1. A random sample of 15 strawberries is taken from a large field and the weight x grams of each strawberry is recorded. The results are summarised below.

$$\sum x = 291 \qquad \sum x^2 = 5968$$

Assume that the weights of strawberries are normally distributed.
Calculate a 95% confidence interval for

(a) (i) the mean of the weights of the strawberries in the field,

(ii) the variance of the weights of the strawberries in the field.

(12)

Strawberries weighing more than 23 g are considered to be less tasty.

(b) Use appropriate confidence limits from part (a) to find the highest estimate of the proportion of strawberries that are considered to be less tasty.

(4)
(Total 16 marks)

2. A doctor wishes to study the level of blood glucose in males. The level of blood glucose is normally distributed. The doctor measured the blood glucose of 10 randomly selected male students from a school. The results, in mmol/litre, are given below.

4.7 3.6 3.8 4.7 4.1 2.2 3.6 4.0 4.4 5.0

(a) Calculate a 95% confidence interval for the mean.

(7)

(b) Calculate a 95% confidence interval for the variance.

(4)

A blood glucose reading of more than 7 mmol/litre is counted as high.

(c) Use appropriate confidence limits from parts (a) and (b) to find the highest estimate of the proportion of male students in the school with a high blood glucose level.

(4)
(Total 15 marks)

3. Brickland and Goodbrick are two manufacturers of bricks. The lengths of the bricks produced by each manufacturer can be assumed to be normally distributed. A random sample of 20 bricks is taken from Brickland and the length, x mm, of each brick is recorded. The mean of this sample is 207.1 mm and the variance is 3.2 mm^2.

(a) Calculate the 98% confidence interval for the mean length of brick from Brickland.

(4)

A random sample of 10 bricks is selected from those manufactured by Goodbrick. The length of each brick, y mm, is recorded. The results are summarised as follows.

$$\sum y = 2046.2 \qquad \sum y^2 = 418\,785.4$$

The variances of the length of brick for each manufacturer are assumed to be the same.

(b) Find a 90% confidence interval for the value by which the mean length of brick made by Brickland exceeds the mean length of brick made by Goodbrick.

(8)

(Total 12 marks)

1. (a) $\bar{x} = \frac{291}{15} = 19.4$ $s = \sqrt{\frac{5968 - 15\bar{x}^2}{14}} = 4.800$ M1 M1

(i) $t_{14} = 2.145$ B1

95% CI $= 19.4 \pm 2.145 \times \frac{4.800}{\sqrt{15}}$ M1 A1ft

$= (16.7, 22.1)$ A1 A1

Note

M1 $\frac{291}{15}$

M1 $\sqrt{\frac{5968 - 15\bar{x}^2}{14}}$

B1 2.145

M1 (19.4) ± t × $\frac{\text{"}\textit{their}\text{ s"}}{\sqrt{15}}$

A1ft 19.4 ± 2.145 × $\frac{\text{"}\textit{their}\text{ s"}}{\sqrt{15}}$

A1 awrt 16.7
A1 awrt 22.1

(ii) 95% CI is given by

$\frac{14 \times 4.800^2}{26.119} < \sigma^2 < \frac{14 \times 4.800^2}{5.629}$ M1B1B1

(12.4, 57.3) accept 12.3 A1 A1 12

Note

M1 $\frac{14 \times s^2}{\chi^2}$
B1 26.119
B1 5.629
A1 awrt 12.4/12.3
A1 awrt 57.3

(b) Require P(X > 23) = P$\left(Z > \frac{23-\mu}{\sigma}\right)$ to be as large as possible

OR $\frac{23-\mu}{\sigma}$ to be as small as possible; both imply highest σ and

μ. $\frac{23-22.1}{\sqrt{57.3..}}$ = 0.124 M1 M1

P(Z > 0.124) = 1 – 0.5478 M1

= 0.4522 A1 4

Note

M1 use of highest mean and sigma
M1 standardising using values of mean and sigma from intervals
M1 finding 1 – P(z > their value)
A1 awrt 0.45

[16]

2. $\bar{x} = 4.01$ B1
s = 0.7992... M1A1

(a) $4.01 \pm t_9(2.5\%)\frac{0.7992...}{\sqrt{10}} = 4.01 \pm 2.262\frac{0.7992...}{\sqrt{10}}$ 2.262 B1

their $\bar{x}$ and s and $\sqrt{10}$ M1A1ft

= 4.5816... and 3.4383... awrt 4.58 and 3.44 A1 7

s^2 = 0.63877...

(b)	$2.700 < \dfrac{9 \times 0.7992..^2}{s^2} < 19.023$	2.7, 19.023	B1B1	
		$9 \times s^2/\sigma^2$	M1	
	$\sigma^2 < 2.13$, $\sigma^2 > 0.302$	both awrt 2.13, 0.302	A1	4
(c)	$P(X > 7) = P\left(Z > \dfrac{7-\mu}{\sigma}\right)$ needs to be as high as possible		M1	
	Therefore μ and σ must be as big as possible		M1	
	$= P\left(Z > \dfrac{7-4.581}{\sqrt{2.13}}\right)$		A1ft	
	$= 1 - 0.9515$			
	$= 0.0485$			
	$= 4.85\%$	4.8 to 4.9	A1	4
	M1 may be implied by them using their highest μ and σ.			

[15]

3. (a)	Confidence interval is given by			
	$\bar{x} \pm t_{19} \times s/\sqrt{n}$		B1	
	2.539			
	i.e.:- $207.1 \pm 2.539 \times \sqrt{\dfrac{3.2}{20}}$		M1	
	Use of $\bar{x} \pm t \times s/\sqrt{n}$			
	i.e.:- 207.1 ± 1.0156		A1	
	All correct			
	<u>i.e.- (206.08…, 208.1156)</u>		A1	4
	AWRT (206, 208)			

(b) $s_p^2 = \dfrac{19 \times 3.2 + 9 \times 10.217\dot{3}}{28}$

$= 5.45557$ A1

AWRT 5.46

Confidence interval is given by

$\bar{x}_B - \bar{x}_G \pm t_{28} \times \sqrt{5.45557\left(\frac{1}{20} + \frac{1}{10}\right)}$ B1

1.701

i.e.:- $(207.1 - 204.62) \pm 1.701\sqrt{5.45557\left(\frac{1}{20} + \frac{1}{10}\right)}$

i.e.:- 2.48 ± 1.53875 M1

Use of $\bar{x} - \bar{y} \pm t\sqrt{s^2\left(\frac{1}{n_x} + \frac{1}{n_y}\right)}$

i.e.:- <u>(0.94125, 4.0187)</u> A1

All correct

AWRT 0.941; 4.02 A1; A1 8

[12]

1. The most able candidates gained full marks for this question. The most common error was in part (a) when they used $\frac{23.04}{\sqrt{15}}$ rather than $\sqrt{\frac{23.04}{15}}$.

Part (b) was well answered. The main error was to use 4.800 instead of 23.04.

In part (c) many candidates knew that they needed to use the highest values from parts (a) and (b) but then either did not square root the "57.3" or used $\sqrt{\frac{\text{"57.3"}}{15}}$ when finding z.

2. Parts (a) and (b) of this question were well answered with many fully correct solutions being given. Part (c) proved to be more difficult with a lot of candidates not being able to start this part of the question. Of those who did realise that μ and σ must be as big as possible a large percentage forgot to use the square root of their answer in part (b).

3. Apart from those candidates that used the Normal distribution rather than the t-distribution part (a) was often correct. Surprisingly few candidates gained full marks in part(b). Common errors were poor arithmetic when calculating the pooled estimate of the variance; using the wrong t-value; using the wrong formula for the confidence interval by dividing the t-value by the standard error.