1. A car manufacturer claims that, on a motorway, the mean number of miles per gallon for the Panther car is more than 70. To test this claim a car magazine measures the number of miles per gallon, x, of each of a random sample of 20 Panther cars and obtained the following statistics.

$$\bar{x} = 71.2 \qquad s = 3.4$$

The number of miles per gallon may be assumed to be normally distributed.

(a) Stating your hypotheses clearly and using a 5% level of significance, test the manufacturer's claim.

(5)

The standard deviation of the number of miles per gallon for the Tiger car is 4.

(b) Stating your hypotheses clearly, test, at the 5% level of significance, whether or not there is evidence that the variance of the number of miles per gallon for the Panther car is different from that of the Tiger car.

(6)
(Total 11 marks)

2. A company manufactures bolts with a mean diameter of 5 mm. The company wishes to check that the diameter of the bolts has not decreased. A random sample of 10 bolts is taken and the diameters, x mm, of the bolts are measured. The results are summarised below.

$$\sum x = 49.1 \qquad \sum x^2 = 241.2$$

Using a 1% level of significance, test whether or not the mean diameter of the bolts is less than 5 mm.

(You may assume that the diameter of the bolts follows a normal distribution.)

(Total 8 marks)

3. A butter packing machine cuts butter into blocks. The weight of a block of butter is normally distributed with a mean weight of 250 g and a standard deviation of 4 g. A random sample of 15 blocks is taken to monitor any change in the mean weight of the blocks of butter.

(a) Find the critical region of a suitable test using a 2% level of significance.

(3)

(b) Assuming the mean weight of a block of butter has increased to 254 g, find the probability of a Type II error.

(5)
(Total 8 marks)

4. Historical records from a large colony of squirrels show that the weight of squirrels is normally distributed with a mean of 1012 g. Following a change in the diet of the squirrels, a biologist is interested in whether or not the mean weight has changed. A random sample of 14 squirrels is weighed and their weights *x*, in grams, recorded. The results are summarised as follows:

$$\sum x = 13\,700 \quad \sum x^2 = 13\,448\,750.$$

Stating your hypotheses clearly test, at the 5% level of significance, whether or not there has been a change in the mean weight of the squirrels.

(Total 7 marks)

5. A machine is set to fill bags with flour such that the mean weight is 1010 grams.

To check that the machine is working properly, a random sample of 8 bags is selected. The weight of flour, in grams, in each bag is as follows.

1010 1015 1005 1000 998 1008 1012 1007

Carry out a suitable test, at the 5% significance level, to test whether or not the mean weight of flour in the bags is less than 1010 grams. (You may assume that the weight of flour delivered by the machine is normally distributed.)

(Total 8 marks)

6. A random sample of 10 mustard plants had the following heights, in mm, after 4 days growth.

5.0, 4.5, 4.8, 5.2, 4.3, 5.1, 5.2, 4.9, 5.1, 5.0

Those grown previously had a mean height of 5.1 mm after 4 days. Using a 2.5% significance level, test whether or not the mean height of these plants is less than that of those grown previously.

(You may assume that the height of mustard plants after 4 days follows a normal distribution.)

(Total 9 marks)

1. (a) H_0: $\mu = 70$ [accept ≤ 70], H_1: $\mu > 70$ B1

$t = \dfrac{71.2-70}{3.4/\sqrt{20}} = 1.58$ M1 A1

critical value $t_{19}(5\%) = 1.729$ B1

not significant, insufficient evidence to confirm manufacturer's claim A1 ft 5

Note

B1 both hypotheses using μ

M1 $\dfrac{71.2-70}{3.4/\sqrt{20}}$

A1 awrt 1.73

A1 correct conclusion ft their t value and CV

(b) H_0: $\sigma^2 = 16$, H_1: $\sigma^2 \neq 16$ B1

test statistic $\dfrac{(n-1)s^2}{\sigma^2}$ =, $\dfrac{219.64}{16}$ = 13.7.. M1 A1

critical values $\chi^2_{19}(5\%)\text{ upper tail} = 32.852$, $\chi^2_{19}(5\%)\text{ lower tail} = 8.907$ not significant B1 B1

Insufficient evidence to suggest that the variance of the miles per gallon of the panther is different from that of the Tiger. A1ft 6

Note

B1 both hypotheses and 16. accept σ = 4 and σ ≠ 4

M1 $\dfrac{(19)\times 3.4^2}{16}$ allow $\dfrac{(19)\times 3.4^2}{4}$

A1 awrt 13.7

B1 32.852

B1 8.907

A1 correct contextual comment

NB those who use $\sigma^2 = 4$ throughout can get B0 M1 A0B1 B1 A1

[11]

2.	$H_0: \mu = 5; H_1: \mu < 5$	both	B1
	CR: $t_9(0.01) > 2.821$		B1
	$\bar{x} = 4.91$		B1
	$s^2 = \frac{1}{9}\left(241.2 - \frac{49.1^2}{10}\right) = 0.0132222$	s= awrt 0.115	M1 A1
	$\frac{\lvert 4.91 - 5 \rvert}{\frac{\sqrt{0.013222}}{\sqrt{10}}} = \pm 2.475$	2.47 – 2.48	M1 A1
	Since 2.475 is not in the critical region there is insufficient evidence to reject H_0 and conclude that the mean diameter of the bolts is not less than (not equal to) 5mm.		A1ft

[8]

3.	(a)	$\frac{\bar{X} - 250}{\frac{4}{\sqrt{5}}} > 2.3263 \text{ or } \frac{\bar{X} - 250}{\frac{4}{\sqrt{5}}} < -2.3263$	±	B1	
		2.3262		M1	
		$\bar{X} > 252.40... \text{ or } \bar{X} < 247.6...$ 252 and 248	awrt	A1	3
	(b)	$P(\bar{X} < 252.4 / \mu = 254) - P(\bar{X} < 247.6 / \mu = 254)$	using their '252.4' and '247.6'	M1	
		$= P\left(Z < \frac{252.4 - 254}{\frac{4}{\sqrt{15}}}\right) - P\left(Z < \frac{247.6 - 254}{\frac{4}{\sqrt{5}}}\right)$	stand using 4√15, 254 their '252.4' or '247.6'	M1	
		$= P(Z < -1.5492) - P(Z < -6.20)$	–1.5492 and –6.20 o.e.	A1	
		$= (1 - 0.9394) - (1 - 1)$		M1	
		$= 0.0606$		A1	5

only need to see one of the standardisation for second M1
if consider only 252.4 and get 0.0606 they get M0M1A0M1A1
ie they can get 3/5

[8]

4. $H_0: \mu = 1012$ $H_1: \mu \neq 1012$ both B1

$\bar{x} = \frac{1317}{14}$ (= 978.57...) M1

$S_x{}^2 = \frac{13448750 - 14\bar{x}^2}{13}$ (= 3255.49) (S_x = 57.056) S^2 or S M1

$t_{13} = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{978.6 - 1012}{57.06/\sqrt{14}} = -2.19...$ awrt –2.19 A1

$t_{13}(57_0)$ 2tail c.v. = –2.160 (±) B1

significant result – there is evidence of a change in mean <u>weight</u> of squirrels

(condone <u>decrease</u>) must mention <u>weight</u> A1ft

[7]

5. Let x represent weight of flour

$\sum = 8055 \therefore \bar{x} = \underline{1006.875}$ M1

Awrt 1006.9

$\sum x^2 = 8110611 \therefore s^2 = \frac{1}{7}\left\{8110611 - \frac{8055^2}{8}\right\} = 33.26785$ M1

$\therefore s = 5.767828\ldots$

awrt 33.7
or awrt 5.77
Allow from calculator

$H_0: \mu = 1010$; $H_1: \mu < 1010$ both B1

CV: $|t| = 1.895$ B1

$t = \frac{|1006.875 - 1010|}{5.767828.../\sqrt{8}} = \pm 1.5324$

Use of $\frac{\bar{x} - \mu}{s/\sqrt{n}}$ M1

awrt – 1.53 A1

Since – 1.53 is not in the critical region ($t < -1.895$)
there is insufficient evidence to reject H_0 and thus the mean
weight of flow delivered by the machine is 10log. A1ft 8

6.	H_0: $\mu = 5.1$, H_1: $\mu < 5.1$	both	B1	
	$v = 9$	9	B1	
	Critical Region $t < -2.262$		B1	
	$\bar{x} = 4.91$	4.91	B1	
	$s^2 = \dfrac{241.89 - 10 \times (4.91)^2}{9} = 0.0899$		M1	
	$s = 0.300$	awrt 0.0899 or 0.300	A1	
	$t = \dfrac{4.91 - 5.1}{\frac{0.3}{\sqrt{10}}} = -2.00$		M1 A1	
	There is no evidence to suggest that the mean height is less than those grown previously	context	A1∫	9

[9]

1. The most able candidates gained full marks for this question. In part (b) a minority of candidates tested whether the variances of the two cars were equal rather than testing whether the variance of the Panther car was equal to 16.

2. This proved to be a good starter question and most candidates gave good solutions. A minority of candidates did not state a conclusion in context.

3. This question proved to be challenging for many candidates. A common mistake was to use the *t* value of 2.624 in part (a) rather than the z value 2.3262. The candidates must learn when to use each test. In part (b) few candidates used both of their values found in part (a) and many did not use $\frac{\sigma}{\sqrt{15}}$.

4. This proved to be a good starter and most candidates gave good solutions. Some failed to express the hypotheses in terms of μ and 1012 and a few did not interpret their conclusion in terms of the mean weight of the squirrels.

5. Many candidates gained full marks for this question. The usual reasons for not doing so were poor arithmetic and not giving the conclusion in context.

6. Candidates had prepared well and solutions given for this question were very good. Hypotheses were stated accurately and the formula was applied well, with clear conclusions that gained full marks.