 to their CR and makes a consistent comment relating this to the manager's suspicion |
| (f) | B1 for a comment that: gives a suitable reason based on lack of independence or the sample not being random so the binomial model is not valid |

**Q2.**

| Qu | Scheme | Marks | AO |
|---|---|---|---|
| (a) | Let $N$ = the number of games Naasir wins $N \sim B(15, \frac{1}{3})$ | M1 | 3.3 |
| (i) | $P(N = 2) = 0.059946\ldots$ **awrt 0.0599** | A1 | 1.1b |
| (ii) | $P(N > 5) = 1 - P(N \leqslant 5) = 0.38162\ldots$ **awrt 0.382** | A1 | 1.1b |
| | | (3) | |
| (b) | $H_0 : p = \frac{1}{3}$ $H_1 : p > \frac{1}{3}$ | B1 | 2.5 |
| | Let $X$ = the number of games Naasir wins $X \sim B(32, \frac{1}{3})$ | M1 | 3.3 |
| | $P(X \geqslant 16) = 1 - P(X \leqslant 15) = 0.03765$ $(< 0.05)$ | A1 | 3.4 |
| | [Significant result so reject $H_0$ (the null model) and conclude:] There is evidence to support Naasir's claim (o.e.) | A1 | 3.5a |
| | | (4) | |
| | | **(7 marks)** | |

| | Notes |
|---|---|
| (a) | M1 for selecting a binomial model with correct $n$ and $p$<br>Award for sight of $B(15, \frac{1}{3})$ (o.e. e.g. in words) or implied by 1 correct answer<br>1st A1 for awrt 0.0599 (from a calculator). Allow 0.05995<br>2nd A1 for awrt 0.382 (from a calculator) |
| (b) | B1 for correctly stating both hypotheses in terms of $p$ or $\pi$<br>Accept $p = 0.\dot{3}$ or any exact equivalent. $H_1 : p \geqslant \frac{1}{3}$ is B0<br>M1 for selecting a suitable model to use for the test.<br>Award for sight of $B(32, \frac{1}{3})$ (o.e. e.g. in words) or implied by 0.03765<br>Can also allow M1 for $P(X \leqslant 15) = 0.962$ or better or $P(X \leqslant 14) = 0.922$ or better<br>1st A1 for use of the model to calculate an appropriate probability using calc.<br>Sight of $P(X \geqslant 16)$ **and** answer awrt 0.0377 |
| **ALT** | **CR** May use CR so award 1st A1 for CR of $X \geqslant 16$ must have seen some probabilities though: 1 of $P(X \leqslant 15) = 0.9623$ or $P(X \leqslant 14) = 0.9224$ or 0.9223<br><br>2nd A1 for conclusion in context that there is support for Naasir's claim<br>Must mention "Naasir" or "his" and "claim" or "method" (o.e.)<br>or e.g. probability of winning a game is $> \frac{1}{3}$ or has increased<br>Dependent on M1 and 1st A1 but can ignore hypotheses but see below<br>If you see $P(X \geqslant 16) = 0.0376$ followed by a correct contextualised conclusion then please award A0A1 |
| **SC** | **Use of 0.3 for $\frac{1}{3}$**<br>If used 0.3 instead of $\frac{1}{3}$ in (a) and score M0A0A0 can condone use of 0.3 in (b)<br>1st A1 ft needs $P(X \geqslant 16) = 0.0138$<br>or CR of $X \geqslant 15$ and sight of 1 of $P(X \geqslant 15) = 0.0327$ or $P(X \geqslant 14) = 0.0694$<br>2nd A1 as before with 0.3 instead $\frac{1}{3}$ (if appropriate) |

**Q3.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **(a)** | The alternative hypothesis should be $H_1: p > 0.15$ | B1 | 2.5 |
| | The calculation of the test statistic should be $P(X \geq 8)$ [= 0.0698] | B1 | 2.3 |
| | | **(2)** | |
| **(b)** | These will affect the conclusion (as the null hypothesis should not be rejected) since $P(X \geq 8)$ [= 0.0698] is greater than 0.05 | B1 | 2.4 |
| | | **(1)** | |
| **(c)** | $P(X \leq 8) = 0.9722\ldots > 0.95$ or $P(X \geq 9) = 0.0277\ldots < 0.05$ | M1 | 2.1 |
| | CR: $\{X \geq 9\}$ | A1 | 1.1b |
| | | **(2)** | |
| **(d)** | awrt **0.0278** | B1ft | 1.1b |
| | | **(1)** | |
| | | | **(6 marks)** |

| Notes | |
|---|---|
| **(a)** | **B1:** Identifying that $\geq$ should be $>$ in the alternative hypothesis<br>**B1:** Identifying that $P(X = 8)$ should be $P(X \geq 8)$<br>Stating $P(X = 8)$ is incorrect on its own is insufficient<br>Check for errors identified and corrected next to the question |
| **(b)** | **B1:** Will affect conclusion **and** correct supporting reason |
| **(c)** | **M1:** For use of tables to find probability associated with critical value [$P(X \leq 8)$ or $P(X \geq 9)$ with B(30, 0.15) (may be implied by either correct probability awrt 0.97 or awrt 0.03) or by the correct CR]<br>**A1:** [30$\geq$]$X \geq 9$ o.e. e.g. $X > 8$<br>Allow '9 or more' or 'CR $\geq$ 9' |
| **(d)** | **B1ft:** awrt 0.0278 (allow awrt 2.78%)<br>or correct ft their one-tailed upper CR from B(30, 0.15) to 3s.f. |

**Q4.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | Let $C$ = the number of successful calls. $C \sim B\left(9, \frac{1}{6}\right)$ | M1 | 3.3 |
| | $P(C \geq 3) = 1 - P(C \leq 2) = 0.1782\ldots$ **awrt 0.178** | A1 | 1.1b |
| | | (2) | |
| (b) | Let $X$ = the number of occasions when at least 3 calls are successful. $P(X = 1) = 5 \times ("0.1782\ldots") \times ("0.8217\ldots")^4$ | M1 | 1.1b |
| | $= 0.4061\ldots$ **awrt 0.406** | A1 | 1.1b |
| | | (2) | |
| (c) | $H_0 : p = \frac{1}{6}$ $H_1 : p > \frac{1}{6}$ | B1 | 2.5 |
| | Let $R$ = the number of successful calls $R \sim B\left(35, \frac{1}{6}\right)$ | M1 | 3.3 |
| | $P(R \geq 11) = 1 - P(R \leq 10) = 0.02\ldots$ | A1 | 3.4 |
| | There is sufficient evidence to support that **Rowan** has more successful sales calls than Afrika. | A1 | 2.2b |
| | | (4) | |
| | | | **(8 marks)** |

| Notes | | |
|---|---|---|
| (a) | **M1:** | For selecting the right model |
| | **A1:** | awrt 0.178 |
| (b) | **M1:** | For $5 \times ("\text{their}(a)") \times ("1 - \text{their}(a)")^4$ |
| | **A1:** | awrt 0.406 |
| (c) | **B1:** | for correctly stating both hypotheses in terms of $p$ or $\pi$ Accept $p = 0.16\dot{}$ |
| | **M1:** | For selecting a suitable model. May be implied by a correct probability or CR |
| | **A1:** | Correct probability statement and answer of 0.02 or better (0.02318…)<br>(CR $R \geq 11$ and either $P(R \leq 9) = 0.9450$ or $P(R \leq 10) = 0.9768$ or $1 - P(R \leq 10) = 0.0232$) |
| | **A1:** | Dependent on M1A1 but can ignore hypotheses. For conclusion in context supporting **Rowan's** belief / **Rowan** is a better sales person |
| | | Do not accept Rowan can reject $H_0$ |

**Q5.**

| Qu | Scheme | Marks | AO |
|---|---|---|---|
| (a) | [$R$ = no. of red beads in Aliya's bracelet] $R \sim B(18, 0.14)$ | B1 (1) | 3.3 |
| (b)(i) | $P(R = 1) = 0.19403\ldots$ awrt **0.194** | B1 | 1.1b |
| (ii) | $P(R \geqslant 4) = 1 - P(R \leqslant 3) = 1 - [0.76184\ldots]$ | M1 | 3.4 |
| | $= 0.2381588\ldots$ awrt **0.238** | A1 (3) | 1.1b |
| (c) | Requires $p = 0.14$ to be constant so need a large number of beads in the sack to ensure that removing 18 beads does not appreciably affect this probability, then it could be suitable. | B1 (1) | 3.5b |
| (d) | $H_0 : p = 0.14$ $H_1 : p \neq 0.14$ | B1 | 2.5 |
| | [$X$ = number of red beads in the sample] $X \sim B(75, 0.14)$ | M1 | 3.3 |
| | $P(X \leqslant 4) = 0.01506\ldots$ or if B(75, 0.14) seen awrt 0.02 | A1 | 3.4 |
| | {0.02 < 0.025 so significant or reject $H_0$ } There is evidence that the proportion of red beads has changed | A1 (4) | 2.2b |
| (e) | $p$-value is $2\times$"0.01506..." = 0.030123... = awrt 0.03 | B1ft (1) | 1.1b |
| | | **(10 marks)** | |

| | Notes |
|---|---|
| (a) | B1 for B(18, 0.14) accept in words e.g. binomial with $n = 18$ and $p = 0.14$ |
| (b)(i) | B1 for awrt 0.194 |
| (ii) | M1 for interpreting "at least 4" Need $1 - P(R \leqslant 3)$ and $1 - p$ [$0 < p < 1$] $P(R = 3) = 0.233..$ OK |
| | A1 for awrt 0.238 |
| (c) | B1 for mention of large number of beads and need for $p = 0.14$ to be constant for it to be suitable. Do NOT accept e.g. "events are independent" |
| (d) | B1 for both hypotheses correct with use of $p$ or $\pi$ |
| | M1 for selecting a suitable model: sight or correct use of B(75, 0.14) May be implied by sight of 0.015 or better or [$P(X > 4)$ =] 0.9849… i.e. 0.985 or better |
| | 1st A1 for use of the correct model awrt 0.015 (accept awrt 0.02 following a correct expression) Allow 1st A1 for awrt 0.985 only if correct comparison with 0.975 is seen. Sight of B(75, 0.14) and $P(X \leqslant 4)$ = awrt 0.02 scores M1A1 |
| | No sight of B(75, 0.14) but sight of awrt 0.015 scores M1(⇒)A1[Condone $P(X = 4)$ =..] |
| | 2nd A1 **(dep on M1A1)** for a correct conclusion in context mentioning "proportion", "red" and "changed" |
| | If there is a statement about $H_0$ or significance it must be compatible. |
| NB | May see CR i.e. $X \leqslant 4$ (mark when prob seen) and $X \geqslant 18$ (prob = 0.01406..) Ignore upper limit |
| | NB for information $P(X = 4) = 0.0104\ldots$ and can only score M1A0A0 if B(75, 0.14) seen |
| (e) | B1ft for awrt 0.03 Allow ft of their probability in (d) provided at least 3sf used |
| | NB an answer of 0.02 in (d) leading to 0.04 in (e) is B0 |
| SC | Use of CR will give significance level of 0.01506…+ 0.01406… = 0.029… score B1 **no ft** |