OCR

Monday 16 June 2014 – Morning

A2 GCE MATHEMATICS

4735/01 Probability & Statistics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4735/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A teacher believes that the calculator paper in a GCSE Mathematics examination was easier than the non-calculator paper. The marks of a random sample of ten students are shown in the table.

Student	A	B	C	D	E	F	G	H	I	J
Mark on paper 1 (non-calculator)	66	79	58	87	67	55	75	62	50	84
Mark on paper 2 (calculator)	57	84	70	90	75	42	82	72	65	82

(i) Use a Wilcoxon signed-rank test, at the 5% significance level, to test the teacher's belief. **[7]**

(ii) State the assumption necessary for this test to be applied. **[1]**

2 During an outbreak of a disease, it is known that 68% of people do not have the disease. Of people with the disease, 96% react positively to a test for diagnosing it, as do m% of people who do not have the disease.

(i) In the case $m = 8$, find the probability that a randomly chosen person has the disease, given that the person reacts positively to the test. **[5]**

(ii) What value of m would be required for the answer to part **(i)** to be 0.95? **[4]**

3 The discrete random variable X has probability generating function $\frac{t}{a-bt}$, where a and b are constants.

(i) Find a relationship between a and b. **[1]**

(ii) Use the probability generating function to find E(X) in terms of a, giving your answer as simply as possible. **[3]**

(iii) Expand the probability generating function as a power series, as far as the term in t^3, giving the coefficients in terms of a and b. **[3]**

(iv) Name the distribution for which $\frac{t}{a-bt}$ is the probability generating function, and state its parameter(s) in terms of a. **[2]**

4 The continuous random variable X has probability density function

$$\mathrm{f}(x) = \begin{cases} x & 0 \leqslant x \leqslant 1, \\ 2-x & 1 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that the moment generating function of X is $\frac{(\mathrm{e}^t-1)^2}{t^2}$. **[6]**

Y_1 and Y_2 are independent observations of a random variable Y. The moment generating function of $Y_1 + Y_2$ is $\frac{(\mathrm{e}^t-1)^2}{t^2}$.

(ii) Write down the moment generating function of Y. **[1]**

(iii) Use the expansion of e^t to find Var(Y). **[5]**

(iv) Deduce the value of Var(X). **[1]**

5 Two discrete random variables X and Y have a joint probability distribution defined by

$$\mathrm{P}(X = x, Y = y) = a(x + y + 1) \qquad \text{for } x = 0, 1, 2 \text{ and } y = 0, 1, 2,$$

where a is a constant.

(i) Show that $a = \frac{1}{27}$. **[2]**

(ii) Find $\mathrm{E}(X)$. **[2]**

(iii) Find $\mathrm{Cov}(X, Y)$. **[5]**

(iv) Are X and Y independent? Give a reason for your answer. **[2]**

(v) Find $\mathrm{P}(X = 1 \,|\, Y = 2)$. **[2]**

6 A Wilcoxon rank-sum test with samples of sizes 11 and 12 is carried out.

(i) What is the least possible value of the test statistic W? **[2]**

(ii) The null hypothesis is that the two samples came from identical populations. Given that the null hypothesis was rejected at the 1% level using a 2-tail test, find the set of possible values of W. **[6]**

7 The continuous random variable X has probability density function

$$\mathrm{f}(x) = \begin{cases} \dfrac{k}{(x+\theta)^5} & \text{for } x \geqslant 0, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a positive constant and θ is a parameter taking positive values.

(i) Find an expression for k in terms of θ. **[2]**

(ii) Show that $\mathrm{E}(X) = \frac{1}{3}\theta$. **[3]**

You are given that $\mathrm{Var}(X) = \frac{2}{9}\theta^2$. A random sample $X_1, X_2, \ldots, X_n$ of n observations of X is obtained. The estimator T_1 is defined as $T_1 = \dfrac{3}{n}\sum_{i=1}^{n} X_i$.

(iii) Show that T_1 is an unbiased estimator of θ, and find the variance of T_1. **[3]**

(iv) A second unbiased estimator T_2 is defined by $T_2 = \frac{1}{3}(X_1 + 3X_2 + 5X_3)$. For the case $n = 3$, which of T_1 and T_2 is more efficient? **[4]**

END OF QUESTION PAPER