Hypothesis Testing Cheat Sheet

Edexcel Stats/Mech Year 1

Hypothesis testing

A hypothesis is a statement made about the value of a population parameter. It can be tested by carrying out an experiment or taking a sample from the population. The statistic calculated from the sample is called the test statistic.

The null hypothesis (H_0) is the hypothesis assumed to be correct. This is rejected if the test statistics is lower than a given threshold, called the significance level.

The alternative hypothesis (H_1) tells us about the parameter if your assumption is shown to be wrong.

Example 1: John wants to see if a coin is unbiased or biased towards coming down heads. He tosses the coin 8 times and counts the number of heads, X, obtained in 8 tosses.

a. Describe the test statistic.
The test statistic is X, the number of heads obtained in 8 tosses.

b. Write down a suitable null hypothesis.
The probability of landing heads for an unbiased coin is 0.5 so
$H_0: p = 0.5$

c. Write down a suitable alternative hypothesis.
The probability for heads is greater than 0.5 if the coin is biased towards heads so:
$H_1: p > 0.5$

Finding critical values

A critical region is a region of the probability distribution which, if the test statistic falls within it, would cause you to reject the null hypothesis. The critical value is the first value to fall inside of the critical region.

The actual significance level of a hypothesis test is the probability of incorrectly rejecting the null hypothesis.

Example 2: A single observation is taken from the binomial distribution $B(6, p)$. The observation is used to test $H_0: p = 0.35$ against $H_1: p > 0.35$

a. Using a 5% significance level, find the critical region for this test.
Assume H_0 is true then $X \sim B(6, 0.35)$
$P(X \geq 4) = 1 - P(X \leq 3)$
$= 1 - 0.8826$
$= 0.1174$
$P(X \geq 5) = 1 - P(X \leq 4)$
$= 1 - 0.9777$
$= 0.0223$
The critical region is 5 or 6.

> You can use the cumulative binomial tables or your calculator

b. State the actual significance level of this test.
P(reject null hypothesis)= $P(X \geq 5)$
$= 0.0223$
$= 2.23\%$

> This is the same as the probability of X falling within the critical region

One-tailed test

A one-tailed test can be used to test if the probability has increased or decreased.

For one-tailed tests,

$$H_1: p > \cdots \text{ or } p < \cdots$$

Example 3: The standard treatment for a particular disease has a $\frac{2}{5}$ probability of success. A researcher has produced a new drug which has been successful with 11 out of 20 patients. He claims that the new drug is more effective than the standard treatment. Test, at 5% significance level, the claim made by the researcher.

1. Define your test statistic, X and parameter, p.
X is the number of patients in the trial for whom the drug was successful.
p is the probability of success for each patient.

2. Formulate a model for the test statistic.
$X \sim B(20, p)$

3. Identify your null and alternative hypotheses.
$H_0: p = 0.4$
$H_1: p > 0.4$

> The researcher claims that the new drug is better so $p > 0.4$

4. Method 1:
Assume H_0 is true and calculate the probability of 11 or more successful treatments
$X \sim B(20, 0.4)$
$P(X \geq 11) = 1 - P(X \leq 10)$
$= 1 - 0.8725$
$= 0.1275$
$= 12.75\%$

5. Compare probability with significance level.
$12.75\% > 5\%$ so, there is not enough evidence to reject H_0

6. Write a conclusion in context.
The new drug is no better than the old one.

OR

Method 2:

1. Work out the critical region and see if 11 lies within it.
$P(X \geq 13) = 1 - P(X \leq 12)$
$= 0.021$
$P(X \geq 12) = 1 - P(X \leq 11)$
$= 0.0565$
The critical region is 13 or more. Since 11 is not in the critical region, we accept H_0.

2. Write a conclusion in context of the question.
There is no evidence that the new drug is better than the old one.

Two-tailed Test

A two-tailed test is used to test if the probability is changed in either direction. The critical region is split at either end of distribution. The significance level at each end is halved.

For two-tailed tests,

$$H_1: p \neq \cdots$$

Example 4: In Enrico's restaurant, the ratio of non-vegetarian to vegetarian meals is found to be 2 to 1. In Manuel's restaurant in a random sample of 10 people ordering meals, 1 ordered a vegetarian meal. Using a 5% significance level, test whether the proportion of people eating vegetarian meals in Manuel's restaurant is different from Enrico's restaurant.

1. The proportion of people eating vegetarian meals at Enrico's is $\frac{1}{3}$.

2. X is the number of people in the sample at Manuel's restaurant who ordered vegetarian meals.
p is the probability that a randomly chosen person at Manuel's orders a vegetarian meal.

3. $H_0: p = \frac{1}{3}$, $H_1: p \neq \frac{1}{3}$

If H_0 is true, $X \sim B\left(10, \frac{1}{3}\right)$

4. Method 1:
$P(X \leq 1) = P(X = 0) + P(X = 1)$
$= \left(\frac{2}{3}\right)^{10} + 10\left(\frac{2}{3}\right)^{9}\left(\frac{1}{3}\right)$
$= 0.01734\ldots + 0.08670\ldots$
$= 0.104$ (3s.f.)

0.104 > 0.025 so insufficient evidence to reject H_0.

Method 2:
Let c_1 and c_2 be the two critical values.
$P(X \leq c_1) \leq 0.025$ and $P(X \geq c_2) \leq 0.025$

For lower tail:
$P(X \leq 0) = 0.017341\ldots < 0.025$
$P(X \leq 1) = 0.10404\ldots > 0.025$
So $c_1 = 0$

For upper tail:
$P(X \geq 6) = 1 - P(X \leq 5)$
$= 0.07656\ldots > 0.025$
$P(X \geq 7) = 1 - P(X \leq 6)$
$= 0.01966\ldots < 0.025$
So $c_2 = 7$

Observed value of 1 is not in critical region so H_0 is not rejected.

5. Conclusion: There is no evidence that proportion of vegetarian meals at Manuel's restaurant is different to Enrico's.