

**Monday 24 June 2013 – Afternoon**

**A2 GCE MATHEMATICS**

**4735/01** Probability & Statistics 4

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4735/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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1

		$S$		
		0	1	2
$F$	0	$\frac{1}{8}$	$\frac{1}{8}$	0
	1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
	2	0	$\frac{1}{8}$	$\frac{1}{8}$

An unbiased coin is tossed three times. The random variables  $F$  and  $S$  denote the total number of heads that occur in the first two tosses and the total number of heads that occur in the last two tosses respectively. The table above shows the joint probability distribution of  $F$  and  $S$ .

(i) Show how the entry  $\frac{1}{4}$  in the table is obtained. [2]

(ii) Find  $\text{Cov}(F, S)$ . [6]

- 2 Two drugs, I and II, for alleviating hay fever are trialled in a hospital on each of 12 volunteer patients. Each received drug I on one day and drug II on a different day. After receiving a drug, the number of times each patient sneezed over a period of one hour was noted. The results are given in the table.

Patient	1	2	3	4	5	6	7	8	9	10	11	12
Drug I	11	34	19	16	10	29	6	17	20	13	4	25
Drug II	12	20	10	18	3	21	9	13	10	19	9	12

The patients may be considered to be a random sample of all hay fever sufferers. A researcher believes that patients taking drug II sneeze less than patients taking drug I.

Test this belief using the Wilcoxon signed rank test at the 5% significance level. [7]

- 3 The continuous random variable  $X$  has probability density function given by

$$f(x) = \begin{cases} \frac{1}{4}xe^{-\frac{1}{2}x} & x \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that the moment generating function of  $X$  is  $(1 - 2t)^{-2}$  for  $t < \frac{1}{2}$ , and state why the condition  $t < \frac{1}{2}$  is necessary. [6]

(ii) Use the moment generating function to find  $\text{Var}(X)$ . [3]

- 4 The effect of water salinity on the growth of a type of grass was studied by a biologist. A random sample of 22 seedlings was divided into two groups  $A$  and  $B$ , each of size 11. Group  $A$  was treated with water of 0% salinity and group  $B$  was treated with water of 0.5% salinity. After three weeks the height (in cm) of each seedling was measured with the following results, which are ordered for convenience.

Group $A$	8.6	9.4	9.7	9.8	10.1	10.5	11.0	11.2	11.8	12.7	12.9
Group $B$	7.4	8.4	8.5	8.8	9.2	9.3	9.5	9.9	10.0	11.1	11.3

Jeffery was asked to test whether the two treatments resulted, on average, in a difference in growth. He chose the Wilcoxon rank sum test.

(i) Justify Jeffery's choice of test. [1]

(ii) Carry out the test at the 5% significance level. [9]

- 5 The discrete random variable  $U$  has probability distribution given by

$$P(U = r) = \begin{cases} \frac{1}{16} \binom{4}{r} & r = 0, 1, 2, 3, 4, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find and simplify the probability generating function (pgf) of  $U$ . [3]

(ii) Use the pgf to find  $E(U)$  and  $\text{Var}(U)$ . [4]

(iii) Identify the distribution of  $U$ , giving the values of any parameters. [2]

(iv) Obtain the pgf of  $Y$ , where  $Y = U^2$ . [2]

(v) State, giving a reason, whether you can obtain the pgf of  $U + Y$  by multiplying the pgf of  $U$  by the pgf of  $Y$ . [1]

- 6 The continuous random variable  $X$  has mean  $\mu$  and variance  $\sigma^2$ , and the independent continuous random variable  $Y$  has mean  $2\mu$  and variance  $3\sigma^2$ . Two observations of  $X$  and three observations of  $Y$  are taken and are denoted by  $X_1, X_2, Y_1, Y_2$  and  $Y_3$  respectively.

(i) Find the expectation of the sum of these 5 observations and hence construct an unbiased estimator,  $T_1$ , of  $\mu$ . [3]

(ii) The estimator  $T_2$ , where  $T_2 = X_1 + X_2 + c(Y_1 + Y_2 + Y_3)$ , is an unbiased estimator of  $\mu$ . Find the value of the constant  $c$ . [2]

(iii) Determine which of  $T_1$  and  $T_2$  is more efficient. [4]

(iv) Find the values of the constants  $a$  and  $b$  for which

$$a(X_1^2 + X_2^2) + b(Y_1^2 + Y_2^2 + Y_3^2)$$

is an unbiased estimator of  $\sigma^2$ . [4]

- 7 Each question on a multiple-choice examination paper has  $n$  possible responses, only one of which is correct. Joni takes the paper and has probability  $p$ , where  $0 < p < 1$ , of knowing the correct response to any question, independently of any other. If she knows the correct response she will choose it, otherwise she will choose randomly from the  $n$  possibilities. The events  $K$  and  $A$  are ‘Joni knows the correct response’ and ‘Joni answers correctly’ respectively.

(i) Show that  $P(A) = \frac{q + np}{n}$ , where  $q = 1 - p$ . [3]

(ii) Find  $P(K|A)$ . [3]

A paper with 100 questions has  $n = 4$  and  $p = 0.5$ . Each correct response scores 1 and each incorrect response scores  $-1$ .

- (iii) (a) Joni answers all the questions on the paper and scores 40. How many questions did she answer correctly? [1]

- (b) By finding the distribution of the number of correct answers, or otherwise, find the probability that Joni scores at least 40 on the paper using her strategy. [6]

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