GCE

Mathematics

Advanced GCE **A2 7890 – 2**

Advanced Subsidiary GCE **AS 3890 – 2**

Mark Schemes for the Units

June 2009

3890-2/7890-2/MS/R/09

Oxford Cambridge and RSA Examinations

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

CONTENTS

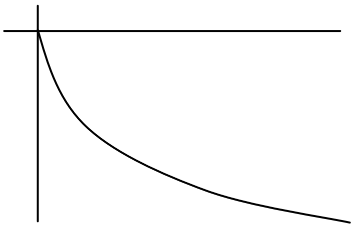
Advanced GCE Mathematics (7890)
Advanced GCE Pure Mathematics (7891)
Advanced GCE Further Mathematics (7892)

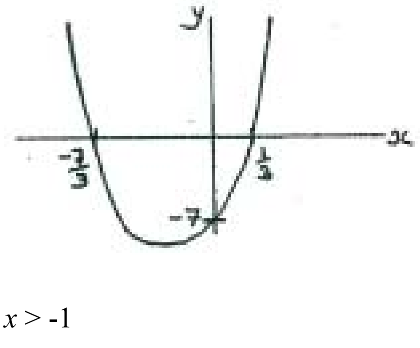
Advanced Subsidiary GCE Mathematics (3890)
Advanced Subsidiary GCE Pure Mathematics (3891)
Advanced Subsidiary GCE Further Mathematics (3892)

MARK SCHEMES FOR THE UNITS

4721 Core Mathematics 1

1 (i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 5x^4 - 2x^{-3}$	**B1** **M1** **A1 3**	$5x^4$ x^{-2} before differentiation or kx^{-3} in $\frac{\mathrm{d}y}{\mathrm{d}x}$ soi $-2x^{-3}$
(ii)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 20x^3 + 6x^{-4}$	**M1** **A1 2** $\boxed{5}$	Attempt to differentiate their (i) – at least one term correct cao
2	$\frac{(8+\sqrt{7})(2-\sqrt{7})}{(2+\sqrt{7})(2-\sqrt{7})}$ $= \frac{9-6\sqrt{7}}{4-7}$ $= -3 + 2\sqrt{7}$	**M1** **A1** **A1** **A1 4** $\boxed{4}$	Multiply numerator and denominator by conjugate Numerator correct and simplified Denominator correct and simplified cao
3 (i)	3^{-2}	**B1 1**	
(ii)	$3^{\frac{1}{3}}$	**B1 1**	
(iii)	$3^{10} \times 3^{30}$ $= 3^{40}$	**M1** **A1 2** $\boxed{4}$	3^{30} or 9^{20} soi
4	$y = 2x - 4$ $4x^2 + (2x-4)^2 = 10$ $8x^2 - 16x + 16 = 10$ $8x^2 - 16x + 6 = 0$ $4x^2 - 8x + 3 = 0$ $(2x-1)(2x-3) = 0$ $x = \frac{1}{2}$, $x = \frac{3}{2}$ $y =$ -3, $y =$ -1	**M1*** **A1** **M1dep*** **A1** **A1** **A1 6** $\boxed{6}$	Attempt to get an equation in 1 variable only Obtain correct 3 term quadratic (aef) Correct method to solve quadratic of form $ax^2 + bx + c = 0$ $(b \neq 0)$ Correct factorisation oe Both x values correct Both y values correct <u>or</u> one correct pair of values www B1 second correct pair of values B1

5 (i)	$(2x^2-5x-3)(x+4)$ $=2x^3+8x^2-5x^2-20x-3x-12$ $=2x^3+3x^2-23x-12$	**M1** **A1** **A1 3**	Attempt to multiply a quadratic by a linear factor or to expand all 3 brackets with an appropriate number of terms (including an x^3 term) Expansion with no more than one incorrect term
(ii)	$2x^4+7x^4$ $=9x^4$ 9	**B1** **B1 2** $\boxed{5}$	$2x^4$ or $7x^4$ soi www $9x^4$ or 9
6 (i)		**B1** **B1 2**	One to one graph only in bottom right hand quadrant Correct graph, passing through origin
(ii)	Translation Parallel to *y*-axis, 5 units	**B1** **B1 2**	
(iii)	$y=-\sqrt{\frac{x}{2}}$	**M1** **A1 2** $\boxed{6}$	$\sqrt{2x}$ or $\sqrt{\frac{x}{2}}$ seen cao
7 (i)	$\left(x-\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2+\frac{1}{4}$ $=\left(x-\frac{5}{2}\right)^2-6$	**B1** **M1** **A1 3**	$a=\frac{5}{2}$ $\frac{1}{4}-a^2$ cao
(ii)	$\left(x-\frac{5}{2}\right)^2-6+y^2=0$ Centre $\left(\frac{5}{2},0\right)$ Radius = $\sqrt{6}$	**B1** **B1** **B1 3** $\boxed{6}$	Correct *x* coordinate Correct *y* coordinate

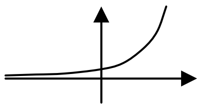
8 (i)	$-42 < 6x < -6$	**M1**	2 equations or inequalities both dealing with all 3 terms
	$-7 < x < -1$	**A1**	-7 and -1 seen oe
		A1 3	$-7 < x < -1$ (or $x > -7$ and $x < -1$)
(ii)	$x^2 > 16$	**B1**	±4 oe seen
	$x > 4$	**B1**	$x > 4$
	or $x < -4$	**B1 3** $\boxed{6}$	$x < -4$ not wrapped, not 'and'
9 (i)	$\sqrt{({}^-1-4)^2+(9-{}^-3)^2}$	**M1**	Correct method to find line length using Pythagoras' theorem
	$=13$	**A1 2**	cao
(ii)	$\left(\frac{4+{}^-1}{2},\frac{{}^-3+9}{2}\right)$	**M1**	Correct method to find midpoint
	$\left(\frac{3}{2},3\right)$	**A1 2**	
(iii)	Gradient of $AB = -\frac{12}{5}$	**B1**	
	$y-3=-\frac{12}{5}(x-1)$	**M1**	Correct equation for line, any gradient, through (1, 3)
	$12x+5y-27=0$	**A1**	Correct equation in any form with gradient simplified
		A1 4 $\boxed{8}$	$12x+5y-27=0$
10 (i)	$(3x+7)(3x-1)=0$	**M1**	Correct method to find roots
	$x=-\frac{7}{3}, x=\frac{1}{3}$	**A1**	Correct factorisation oe
		A1 3	Correct roots
(ii)	$\frac{dy}{dx}=18x+18$	**M1**	Attempt to differentiate y
	$18x+18=0$	**M1**	Uses $\frac{dy}{dx}=0$
	$x=-1$	**A1**	
	$y=-16$	**A1 ft 4**	
(iii)		**B1**	Positive quadratic curve
		B1	y intercept (0, -7)
		B1 3	Good graph, with correct roots indicated and minimum point in correct quadrant
(iv)	$x > -1$	**B1 1** $\boxed{11}$	

11 (i)	Gradient of normal = $-\frac{2}{3}$	**B1**	
	$\frac{dy}{dx}=\frac{1}{2}kx^{-\frac{1}{2}}$	**M1*** **A1**	Attempt to differentiate equation of curve $\frac{1}{2}kx^{-\frac{1}{2}}$
	When $x = 4$, $\frac{dy}{dx}=\frac{k}{4}$	**M1dep***	Attempt to substitute $x = 4$ into their $\frac{dy}{dx}$ soi
	$\therefore \frac{k}{4}=\frac{3}{2}$ $k=6$	**M1dep*** **A1 6**	Equate their gradient expression to negative reciprocal of their gradient of normal cao
(ii)	P is point (4, 12)	**B1 ft**	
	Q is point (22, 0)	**M1** **A1**	Correct method to find coordinates of Q Correct x coordinate
	Area of triangle $=\frac{1}{2}\times 12\times 22$ = 132 sq. units	**M1** **A1 5** $\boxed{\textbf{11}}$	Must use y coordinate of P and x coordinate of Q

4722 Core Mathematics 2

1	**(i)**	$\cos\theta = \frac{6.4^2+7.0^2-11.3^2}{2\times6.4\times7.0}$	M1		Attempt use of cosine rule (any angle)
		$= -0.4211$	A1		Obtain one of 115°, 34.2°, 30.9°, 2.01, 0.597, 0.539
		$\theta = 115^\circ$ or 2.01 rads	A1	**3**	Obtain 115° or 2.01 rads, or better
	(ii)	area = $\frac{1}{2}\times7\times6.4\times\sin115$	M1		Attempt triangle area using (½)*ab*sin*C*, or equiv
		= 20.3 cm^2	A1	**2**	Obtain 20.3 (cao)
			$\boxed{5}$		
2	**(i)**	$a + 9d = 2(a + 3d)$	M1*		Attempt use of $a + (n - 1)d$ or $a + nd$ at least once for u_4, u_{10} or u_{20}
		$a = 3d$	A1		Obtain $a = 3d$ (or unsimplified equiv) and $a + 19d = 44$
		$a + 19d = 44 \Rightarrow 22d = 44$	M1dep*		Attempt to eliminate one variable from two simultaneous equations in a and d, from u_4, u_{10}, u_{20} and no others
		$d = 2, a = 6$	A1	**4**	Obtain $d = 2$, $a = 6$
	(ii)	$S_{50} = {}^{50}/_2\ (2\text{x}6 + 49\text{x}2)$	M1		Attempt S_{50} of AP, using correct formula, with $n = 50$, allow $25(2a + 24d)$
		$= 2750$	A1	**2**	Obtain 2750
			$\boxed{6}$		
3		$\log 7^x = \log 2^{x+1}$	M1		Introduce logarithms throughout, or equiv with base 7 or 2
		$x\log 7 = (x+1)\log 2$	M1		Drop power on at least one side
			A1		Obtain correct linear equation (allow with no brackets)
		$x(\log 7 - \log 2) = \log 2$	M1		**Either** expand bracket and attempt to gather x terms, **or** deal correctly with algebraic fraction
		$x = 0.553$	A1	**5**	Obtain $x = 0.55$, or rounding to this, with no errors seen
			$\boxed{5}$		
4	**(i)**	$(x^2-5)^3 = (x^2)^3 + 3(x^2)^2(-5) + 3(x^2)(-5)^2 + (-5)^3$	M1*		Attempt expansion, with product of powers of x^2 and $\pm$ 5, at least 3 terms
		$= x^6 - 15x^4 + 75x^2 - 125$	M1*		Use at least 3 of binomial coeffs of 1, 3, 3, 1
			A1dep*		Obtain at least two correct terms, coeffs simplified
			A1	**4**	Obtain fully correct expansion, coeffs simplified
		OR			
		$(x^2 - 5)^3 = (x^2 - 5)(x^4 - 10x^2 + 25)$	M2		Attempt full expansion of all 3 brackets
		$= x^6 - 15x^4 + 75x^2 - 125$	A1		Obtain at least two correct terms
			A1		Obtain full correct expansion
	(ii)	$\int (x^2-5)^3\,dx = \frac{1}{7}x^7 - 3x^5 + 25x^3 - 125x + c$	M1		Attempt integration of terms of form kx^n
			A1√		Obtain at least two correct terms, allow unsimplified coeffs
			A1		Obtain $\frac{1}{7}x^7 - 3x^5 + 25x^3 - 125x$
			B1	**4**	+ c, and no dx or ∫ sign
			$\boxed{8}$		

5	**(i)**	$2x = 30^\circ, 150^\circ$	M1		Attempt $\sin^{-1} 0.5$, then divide or multiply by 2
		$x = 15^\circ, 75^\circ$	A1		Obtain 15° (allow $^{\pi}/_{12}$ or 0.262)
			A1	**3**	Obtain 75° (not radians), and no extra solutions in range
	(ii)	$2(1 - \cos^2 x) = 2 - \sqrt{3}\cos x$	M1		Use $\sin^2 x = 1 - \cos^2 x$
		$2\cos^2 x - \sqrt{3}\cos x = 0$	A1		Obtain $2\cos^2 x - \sqrt{3}\cos x = 0$ or equiv (no constant terms)
		$\cos x\,(2\cos x - \sqrt{3}) = 0$	M1		Attempt to solve quadratic in $\cos x$
		$\cos x = 0, \cos x = ½\sqrt{3}$	A1		Obtain 30° (allow $^{\pi}/_{6}$ or 0524), and no extra solns in
range					
		$x = 90^\circ$, $x = 30^\circ$	B1	**5**	Obtain 90° (allow $^{\pi}/_{2}$ or 1.57), from correct quadratic only
				SR	answer only B1 one correct solution
					B1 second correct solution, and no others
				$\boxed{8}$	
6		$\int \left(3x^2 + a\right)\,\mathrm{d}x = x^3 + ax + c$	M1		Attempt to integrate
			A1		Obtain at least one correct term, allow unsimplified
			A1		Obtain $x^3 + ax$
		$(-1, 2) \Rightarrow -1 - a + c = 2$	M1		Substitute at least one of (–1, 2) or (2, 17) into integration attempt involving a and c
		$(2, 17) \Rightarrow 8 + 2a + c = 17$	A1		Obtain two correct equations, allow unsimplified
			M1		Attempt to eliminate one variable from two equations in a and c
		$a = 2, c = 5$	A1		Obtain $a = 2$, $c = 5$, from correct equations
		Hence $y = x^3 + 2x + 5$	A1	**8**	State $y = x^3 + 2x + 5$
				$\boxed{8}$	
7	**(i)**	$f(-2) = -16 + 36 - 22 - 8$	M1		Attempt f(-2), or equiv
		$= -10$	A1	**2**	Obtain -10
	(ii)	$f(½) = ¼ + 2¼ + 5½ - 8 = 0$ AG	M1		Attempt f(½) (no other method allowed)
			A1	**2**	Confirm f(½) = 0, extra line of working required
	(iii)	$f(x) = (2x - 1)(x^2 + 5x + 8)$	M1		Attempt complete division by $(2x - 1)$ or $(x - ½)$ or equiv
			A1		Obtain $x^2 + 5x + c$ or $2x^2 + 10x + c$
			A1	**3**	State $(2x - 1)(x^2 + 5x + 8)$ or $(x - ½)(2x^2 + 10x + 16)$
	(iv)	f(x) has one real root ($x = ½$)	B1√		State 1 root, following their quotient, ignore reason
		because $b^2 - 4ac = 25 - 32 = -7$			
		hence quadratic has no real roots as $-7 < 0$,	B1√	**2**	Correct calculation, eg discriminant or quadratic formula, following their quotient, or cubic has max at (-2.15, -9.9)
				$\boxed{9}$	

8 **(i)**	½ × r^2 × 1.2 = 60	M1		Attempt (½) $r^2\theta = 60$
	$r = 10$	A1		Obtain $r = 10$
	$r\theta = 10 \times 1.2 = 12$	B1√		State or imply arc length is $1.2r$, following their r
	perimeter = 10 + 10 + 12 = 32 cm	A1	**4**	Obtain 32
(ii)(a)	$u_5 = 60 \times 0.6^4$	M1		Attempt u_5 using ar^4, or list terms
	= 7.78	A1	**2**	Obtain 7.78, or better
(b)	$S_{10} = \frac{60\left(1-0.6^{10}\right)}{1-0.6}$	M1		Attempt use of correct sum formula for a GP, or sum terms
	= 149	A1	**2**	Obtain 149, or better (allow 149.0 – 149.2 inclusive)
(c)	common ratio is less than 1, so series is convergent and hence sum to infinity exists	B1		series is convergent or $-1 < r < 1$ (allow $r < 1$) or reference to areas getting smaller / adding on less each time
	$S_\infty = \frac{60}{1-0.6}$	M1		Attempt S_∞ using $\frac{a}{1-r}$
	= 150	A1	**3**	Obtain $S_\infty = 150$
				SR B1 only for 150 with no method shown
			$\boxed{11}$	
9 **(i)**		B1		Sketch graph showing exponential growth (both quadrants)
		B1	**2**	State or imply (0, 4)
(ii)	$4k^x = 20k^2$			
	$k^x = 5k^2$	M1		Equate $4k^x$ to $20k^2$ and take logs (any, or no, base)
	$x = \log_k 5k^2$			
	$x = \log_k 5 + \log_k k^2$	M1		Use $\log ab = \log a + \log b$
	$x = 2\log_k k + \log_k 5$	M1		Use $\log a^b = b \log a$
	$x = 2 + \log_k 5$ AG	A1	**4**	Show given answer correctly
OR	$4k^x = 20k^2$			
	$k^x = 5k^2$	M1		Attempt to rewrite as single index
	$k^{x-2} = 5$	A1		Obtain $k^{x-2} = 5$ or equiv eg $4k^{x-2} = 20$
	$x - 2 = \log_k 5$	M1		Take logs (to any base)
	$x = 2 + \log_k 5$ AG	A1		Show given answer correctly
(iii) (a)	area $\approx \frac{1}{2}\times\frac{1}{2}\times\left(4k^0+8k^{\frac{1}{2}}+4k^1\right)$	M1		Attempt y-values at $x = 0$, ½ and 1, and no others
		M1		Attempt to use correct trapezium rule, 3 y-values, h = ½
	$\approx 1+2k^{\frac{1}{2}}+k$	A1	**3**	Obtain a correct expression, allow unsimplified
(b)	$1+2k^{\frac{1}{2}}+k=16$	M1		Equate attempt at area to 16
	$\left(k^{\frac{1}{2}}+1\right)^2=16$	M1		Attempt to solve 'disguised' 3 term quadratic
	$k^{\frac{1}{2}}=3$			
	$k=9$	A1	**3**	Obtain $k = 9$ only
			$\boxed{12}$	

4723 Core Mathematics 3

1 (i)	State $y = \sec x$	B1		
(ii)	State $y = \cot x$	B1		
(iii)	State $y = \sin^{-1} x$	B1	3	
			$\boxed{3}$	
2	Either: State or imply $\int \pi(2x-3)^4 \, \mathrm{d}x$	B1		or unsimplified equiv
	Obtain integral of form $k(2x-3)^5$	M1		any constant k involving π or not
	Obtain $\frac{1}{10}(2x-3)^5$ or $\frac{1}{10}\pi(2x-3)^5$	A1		
	Attempt evaluation using 0 and $\frac{3}{2}$	M1		subtraction correct way round
	Obtain $\frac{243}{10}\pi$	A1	5	or exact equiv
	Or: State or imply $\int \pi(2x-3)^4 \, \mathrm{d}x$	B1		or unsimplified equiv
	Expand and obtain integral of order 5	M1		with at least three terms correct
	Ob'n $\frac{16}{5}x^5 - 24x^4 + 72x^3 - 108x^2 + 81x$	A1		with or without π
	Attempt evaluation using (0 and) $\frac{3}{2}$	M1		
	Obtain $\frac{243}{10}\pi$	A1	(5)	or exact equiv
			$\boxed{5}$	
3 (i)	Attempt use of identity for $\sec^2 \alpha$	M1		using $\pm\tan^2\alpha \pm 1$
	Obtain $1 + (m+2)^2 - (1+m^2)$	A1		absent brackets implied by subsequent correct working
	Obtain $4m + 4 = 16$ and hence $m = 3$	A1	3	
(ii)	Attempt subn in identity for $\tan(\alpha+\beta)$	M1		using $\dfrac{\pm\tan\alpha \pm \tan\beta}{1 \pm \tan\alpha\tan\beta}$
	Obtain $\dfrac{5+3}{1-15}$ or $\dfrac{m+2+m}{1-m(m+2)}$	A1√		following their m
	Obtain $-\frac{4}{7}$	A1	3	or exact equiv
			$\boxed{6}$	
4 (i)	Obtain $\frac{1}{3}e^{3x} + e^{x}$	B1		
	Substitute to obtain $\frac{1}{3}e^{9a} + e^{3a} - \frac{1}{3}e^{3a} - e^{a}$	B1		or equiv
	Equate definite integral to 100 and attempt rearrangement	M1		as far as $e^{9a} = \ldots$
	Introduce natural logarithm	M1		using correct process
	Obtain $a = \frac{1}{9}\ln(300 + 3e^{a} - 2e^{3a})$	A1	5	AG; necessary detail needed
(ii)	Obtain correct first iterate	B1		allow for 4 dp rounded or truncated
	Show correct iteration process	M1		with at least one more step
	Obtain at least three correct iterates in all	A1		allowing recovery after error
	Obtain 0.6309	A1	4	following at least three correct steps; answer required to exactly 4 dp
	[0.6 → 0.631269 → 0.630884 → 0.630889]			
			$\boxed{9}$	

5	**(i)**	Either: Show correct process for comp'n	M1		correct way round and in terms of x
		Obtain $y = 3(3x+7)-2$	A1		or equiv
		Obtain $x = -\frac{19}{9}$	A1	3	or exact equiv; condone absence of $y = 0$
		Or: Use fg(x) = 0 to obtain $g(x) = \frac{2}{3}$	B1		
		Attempt solution of $g(x) = \frac{2}{3}$	M1		
		Obtain $x = -\frac{19}{9}$	A1	(3)	or exact equiv; condone absence of $y = 0$
	(ii)	Attempt formation of one of the equations $3x+7 = \frac{x-7}{3}$ or $3x+7 = x$ or $\frac{x-7}{3} = x$	M1		or equiv
		Obtain $x = -\frac{7}{2}$	A1		or equiv
		Obtain $y = -\frac{7}{2}$	A1√	3	or equiv; following their value of x
	(iii)	Attempt solution of modulus equation	M1		squaring both sides to obtain 3-term quadratics or forming linear equation with signs of $3x$ different on each side
		Obtain $-12x+4 = 42x+49$ or $3x-2 = -3x-7$	A1		or equiv
		Obtain $x = -\frac{5}{6}$	A1		or exact equiv; as final answer
		Obtain $y = \frac{9}{2}$	A1	4	or equiv; and no other pair of answers
				$\boxed{10}$	
6	**(i)**	Obtain derivative $k(37+10y-2y^2)^{-\frac{1}{2}}f(y)$	M1		any constant k; any linear function for f
		Obtain $\frac{1}{2}(10-4y)(37+10y-2y^2)^{-\frac{1}{2}}$	A1	2	or equiv
	(ii)	Either: Sub'te $y = 3$ in expression for $\frac{dx}{dy}$	*M1		
		Take reciprocal of expression/value	*M1		and without change of sign
		Obtain –7 for gradient of tangent	A1		
		Attempt equation of tangent	M1		dep *M *M
		Obtain $y = -7x+52$	A1	5	and no second equation
		Or: Sub'te $y = 3$ in expression for $\frac{dx}{dy}$	M1		
		Attempt formation of eq'n $x = m'y+c$	M1		where m' is attempt at $\frac{dx}{dy}$
		Obtain $x-7 = -\frac{1}{7}(y-3)$	A1		or equiv
		Attempt rearrangement to required form	M1		
		Obtain $y = -7x+52$	A1	(5)	and no second equation
				$\boxed{7}$	

7 (i)	State $R = 10$	B1		or equiv
	Attempt to find value of α	M1		implied by correct answer or its complement; allow sin/cos muddles
	Obtain 36.9 or $\tan^{-1}\frac{3}{4}$	A1	3	or greater accuracy 36.8699…
(ii)(a)	Show correct process for finding one angle	M1		
	Obtain (64.16 + 36.87 and hence) 101	A1		or greater accuracy 101.027…
	Show correct process for finding second angle	M1		
	Obtain (115.84 + 36.87 and hence) 153	A1√	4	following their value of α ; or greater accuracy 152.711…; and no other between 0 and 360
(b)	Recognise link with part **(i)**	M1		signalled by 40 … − 20 …
	Use fact that maximum and minimum values of sine are 1 and –1	M1		may be implied; or equiv
	Obtain 60	A1	3	
			$\boxed{10}$	
8 (i)	Refer to translation and stretch	M1		in either order; allow here equiv informal terms such as 'move', …
	State translation in x direction by 6	A1		or equiv; now with correct terminology
	State stretch in y direction by 2	A1	3	or equiv; now with correct terminology
	[SC: if M0 but one transformation completely correct, give B1]			
(ii)	State $2\ln(x-6) = \ln x$	B1		or $2\ln(a-6) = \ln a$ or equiv
	Show correct use of logarithm property	*M1		
	Attempt solution of 3-term quadratic	M1		dep *M
	Obtain 9 only	A1	4	following correct solution of equation
(iii)	Attempt evaluation of form $k(y_0 + 4y_1 + y_2)$	M1		any constant k; maybe with $y_0 = 0$ implied
	Obtain $\frac{1}{3}\times 1(2\ln 1 + 8\ln 2 + 2\ln 3)$	A1		or equiv
	Obtain 2.58	A1	3	or greater accuracy 2.5808…
			$\boxed{10}$	
9 (a)	Attempt use of quotient rule	*M1		or equiv; allow numerator wrong way round and denominator errors
	Obtain $\frac{(kx^2+1)2kx-(kx^2-1)2kx}{(kx^2+1)^2}$	A1		or equiv; with absent brackets implied by subsequent correct working
	Obtain correct simplified numerator $4kx$	A1		
	Equate numerator of first derivative to zero	M1		dep *M
	State $x = 0$ or refer to $4kx$ being linear or observe that, with $k \neq 0$, only one sol'n	A1√	5	AG or equiv; following numerator of form $k'kx = 0$, any constant k'

(b)	Attempt use of product rule	*M1		
	Obtain $me^{mx}(x^2+mx)+e^{mx}(2x+m)$	A1		or equiv
	Equate to zero and either factorise with factor e^{mx} or divide through by e^{mx}	M1		dep *M
	Obtain $mx^2+(m^2+2)x+m=0$ or equiv and observe that e^{mx} cannot be zero	A1		
	Attempt use of discriminant	M1		using correct b^2-4ac with their a, b, c
	Simplify to obtain m^4+4	A1		or equiv
	Observe that this is positive for all m and hence two roots	A1	7	or equiv; AG
			$\boxed{12}$	

4724 Core Mathematics 4

1	Long Division For leading term $3x^2$ in quotient	B1	
	Suff evid of div process (ax^2 , mult back, attempt sub)	M1	
	(Quotient) = $3x^2-4x-5$	A1	
	(Remainder) = $-x+2$	A1	
	Identity $3x^4-x^3-3x^2-14x-8=Q\left(x^2+x+2\right)+R$	*M1	
	$Q=ax^2+bx+c, R=dx+e$ & attempt ≥ 3 ops.	dep*M1	If $a=3$,this ⇒ 1 operation
	$a=3, b=-4, c=-5$	A1	dep*M1; Q = ax^2+bx+c
	$d=-1, e=2$	A1	
	Inspection Use 'Identity' method; if *R* = *e*, check cf(*x*) correct before awarding 2nd M1		
		$\boxed{4}$	
2	Indefinite Integral Attempt to connect $\mathrm{d}x$ & $\mathrm{d}\theta$	*M1	Incl $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ or $\frac{\mathrm{d}\theta}{\mathrm{d}x}$; not $\mathrm{d}x=\mathrm{d}\theta$
	Reduce to $\int 1-\tan^2\theta\,(\mathrm{d}\theta)$	A1	A0 if $\frac{\mathrm{d}\theta}{\mathrm{d}x}=\sec^2\theta$; but allow all following A marks
	Use $\tan^2\theta=(1,-1)+\left(\sec^2\theta,-\sec^2\theta\right)$	dep*M1	
	Produce $\int 2-\sec^2\theta\,(\mathrm{d}\theta)$	A1	
	Correct √ integration of function of type $d+e\sec^2\theta$	√A1	including $d=0$
	EITHER Attempt limits change (allow degrees here)	M1	(This is 'limits' aspect; the
	OR Attempt integ, re-subst & use original ($\sqrt{3}$,1)		integ need not be accurate)
	$\frac{1}{6}\pi-\sqrt{3}+1$ isw Exact answer required	A1	
		$\boxed{7}$	

3	**(i)**	$\left(1+\frac{x}{a}\right)^{-2} = 1+(-2)\frac{x}{a}+\frac{-2.-3}{2}\left(\frac{x}{a}\right)^2+\ldots$	M1	Check 3rd term; accept $\frac{x^2}{a}$
		$= 1-\frac{2x}{a}+\ldots$ or $1+\left(-\frac{2x}{a}\right)$	B1	or $1-2xa^{-1}$ (Ind of M1)
		$\ldots+\frac{3x^2}{a^2}+\ldots$ (or $3\left(\frac{x}{a}\right)^2$ or $3x^2a^{-2}$)	A1	Accept $\frac{6}{2}$ for 3
		$(a+x)^{-2} = \frac{1}{a^2}\left\{\text{their expansion of } \left(1+\frac{x}{a}\right)^{-2}\right\}$ mult out	√A1 **4**	$\frac{1}{a^2}-\frac{2x}{a^3}+\frac{3x^2}{a^4}$ **;** accept eg a^{-2}
	(ii)	Mult out $(1-x)$(their exp) to produce all terms/cfs(x^2)	M1	Ignore other terms
		Produce $\frac{3}{a^2}+\frac{2}{a}(=0)$ or $\frac{3}{a^4}+\frac{2}{a^3}$ $(=0)$ or AEF	A1	Accept x^2 if in both terms
		$a=-\frac{3}{2}$ www seen anywhere in (i) or (ii)	A1 **3**	Disregard any ref to $a=0$
			$\boxed{7}$	
4	**(i)**	Differentiate as a product, $u\,\mathrm{d}v+v\,\mathrm{d}u$	M1	or as 2 separate products
		$\frac{\mathrm{d}}{\mathrm{d}x}(\sin 2x)=2\cos 2x$ <u>or</u> $\frac{\mathrm{d}}{\mathrm{d}x}(\cos 2x)=-2\sin 2x$	B1	
		$\mathrm{e}^x(2\cos 2x+4\sin 2x)+\mathrm{e}^x(\sin 2x-2\cos 2x)$	A1	terms may be in diff order
		Simplify to $5\,\mathrm{e}^x\sin 2x$ www	A1 **4**	Accept $10\mathrm{e}^x\sin x\cos x$
	(ii)	Provided result (i) is of form $k\,\mathrm{e}^x\sin 2x$, k const		
		$\int \mathrm{e}^x\sin 2x\,\mathrm{d}x=\frac{1}{k}\mathrm{e}^x(\sin 2x-2\cos 2x)$	B1	
		$[\,\mathrm{e}^x(\sin 2x-2\cos 2x)]_0^{\frac{1}{4}\pi}=\mathrm{e}^{\frac{1}{4}\pi}+2$	B1	
		$\frac{1}{5}\left(\mathrm{e}^{\frac{1}{4}\pi}+2\right)$	B1 **3**	Exact form to be seen
		SR Although 'Hence', award M2 for double integration by parts and solving + A1 for correct answer.		
			$\boxed{7}$	

5 **(i)**	$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ aef used	M1	
	$=\frac{4t+3t^2}{2+2t}$	A1	
	Attempt to find t from one/both equations	M1	<u>or</u> diff (ii) cartesian eqn $\rightarrow$ M1
	State/imply $t=-3$ is only solution of both equations	A1	subst $(3,-9)$,solve for $\frac{dy}{dx}\rightarrow$ M1
	Gradient of curve = $-\frac{15}{4}$ or $\frac{-15}{4}$ or $\frac{15}{-4}$	A1 **5**	grad of curve = $-\frac{15}{4}\rightarrow$ A1

[**SR** If $t=1$ is given as solution & not disqualified, award A0 + √A1 for grad = $-\frac{15}{4}$ & $\frac{7}{4}$;

If $t=1$ is given/used as only solution, award A0 + √A1 for grad = $\frac{7}{4}$]

(ii)	$\frac{y}{x}=t$	B1	
	Substitute into either parametric eqn	M1	
	Final answer $x^3=2xy+y^2$	A2 **4**	

[**SR** Any correct unsimplified form (involving fractions or common factors) $\rightarrow$ A1]

$\boxed{9}$

---

6 **(i)**	$4x\equiv A(x-3)^2+B(x-3)(x-5)+C(x-5)$	M1	
	$A=5$	A1	'cover-up' rule, award B1
	$B=-5$	A1	
	$C=-6$	A1 **4**	'cover-up' rule, award B1

Cands adopting other alg. manip. may be awarded M1 for a full satis method + 3 @ A1

(ii)	$\int\frac{A}{x-5}dx=A\ln(5-x)$ or $A\ln\lvert 5-x\rvert$ or $A\ln\lvert x-5\rvert$	√B1	but <u>not</u> $A\ln(x-5)$
	$\int\frac{B}{x-3}dx=B\ln(3-x)$ or $B\ln\lvert 3-x\rvert$ or $B\ln\lvert x-3\rvert$	√B1	but <u>not</u> $B\ln(x-3)$

<u>If candidate is awarded B0,B0,</u> then award **SR** √ B1 for $A\ln(x-5)$ **and** $B\ln(x-3)$

	$\int\frac{C}{(x-3)^2}dx=-\frac{C}{x-3}$	√B1	
	$5\ln\frac{3}{4}+5\ln 2$ aef, isw √ $A\ln\frac{3}{4}-B\ln 2$	√ B1	Allow if **SR** B1 awarded
	-3 √ $\frac{1}{2}C$	√B1 **5**	

[Mark at earliest correct stage & isw; no ln 1] $\boxed{9}$

7	**(i)**	Attempt scalar prod {**u**.(4**i** + **k**) or **u**.(4**i** + 3**j** + 2**k**)}= 0	M1	where **u** is the given vector
		Obtain $\frac{12}{13}+c=0$ <u>or</u> $\frac{12}{13}+3b+2c=0$	A1	
		$c=-\frac{12}{13}$	A1	
		$b=\frac{4}{13}$	A1	cao No ft
		Evaluate $\left(\frac{3}{13}\right)^2+(\text{their } b)^2+(\text{their } c)^2$	M1	Ignore non-mention of $\sqrt{\ }$
		Obtain $\frac{9}{169}+\frac{144}{169}+\frac{16}{169}=1$ AG	A1 **6**	Ignore non-mention of $\sqrt{\ }$
	(ii)	Use $\cos\theta=\frac{\boldsymbol{x}.\boldsymbol{y}}{\lvert\boldsymbol{x}\rvert\lvert\boldsymbol{y}\rvert}$	M1	
		Correct method for finding scalar product	M1	
		36° (35.837653…) Accept 0.625 (rad)	A1 **3**	From $\frac{18}{\sqrt{17}\sqrt{29}}$
		SR If 4**i**+**k** = (4,1,0) in (i) & (ii), mark as scheme but allow final A1 for 31°(31.160968) or 0.544		
			$\boxed{9}$	
8	**(i)**	$\frac{\mathrm{d}}{\mathrm{d}x}\left(y^2\right)=2y\frac{\mathrm{d}y}{\mathrm{d}x}$	B1	
		$\frac{\mathrm{d}}{\mathrm{d}x}(uv)=u\ \mathrm{d}v+v\ \mathrm{d}u$ used on $(-7)xy$	M1	
		$\frac{\mathrm{d}}{\mathrm{d}x}\left(14x^2-7xy+y^2\right)=28x-7x\frac{\mathrm{d}y}{\mathrm{d}x}-7y+2y\frac{\mathrm{d}y}{\mathrm{d}x}$	A1	(= 0)
		$2y\frac{\mathrm{d}y}{\mathrm{d}x}-7x\frac{\mathrm{d}y}{\mathrm{d}x}=7y-28x\rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{28x-7y}{7x-2y}$ www AG	A1 **4**	As AG, intermed step nec
	(ii)	Subst x = 1 into eqn curve & solve quadratic eqn in y	M1	$(y=3 \text{ or } 4)$
		Subst x = 1 and (one of) their y-value(s) into given $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=7 \text{ or } 0\right)$
		Find eqn of tgt, with their $\frac{\mathrm{d}y}{\mathrm{d}x}$, going through (1, their y)	*M1	using (one of) y value(s)
		Produce either $y=7x-4$ or $y=4$	A1	
		Solve simultaneously their two equations	dep*M1	provided they have two
		Produce $x=\frac{8}{7}$	A1 **6**	
			$\boxed{10}$	

9	**(i)**	$\frac{20}{k_1}$ (seconds)	B1 **1**	
	(ii)	$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -k_2(\theta - 20)$	B1 **1**	
	(iii)	Separate variables or invert each side	M1	Correct eqn or very similar
		Correct int of each side (+ c)	A1,A1	for each integration
		Subst $\theta = 60$ when $t = 0$ into eqn containing 'c'	M1	or $\theta = 60$ when t = their $(\mathbf{i})$
		c $(\text{or} - c)$ = ln 40 or $\frac{1}{k_2}\ln 40$ or $\frac{1}{k_2}\ln 40k_2$	A1	Check carefully their 'c'
		Subst their value of c and $\theta = 40$ back into equation	M1	Use scheme on LHS
		$t = \frac{1}{k_2}\ln 2$	A1	Ignore scheme on LHS
		Total time = $\frac{1}{k_2}\ln 2 + \text{their (i)}$ (seconds)	√A1 **8**	

SR If the negative sign is omitted in part **(ii)**, allow all marks in **(iii)** with ln 2 replaced by $\ln\frac{1}{2}$.

SR If definite integrals used, allow M1 for eqn where $t = 0$ and $\theta = 60$ correspond; a second M1 for eqn where $t = t$ and $\theta = 40$ correspond & M1 for correct use of limits. Final answer scores 2.

$\boxed{\mathbf{10}}$

4725 Further Pure Mathematics 1

1.		B1		State correct value of S_{250} or S_{100}
		M1		Subtract $S_{250} - S_{100}$ (or S_{101} or S_{99})
	984390625 – 25502500 = 958888125	A1	3	Obtain correct exact answer
			3	
2.	$3a + 5b = 1$, $a + 2b = 1$	M1		Obtain a pair of simultaneous equations
		M1		Attempt to solve
	$a = -3$, $b = 2$	A1 A1	4	Obtain correct answers.
			4	
3.	(i) 11 – 29i	B1 B1	2	Correct real and imaginary parts
	(ii) 1 + 41i	B1 B1	2	Correct real and imaginary parts
			4	
4.	Either $p + q = -1, pq = -8$	B1		Both values stated or used
	$\frac{p+q}{pq}$	B1		Correct expression seen
		M1		Use their values in their expression
	$-\frac{7}{8}$	A1	4	Obtain correct answer
			4	
	Or $\frac{1}{p} + \frac{1}{q} = 8$	B1		Substitute $x = \frac{1}{u}$ and use new quadratic
	$p + q = 1$	B1		Correct value stated
		M1		Use their values in given expression
	$-\frac{7}{8}$	A1		Obtain correct answer
	Or $\frac{-1 \pm \sqrt{33}}{2}$	M1		Find roots of given quadratic equation
		A1		Correct values seen
	$-\frac{7}{8}$	M1		Use their values in given expression
		A1		Obtain correct answer
5.	(i) $u^3 = \{(-)(5u + 7)\}^2$	M1		Use given substitution and rearrange
		A1		Obtain correct expression, or equivalent
	$u^3 - 25u^2 - 70u - 49 = 0$	A1	3	Obtain correct final answer
	(ii)	M1		Use coefficient of u of their cubic or identity connecting the symmetric functions and substitute values from given equation
	-70	A1 ft	2	Obtain correct answer
			5	

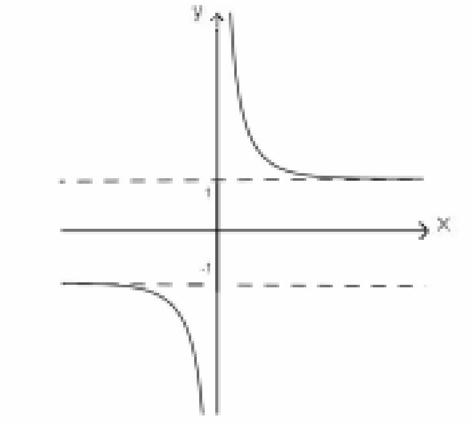
6.	(i) $3\sqrt{2}, -\frac{\pi}{4}$ or -45° AEF	B1 B1	2	State correct answers
	(ii)(a)	B1B1 B1 ft	3	Circle, centre (3, -3), through O ft for $(\pm 3, \pm 3)$ only
	(ii)(b)	B1 B1 B1	3	Straight line with +ve slope, through (3, -3) or their centre Half line only starting at centre
	(iii)	B1ft B1ft B1ft	3 **11**	Area above horizontal through a, below (ii) (b) Outside circle
7.	(i)	M1 A1	2	Show that terms cancel in pairs Obtain given answer correctly
	(ii)	M1 A1	2	Attempt to expand and simplify Obtain given answer correctly
	(iii)	B1 B1		Correct $\sum r$ stated $\sum 1 = n$
		M1* *DM1		Consider sum of 4 separate terms on RHS Required sum is LHS – 3 terms
	$(n+1)^4 - 1 - n(n+1)(2n+1) - 2n(n+1) - n$	A1		Correct unsimplified expression
	$4\sum_{r=1}^{n} r^3 = n^2(n+1)^2$	A1	6 **10**	Obtain given answer correctly
8.	(i)	B1 B1 B1	3	Find coordinates (0, 0) (3, 1) (2, 1) (5, 2) found Accurate diagram sketched
	(ii) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$	B1 B1	2	Each column correct
	(iii) *Either* $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$	B1 M1 A1ft		Correct inverse for their (ii) stated Post multiply **C** by inverse of (ii) Correct answer found
	Or	M1 A2ft		Set up 4 equations for elements from correct matrix multiplication All elements correct, -1 each error
		B1 B1 B1	6 **11**	Shear, x axis invariant or parallel to x-axis eg image of (1, 1) is (3, 1) **SR** allow s.f. 2 or shearing angle of correct angle to appropriate axis

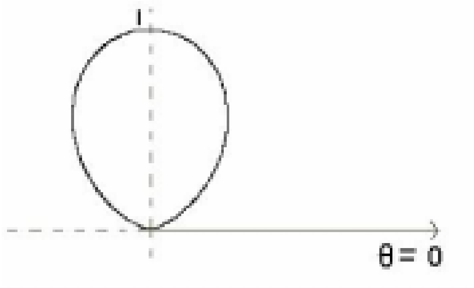
9.	(i) $a\begin{vmatrix}a & 1\\ 1 & 2\end{vmatrix} - \begin{vmatrix}1 & 1\\ 1 & 2\end{vmatrix} + \begin{vmatrix}1 & a\\ 1 & 1\end{vmatrix}$	M1		Correct expansion process shown
		A1		Obtain correct unsimplified expression
	$2a^2 - 2a$	A1	3	
				Obtain correct answer
	(ii)	M1		
	$a = 0$ or 1	A1ft		Equate their det to 0
		A1ft	3	Obtain correct answers, ft solving a quadratic
	(iii) (a)	B1 B1		Equations consistent, but non unique solutions
	(b)	B1		Correct equations seen &
		B1	4	inconsistent, no solutions
			10	
10.	i)	M1		Attempt to find next 2 terms
	$u_2 = 7$ $u_3 = 19$	A1		Obtain correct answers
		A1	3	Show given result correctly
	(ii)	M1		Expression involving a power of 3
	$u_n = 2(3^{n-1}) + 1$	A1	2	Obtain correct answer
	(iii)	B1ft		Verify result true when $n = 1$ or $n = 2$
	$u_{n+1} = 3(2(3^{n-1})+1) - 2$	M1		Expression for u_{n+1} using recurrence relation
		A1		Correct unsimplified answer
	$u_{n+1} = 2(3^n) + 1$	A1		Correct answer in correct form
		B1		Statement of induction conclusion
			5 **10**	

4726 Further Pure Mathematics 2

1(i)	Attempt area = $\pm\Sigma(0.3y)$ for at least three y values	M1	May be implied
	Get 1.313(1..) or 1.314	A1	Or greater accuracy
(ii)	Attempt ± sum of areas (4 or 5 values)	M1	May be implied
	Get 0.518(4..)	A1	Or greater accuracy **SC** If answers only seen, 1.313(1..) or 1.314 B2 0.518(4..) B2 −1.313(1..) or −1.314 B1 −0.518(4..) B1
	Or		
	Attempt answer to	M1	
	part **(i)**−final rectangle	A1	
	Get 0.518(4..)		
(iii)	Decrease width of strips	B1	Use more strips or equivalent
2	Attempt to set up quadratic in x	M1	Must be quadratic; = 0 may be implied
	Get $x^2(y-1) - x(2y+1) + (y-1)=0$	A1	
	Use $b^2 \geq 4ac$ for real x on their quadratic	M1	Allow =,>,<,≤ here; may be implied
	Clearly solve to AG	A1	If other (in)equalities used, the step to AG must be clear **SC** Reasonable attempt to diff. using prod/quot rule M1 Solve correct $dy/dx=0$ to get $x= -1$, $y = \frac{1}{4}$ A1 Attempt to justify inequality e.g. graph or to show $d^2y/dx^2>0$ M1 Clearly solve to AG A1
3(i)	Reasonable attempt at chain rule	M1	Product in answer
	Reasonable attempt at product/quotient rule	M1	Sum of two parts
	Correctly get $f'(0) =1$	A1	
	Correctly get $f''(0) = 1$	A1	**SC** Use of $\ln y = \sin x$ follows same scheme
(ii)	Reasonable attempt at Maclaurin with their values	M1	In $af(0) + bf'(0)x + cf''(0)x^2$
	Get $1 + x + \frac{1}{2}x^2$	A1√	From their f(0), f′(0), f″(0) in a correct Maclaurin; all non-zero terms
4	Attempt to divide out.	M1	Or $A+B/(x-2)+(Cx(+D))/(x^2+4)$; allow $A=1$ and/or $B=1$ quoted
	Get $x^3= A(x-2)(x^2+4)+B(x^2+4)+(Cx+D)(x-2)$	M1	Allow √ mark from their Part Fract; allow $D=0$ but not $C=0$
	State/derive/quote $A=1$	A1	
	Use x values and/or equate coeff	M1	To potentially get all their constants

	Get B=1, C=1, D=-2	A1	For one other correct from cwo
		A1	For all correct from cwo
5(i)	Derive/quote $\mathrm{d}\theta=2\mathrm{d}t/(1+t^2)$	B1	May be implied
	Replace their cos θ and their dθ, both in terms of t	M1	Not $\mathrm{d}\theta = \mathrm{d}t$
	Clearly get $\int(1-t^2)/(1+t^2)\,\mathrm{d}t$ or equiv	A1	Accept limits of t quoted here
	Attempt to divide out	M1	Or use AG to get answer above
	Clearly get/derive AG	A1	
			SC Derive $\mathrm{d}\theta = 2\cos^2\tfrac{1}{2}\theta\,\mathrm{d}t$ B1 Replace cosθ in terms of half-angles and their dθ ($\neq$dt) M1 Get $\int 2\cos^2\tfrac{1}{2}\theta - 1\,\mathrm{d}t$ or $\int 1 - 1/2\cos^2\tfrac{1}{2}\theta\,.2/(1+t^2)\,\mathrm{d}t$ A1 Use $\sec^2\tfrac{1}{2}\,\theta = 1+t^2$ M1 Clearly get/derive AG A1
(ii)	Integrate to $a\tan^{-1}bt - t$	M1	
	Get$\tfrac{1}{2}\pi - 1$	A1	
6	Get $k\sinh^{-1}k_1x$	M1	For either integral; allow attempt at ln version here
	Get $\tfrac{1}{3}\sinh^{-1}\tfrac{3}{4}x$	A1	Or ln version
	Get $\tfrac{1}{2}\sinh^{-1}\tfrac{2}{3}x$	A1	Or ln version
	Use limits in their answers	M1	
	Attempt to use correct ln laws to set up a solvable equation in a	M1	
	Get $a = 2^{\frac{1}{3}}.\ 3^{\frac{1}{2}}$	A1	Or equivalent

7(i)		B1	y-axis asymptote; equation may be implied if clear
		B1	Shape
		B1	$y = \pm 1$ asymptotes; may be implied if seen as on graph
(ii)	Reasonable attempt at product rule, giving two terms	M1	
	Use correct Newton-Raphson at least once with their $f'(x)$ to produce an x_2	M1	May be implied
	Get $x_2 = 2.0651$	A1√	One correct at any stage if reasonable
	Get $x_3 = 2.0653$, $x_4 = 2.0653$	A1	cao; or greater accuracy which rounds
(iii)	Clearly derive coth $x = ½x$	B1	AG; allow derivation from AG Two roots only
	Attempt to find second root e.g. symmetry	M1	
	Get ± 2.0653	A1√	± their iteration in part **(ii)**
8(i)	**(a)** Get $½(e^{\ln a} + e^{-\ln a})$	M1	
	Use $e^{\ln a} = a$ and $e^{-\ln a} = {}^{1}/_{a}$	M1	
	Clearly derive AG	A1	
	(b) Reasonable attempt to multiply out their attempts at exponential definitions of cosh and sinh	M1	4 terms in each
	Correct expansion seen as $e^{(x+y)}$ etc.	A1	
	Clearly tidy to AG	A1	With $e^{-(x-y)}$ seen or implied
(ii)	Use $x = y$ and cosh0 = 1 to get AG	B1	
(iii)	Attempt to expand and equate coefficients	M1	($13 = R \cosh \ln a = R(a^2+1)/2a$ $5 = R \sinh \ln a = R(a^2-1)/2a$)
	Attempt to eliminate R (or a) to set up a solvable equation in a (or R)	M1	**SC** If exponential definitions used, $8e^{x} + 18e^{-x} = Re^{x}/a + Rae^{-x}$ and same scheme follows
	Get $a = {}^{3}/_{2}$ (or $R = 12$)	A1	
	Replace for a (or R) in relevant equation to set up solvable equation in R (or a)	M1	
	Get $R=12$ (or $a = {}^{3}/_{2}$)	A1	Ignore if $a={}^{2}/_{3}$ also given
(iv)	Quote/derive $(\ln{}^{3}/_{2}, 12)$	B1√ B1√	On their R and a
9(i)	Use $\sin\theta.\sin^{n-1}\theta$ and parts	M1	Reasonable attempt with 2 parts, one yet to be integrated

	Get $-\cos\theta.\sin^{n-1}\theta+(n\text{-}1)\int\sin^{n\text{-}2}\theta.\cos^2\theta\ d\theta$	A1	Signs need to be carefully considered
	Replace $\cos^2 = 1 - \sin^2$	M1	
	Clearly use limits and get AG	A1	
(ii)	**(a)** Solve for r=0 for at least one θ	M1	θ need not be correct
	Get (θ) = 0 and π	A1	Ignore extra answers out of range
		B1	General shape (symmetry stated or approximately seen)
		B1	Tangents at θ=0, π and max r seen
	(b) Correct formula used; correct r	M1	May be $\int\ r^2\ d\theta$ with correct limits
	Use $6I_6 = 5I_4$, $4I_4 = 3I_2$	M1	At least one
	Attempt I_0 (or I_2)	M1	(I_0 = ½π)
	Replace their values to get I_6	M1	
	Get 5π/32	A1	
	Use symmetry to get 5π/32	A1	May be implied but correct use of limits must be given somewhere in answer
	Or		
	Correct formula used; correct r	M1	
	Reasonable attempt at formula $(2\mathrm{i}\sin\theta)^6 = (z - {}^1/z)^6$	M1	
	Attempt to multiply out both sides (7 terms)	M1	
	Get correct expansion	A1	
	Convert to trig. equivalent and integrate their expression	M1	cwo
	Get 5π/32	A1	
	Or		
	Correct formula used; correct r	M1	
	Use double-angle formula and attempt to cube (4 terms)	M1	
	Get correct expression	A1	
	Reasonable attempt to put $\cos^2 2\theta$ into integrable form and integrate	M1	
	Reasonable attempt to integrate $\cos^3 2\theta$ as e.g.$\cos^2 2\theta.\cos 2\theta$	M1	cwo
	Get 5π/32	A1	

4727 Further Pure Mathematics 3

1	$\left(\frac{1}{2}\sqrt{3}+\frac{1}{2}\mathrm{i}\right)^{\frac{1}{3}}=\left(\cos\frac{1}{6}\pi+\mathrm{i}\sin\frac{1}{6}\pi\right)^{\frac{1}{3}}$	B1		For $\arg z=\frac{1}{6}\pi$ seen or implied
	$=\cos\frac{1}{18}\pi+\mathrm{i}\sin\frac{1}{18}\pi$,	M1		For dividing $\arg z$ by 3
	$\cos\frac{13}{18}\pi+\mathrm{i}\sin\frac{13}{18}\pi$,	A1		For any one correct root
	$\cos\frac{25}{18}\pi+\mathrm{i}\sin\frac{25}{18}\pi$	A1	**4**	For 2 other roots and no more in range $0 \leqslant \theta < 2\pi$
			$\boxed{4}$	
2 (i)	$\frac{1}{5}\mathrm{e}^{-\frac{1}{3}\pi\mathrm{i}}$	B1	**1**	For stating correct inverse in the form $r\,\mathrm{e}^{\mathrm{i}\theta}$
(ii)	$r_1\mathrm{e}^{\mathrm{i}\theta}\times r_2\mathrm{e}^{\mathrm{i}\phi}=r_1r_2\,\mathrm{e}^{\mathrm{i}(\theta+\phi)}$	M1 A1	 **2**	For stating 2 distinct elements multiplied For showing product of correct form
(iii)	$Z^2=\mathrm{e}^{2\mathrm{i}\gamma}$	B1		For $\mathrm{e}^{2\mathrm{i}\gamma}$ seen or implied
	$\Rightarrow\ \mathrm{e}^{2\mathrm{i}\gamma-2\pi\mathrm{i}}$	B1	**2**	For correct answer. aef
			$\boxed{5}$	
3 (i)	$[6-4\lambda,\,-7+8\lambda,\,-10+7\lambda]$ on p	B1		For point on l seen or implied
	$\Rightarrow 3(6-4\lambda)-4(-7+8\lambda)-2(-10+7\lambda)=8$	M1		For substituting into equation of p
	$\Rightarrow\ \lambda=1\ \Rightarrow\ (2,1,-3)$	A1	**3**	For correct point. Allow position vector
(ii)	METHOD 1			
	$\mathbf{n}=[-4,8,7]\times[3,-4,-2]$	M1* M1 (*dep)		For direction of l and normal of p seen For attempting to find $\mathbf{n}_1\times\mathbf{n}_2$
	$\mathbf{n}=k[12,13,-8]$	A1		For correct vector
	$(2,1,-3)$ OR $(6,-7,-10)$	M1		For finding scalar product of their point on l with their attempt at **n**, or equivalent
	$\Rightarrow\ 12x+13y-8z=61$	A1	**5**	For correct equation, aef cartesian
	METHOD 2			
	$\mathbf{r}=[2,1,-3]$ *OR* $[6,-7,-10]$ $+\lambda[-4,8,7]+\mu[3,-4,-2]$	M1 A1√		For stating eqtn of plane in parametric form (may be implied by next stage), using $[2,1,-3]$ (ft from **(i)**) Or $[6,-7,-10]$, $\mathbf{n}_1$ and $\mathbf{n}_2$ (as above)
	$x=\ \ 2-4\lambda+3\mu$	M1		For writing as 3 linear equations
	$y=\ \ 1+8\lambda-4\mu$ $z=-3+7\lambda-2\mu$	M1		For attempting to eliminate λ and μ
	$\Rightarrow\ 12x+13y-8z=61$	A1		For correct equation aef cartesian
	METHOD 3			
	$3(6+3\mu)-4(-7-4\mu)-2(-10-2\mu)=8$	M1		For finding foot of perpendicular from point on l to p
	$\Rightarrow\ \mu=-2\ \Rightarrow\ (0,1,-6)$	A1		For correct point or position vector
	From 3 points $(2,1,-3),\ (6,-7,-10),\ (0,1,-6)$, **n** = vector product of 2 of $[2,0,3],\ [6,-8,-4],\ [-4,8,7]$	M1		Use vector product of 2 vectors in plane
	$\Rightarrow\ \mathbf{n}=k[12,13,-8]$			
	$(2,1,-3)$ OR $(6,-7,-10)$	M1		For finding scalar product of their point on l with their attempt at **n**, or equivalent
	$\Rightarrow\ 12x+13y-8z=61$	A1		For correct equation aef cartesian
			$\boxed{8}$	

4	(i)	IF $e^{\int \frac{1}{1-x^2}\,dx} = e^{\frac{1}{2}\ln\frac{1+x}{1-x}} = \left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$	M1 A1 **2**	For IF stated or implied. Allow $\pm\int$ and omission of dx For integration and simplification to **AG** (intermediate step must be seen)
	(ii)	$\frac{d}{dx}\left(y\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}\right) = (1+x)^{\frac{1}{2}}$	M1*	For multiplying both sides by IF
		$y\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}} = \frac{2}{3}(1+x)^{\frac{3}{2}} + c$	M1 A1	For integrating RHS to $k(1+x)^n$ For correct equation (including + c)
				In either order:
		$(0, 2) \Rightarrow 2 = \frac{2}{3} + c \Rightarrow c = \frac{4}{3}$	M1 (*dep) M1 (*dep)	For substituting $(0, 2)$ into their GS (including +c) For dividing solution through by IF, including dividing c or their numerical value for c
		$y = \frac{2}{3}(1+x)(1-x)^{\frac{1}{2}} + \frac{4}{3}\left(\frac{1-x}{1+x}\right)^{\frac{1}{2}}$	A1 **6**	For correct solution aef (even unsimplified) in form $y = f(x)$
			$\boxed{8}$	
5	(i)	$m^2 - 6m + 9\ (= 0) \Rightarrow m = 3$	M1 A1	For attempting to solve correct auxiliary equation For correct m
		CF = $(A + Bx)e^{3x}$	A1 **3**	For correct CF
	(ii)	ke^{3x} and kxe^{3x} both appear in CF	B1 **1**	For correct statement
	(iii)	$y = kx^2e^{3x} \Rightarrow y' = 2kxe^{3x} + 3kx^2e^{3x}$	M1 A1	For differentiating kx^2e^{3x} twice For correct y' aef
		$\Rightarrow y'' = 2ke^{3x} + 12kxe^{3x} + 9kx^2e^{3x}$	A1	For correct y'' aef
		$\Rightarrow$ $ke^{3x}\left(2 + 12x + 9x^2 - 12x - 18x^2 + 9x^2\right) = e^{3x}$	M1	For substituting y'', y', y into DE
		$\Rightarrow k = \frac{1}{2}$	A1 **5**	For correct k
			$\boxed{9}$	

6 (i)	METHOD 1			
	$\mathbf{n}_1 = [1, 1, 0] \times [1, -5, -2]$	M1		For attempting to find vector product of the pair of direction vectors
	$= [-2, 2, -6] = k[1, -1, 3]$	A1		For correct $\mathbf{n}_1$
	Use $(2, 2, 1)$	M1		For substituting a point into equation
	$\Rightarrow \mathbf{r}.[-2, 2, -6] = -6 \Rightarrow \mathbf{r}.[1, -1, 3] = 3$	A1	**4**	For correct equation. aef in this form
	METHOD 2			
	$x = 2 + \lambda + \mu$ $y = 2 + \lambda - 5\mu$ $z = 1 \quad - 2\mu$	M1 M1		For writing as 3 linear equations For attempting to eliminate λ and μ
	$\Rightarrow x - y + 3z = 3$	A1		For correct cartesian equation
	$\Rightarrow \mathbf{r}.[1, -1, 3] = 3$	A1		For correct equation. aef in this form
(ii)	For $\mathbf{r} = \mathbf{a} + t\mathbf{b}$			
	METHOD 1			
	$\mathbf{b} = [1, -1, 3] \times [7, 17, -3]$	M1		For attempting to find $\mathbf{n}_1 \times \mathbf{n}_2$
	$= k[2, -1, -1]$	A1√		For a correct vector. ft from $\mathbf{n}_1$ in **(i)**
	e.g. x, y or $z = 0$ in $\left\{\begin{array}{l} x - y + 3z = 3 \\ 7x + 17y - 3z = 21 \end{array}\right.$	M1		For attempting to find a point on the line
	$\Rightarrow \mathbf{a} = \left[0, \frac{3}{2}, \frac{3}{2}\right]$ OR $[3, 0, 0]$ OR $[1, 1, 1]$	A1√		For a correct vector. ft from equation in **(i)** **SR** a correct vector may be stated without working
	Line is (e.g.) $\mathbf{r} = [1, 1, 1] + t[2, -1, -1]$	A1√	**5**	For stating equation of line ft from **a** and **b** **SR** for $\mathbf{a} = [2, 2, 1]$ stated award M0
	METHOD 2			
	Solve $\left\{\begin{array}{l} x - y + 3z = 3 \\ 7x + 17y - 3z = 21 \end{array}\right.$ by eliminating one variable (e.g. z)	M1		In either order: For attempting to solve equations
	Use parameter for another variable (e.g. x) to find other variables in terms of t	M1		For attempting to find parametric solution
	(eg) $y = \frac{3}{2} - \frac{1}{2}t$, $z = \frac{3}{2} - \frac{1}{2}t$	A1√ A1√		For correct expression for one variable For correct expression for the other variable ft from equation in **(i)** for both
	Line is (eg) $\mathbf{r} = \left[0, \frac{3}{2}, \frac{3}{2}\right] + t[2, -1, -1]$	A1√		For stating equation of line. ft from parametric solutions
	METHOD 3			
	eg x, y or $z = 0$ in $\left\{\begin{array}{l} x - y + 3z = 3 \\ 7x + 17y - 3z = 21 \end{array}\right.$	M1		For attempting to find a point on the line
	$\Rightarrow \mathbf{a} = \left[0, \frac{3}{2}, \frac{3}{2}\right]$ *OR* $[3, 0, 0]$ *OR* $[1, 1, 1]$	A1√		For a correct vector. ft from equation in **(i)** **SR** a correct vector may be stated without working **SR** for $\mathbf{a} = [2, 2, 1]$ stated award M0
	eg $[3, 0, 0] - [1, 1, 1]$	M1		For finding another point on the line and using it with the one already found to find **b**
	$\mathbf{b} = k[2, -1, -1]$	A1√		For a correct vector. ft from equation in **(i)**
	Line is (eg) $\mathbf{r} = [1, 1, 1] + t[2, -1, -1]$	A1√		For stating equation of line. ft from **a** and **b**

6 (ii) contd	METHOD 4			
	A point on Π_1 is $[2+\lambda+\mu,\ 2+\lambda-5\mu,\ 1-2\mu]$	M1		For using parametric form for Π_1 and substituting into Π_2
	On $\Pi_2 \Rightarrow$ $[2+\lambda+\mu,\ 2+\lambda-5\mu,\ 1-2\mu]\,.\,[7,17,-3]=21$	A1		For correct unsimplified equation
	$\Rightarrow\ \lambda-3\mu=-1$	A1		For correct equation
	Line is (e.g.) $\mathbf{r}=[2,2,1]+(3\mu-1)[1,1,0]+\mu[1,-5,-2]$	M1		For substituting into Π_1 for λ *or* μ
	$\Rightarrow\ \mathbf{r}=[1,1,1]\ or\ \left[\frac{7}{3},\frac{1}{3},\frac{1}{3}\right]+t\,[2,-1,-1]$	A1		For stating equation of line
			$\boxed{9}$	
7 (i)	$\cos 3\theta+\mathrm{i}\sin 3\theta=\ c^3+3\mathrm{i}\,c^2s-3cs^2-\mathrm{i}\,s^3$	M1		For using de Moivre with $n=3$
	$\Rightarrow \cos 3\theta=c^3-3cs^2$ and $\sin 3\theta=3c^2s-s^3$	A1		For both expressions in this form (seen or implied) **SR** For expressions found without de Moivre M0 A0
	$\Rightarrow \tan 3\theta=\dfrac{3c^2s-s^3}{c^3-3cs^2}$	M1		For expressing $\dfrac{\sin 3\theta}{\cos 3\theta}$ in terms of *c* and *s*
	$\tan 3\theta=\dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}=\dfrac{\tan\theta(3-\tan^2\theta)}{1-3\tan^2\theta}$	A1	**4**	For simplifying to **AG**
(ii) (a)	$\theta=\frac{1}{12}\pi \Rightarrow\ \tan 3\theta=1$ $\Rightarrow\ 1-3t^2=t(3-t^2)\ \Rightarrow$ $t^3-3t^2-3t+1=0$	B1	**1**	For both stages correct **AG**
(b)	$(t+1)(t^2-4t+1)=0$	M1 A1		For attempt to factorise cubic For correct factors
	$\Rightarrow\ (t=-1),\ \ t=2\pm\sqrt{3}$	A1		For correct roots of quadratic
	– sign for smaller root $\Rightarrow$ $\tan\frac{1}{12}\pi=2-\sqrt{3}$	A1	**4**	For choice of – sign and correct root **AG**
(iii)	$\mathrm{d}t=(1+t^2)\,\mathrm{d}\theta$	B1		For differentiation of substitution and use of $\sec^2\theta=1+\tan^2\theta$
	$\Rightarrow \int_0^{\frac{1}{12}\pi}\tan 3\theta\ \mathrm{d}\theta$	B1		For integral with correct θ limits seen
	$=\left[\frac{1}{3}\ln\left(\sec 3\theta\right)\right]_0^{\frac{1}{12}\pi}=\frac{1}{3}\ln\left(\sec\frac{1}{4}\pi\right)$	M1		For integrating to $k\ln\left(\sec 3\theta\right)$ OR $k\ln\left(\cos 3\theta\right)$
	$=\frac{1}{3}\ln\sqrt{2}=\frac{1}{6}\ln 2$	M1		For substituting limits and $\sec\frac{1}{4}\pi=\sqrt{2}$ OR $\cos\frac{1}{4}\pi=\frac{1}{\sqrt{2}}$ seen
		A1	**5**	For correct answer aef
			$\boxed{14}$	

8 **(i)**	$a^2 = (ap)^2 = apap \Rightarrow a = pap$	B1		For use of given properties to obtain **AG**
	$p^2 = (ap)^2 = apap \Rightarrow p = apa$	B1	**2**	For use of given properties to obtain **AG** **SR** allow working from **AG** to obtain relevant properties
(ii)	$(p^2)^2 = p^4 = e \Rightarrow$ order $p^2 = 2$	B1		For correct order with no incorrect working seen
	$(a^2)^2 = (p^2)^2 = e \Rightarrow$ order $a = 4$	B1		For correct order with no incorrect working seen
	$(ap)^4 = a^4 = e \Rightarrow$ order $ap = 4$	B1		For correct order with no incorrect working seen
	$(ap^2)^2 = ap^2ap^2 = ap\,.\,a\,.\,p = a^2$ *OR* $ap^2 = a.a^2 = a^3 \Rightarrow$	M1		For relevant use of **(i)** or given properties
	$(ap^2)^2 = a^6 = a^2$ $\Rightarrow$ order $ap^2 = 4$	A1	**5**	For correct order with no incorrect working seen
(iii)	METHOD 1 $p^2 = a^2,\ ap^2 = a^3$	M2		For use of the given properties to simplify p^2 and ap^2
	$\Rightarrow \{e, a, p^2, ap^2\} = \{e, a, a^2, a^3\}$	A1		For obtaining a^2 and a^3
	which is a cyclic group	A1	**4**	For justifying that the set is a group
	METHOD 2 $\begin{array}{c\|cccc} & e & a & p^2 & ap^2 \\ \hline e & e & a & p^2 & ap^2 \\ a & a & p^2 & ap^2 & e \\ p^2 & p^2 & ap^2 & e & a \\ ap^2 & ap^2 & e & a & p^2 \end{array}$	M1 A1		For attempting closure with all 9 non-trivial products seen For all 16 products correct
	Completed table is a cyclic group	B2		For justifying that the set is a group
	METHOD 3 $\begin{array}{c\|cccc} & e & a & p^2 & ap^2 \\ \hline e & e & a & p^2 & ap^2 \\ a & a & p^2 & ap^2 & e \\ p^2 & p^2 & ap^2 & e & a \\ ap^2 & ap^2 & e & a & p^2 \end{array}$	M1 A1		For attempting closure with all 9 non-trivial products seen For all 16 products correct
	Identity = e	B1		For stating identity
	Inverses exist since EITHER: e is in each row/column OR: p^2 is self-inverse; a, ap^2 form an inverse pair	B1		For justifying inverses ($e^{-1} = e$ may be assumed)

(iv)	METHOD 1 e.g. $\left.\begin{array}{l} a\,.\,ap = a^2p = p^3 \\ ap\,.\,a = p \end{array}\right\} \Rightarrow$ not commutative	M1 M1 B1 A1	**4**	For attempting to find a non-commutative pair of elements, at least one involving *a* (may be embedded in a full or partial table) For simplifying elements both ways round For a correct pair of non-commutative elements For stating *Q* non-commutative, with a clear argument
	METHOD 2 Assume commutativity, so (eg) $ap = pa$ **(i)** $\Rightarrow$ $p = ap.a \Rightarrow p = pa.a = pa^2 = pp^2 = p^3$ But p and p^3 are distinct $\Rightarrow Q$ is non-commutative	M1 M1 B1 A1		For setting up proof by contradiction For using **(i)** and/or given properties For obtaining and stating a contradiction For stating *Q* non-commutative, with a clear argument
		$\boxed{15}$		

4728 Mechanics 1

1 i	$x^2 + (3x)^2 = 6^2$ $10x^2 = 36$ $x = 1.9(0)$ (1.8973..)	M1 A1 A1 [3]	Using Pythagoras, 2 squared terms May be implied Not surd form unless rationalised (3√10)/5, (6√10)/10
ii	$\tan\theta = 3x/x$ (= 3×1.9/1.9) = 3 $\theta = 71.6^\circ$ (71.565..)	M1 A2 [3]	Must target correct angle. Accept sin θ = 3×1.9/6 or cosθ = 1.9/6 which give θ=71.8°, θ=71.5° respectively, A1. **SR** $\theta = 71.6^\circ$ from tan$\theta = 3x/x$ if x is incorrect; x used A1, no evidence of x used A2
2 i		B1 B1 [2]	Inverted V shape with straight lines. Starts at origin, ends on t-axis, or horizontal axis if no labelling evident
ii	$6 = 3v/2$ $v = 4$ ms^{-1}	M1 A1 A1 [3]	Not awarded if special (right angled, isosceles) triangle assumed, or $s = (u+v)t/2$, or max v at specific t.
iii	T accn = 4/2.4 or s accn = 16/(2x2.4) T accn = 1 2/3 s or s accn = 10/3 Deceleration = 4/(3 - 1 2/3) or 16/2(6-10/3) Deceleration = 3 ms^{-2}	M1* A1 D*M1 A1 [4]	Uses $t = v/a$ or $s = v^2/2a$. May be implied Accept 4/(3 - 1.67) or 16/2(6-3.33) Accept 3.01; award however v = 4 obtained in (ii). a = -3 gets A0.
3 i	0.8gsin30 0.8x0.2 0.8×9.8sin30 - T = 0.8x0.2 T = 3.76 N AG	B1 B1 M1 A1 [4]	Not for 3.92 stated without justification Or 0.16 Uses N2L // to slope, 3 non-zero terms, inc ma Not awarded if initial B1 withheld.
ii	3.76 - F = 3×0.2 F = 3.16 3.16 = μx3×9.8 μ = 0.107 (0.10748)	M1 A1 A1 M1 A1 [5]	Uses N2L, B alone, 3 non-zero terms Needs *correct value* of T. May be implied. Uses $F=\mu R$ (Accept with R = 3, but not with R=0.8g(cos30), F=0.6, F=3.76, $F = f$(mass P)) Not 0.11, 0.108 (unless it comes from using g=9.81 consistently through question.

4 i	$v^2 = 7^2 - 2\times9.8\times2.1$ $v = 2.8$ ms^{-1}	M1 A1 A1 [3]	Uses $v^2 = u^2 - 2gs$. Accept $7^2 = u^2 + 2gs$
ii	$v = 0$ $0^2 = 7^2 - 2\times9.8s$ $s = 2.5$ m	B1 M1 A1 [3]	Velocity = 0 at greatest height Uses $0 = u^2 - 2gs$. Accept $7^2 = 2\times9.8s$.
iii	$v = -5.7$ (or $t = 0.71$ oef to reach greatest height) $-5.7 = 7 - 9.8t$ or $5.7 = (0+)\ 9.8T$ $t = 1.3(0)$ s (1.2959..)	B1 M1 A1 [3]	Allows for change of direction Uses $v = u$ + or – gt. Not 1.29 unless obtained from g=9.81 consistently
5 i	$0.5\times6 = 0.5v + m(v+1)$ $3 = 0.5v + mv + m$ $v(m + 0.5) = -m + 3$ AG	M1 A1 A1 [3]	Uses CoLM. Includes g throughout MR-1
ii	Momentum before = +/- $(4m - 0.5\times2)$ +/- $(4m - 0.5\times2) = mv + 0.5(v+1)$ $4m - 0.5\times2 = mv + 0.5(v+1)$ $v(m+0.5) = 4m - 1.5$	B1 M1 A1 A1 [4]	Includes g throughout MR-1 Needs opposite directions in CoLM on “before” side only. RHS in format am + b or b + am. Ignore values for a and b if quoted.
iii	$4m - 1.5 = - m + 3$ $5m = 4.5$ $m = 0.9$ kg AG $0.9 + v(0.9+0.5) = 3$ or $4\times0.9 - 1.5 = v(0.9+0.5)$ $v = (3-0.9)/(0.9+0.5) = 2.1/1.4$ $v = 1.5$ ms^{-1}	M1 A1 M1 A1 [4]	Attempts to obtain eqn in 1 variable from answers in (i) and (ii) Ignore $m = -0.5$ if seen Substitutes for m=0.9 in any m, v equation obtained earlier.
6 ia b	Perp = 10cos20 (= 9.3967 or 9.4) // = 10sin20 (= 3.4202) μ = 10sin20/10cos20 = tan20 (= 3.42/9.4) $\mu = 0.364$ (0.36397..) AG	B1 B1 [2] M1 A1 [2]	Includes g, MR -1 in part (i). Accept –ve values. Must use $\lvert F\rvert = \mu\lvert R\rvert$ Accept after inclusion of g twice
ii	*No misread, and resolving of 10 and T required* R = 10cos20 + Tcos45 F = Tcos45 - 10sin 20 or Tcos45 =μR + 10sin20 Tcos45 – 3.42 = 0.364(9.4 + Tcos45) $0.707T - 3.42 = 3.42 + 0.257T$ $0.45T = 6.84$ $T = 15.2$ N (15.209..)	M1* A1 M1* A1 D*M1 A1 A1 [7]	3 term equation perp plane, 2 unknowns 9.4 + 0.707T (accept 9.4+.71T) 3 term equation // plane, 2 unknowns 0.707T - 3.42 (accept 0.71T - 3.4) Substitutes for F and R in F=0.364R *Award final A1 only for T = 149 N after using 10g for weight*

7 i	$a = \mathrm{d}v/\mathrm{d}t$ $a = 6 - 2t$ ms^{-2}	M1 A1 [2]	Differentiation attempt. Answer 6-t implies division by t
ii	$s = \int v\mathrm{d}t$ $s = \int 6t - t^2\ \mathrm{d}t$ $s = 3t^2 - t^3/3\ (+c)$ $t = 0, v = 0, c = 0$ $t = 3, s = 3\text{x}3^2 - 3^3/3$ $s =$ 18 m AG	M1* A1 B1 D*M1 A1 [5]	Integration attempt on v Award if limits 0,3 used Requires earlier integration Does not require B1 to be earned.
iii	Distance remaining (= 100 -18) = 82 Total time = 3 + 82/9 T = 12.1 s (12 1/9)	B1 M1 A1 [3]	 Numerator not 100 Not 109/9
iv	Distance before slows = 18 + (22 - 3)x9 Distance while decelerating = 200 - 189 = 11 $11 = 9t - 0.3t^2$ or $11 = (9+8.23)t/2$ or $8.23 = 9-0.6t$ t = 1.28 (1.2765.., accept 1.3) T = 23.3 s (23.276..)	M1* A1 D*M1 A1 D*M1 A1 A1 [7]	(=189 m) Two sub-regions considered Accept 10.99. 10.9 penalise -1PA. Uses $s = ut - 0.5\text{x}0.6t^2$, or $v^2 = u^2 - 2\text{x}0.6s$ with $s=(u+v)t/2$ or $v=u+at$ Finds t. (If QE, it must have 3 terms and smaller positive root chosen.)

4729 Mechanics 2

1 (i)	$\frac{1}{2}\times75\times12^2$ or $\frac{1}{2}\times75\times3^2$ (either KE)	B1	M1 $12^2 = 3^2 + 2a \times 180$
	$75\times9.8\times40$ (PE)	B1	A1 $a = 0.375$ (3/8)
	$R\times180$ (change in energy = 24337)	B1	M1 $75 \times 9.8 \times \sin\theta - R = 75a$
	$\frac{1}{2}\times75\times12^2=\frac{1}{2}\times75\times3^2+75\times9.8\times40-R\times180$	M1	A1 $R = 135$
	$R = 135$ N	A1 **5**	(max 4 for no energy) **5**

2 (i)	$R = F = P/v = 44\,000/v = 1400$	M1	
	$v = 31.4$ m s^{-1}	A1 **2**	
(ii)	$44\,000/v = 1400 + 1100 \times 9.8 \times 0.05$	M1 A1	must have g
	$v = 22.7$ m s^{-1}	A1 **3**	
(iii)	$22\,000/10 + 1100\times9.8\times0.05 - 1400$	M1	
	$= 1100a$	A1	
	$a = 1.22$ m s^{-2}	A1 **3**	**8**

3 (i)	$\cos\theta = 5/13$ or $\sin\theta = 12/13$ or $\theta = 67.4°$	B1	any one of these
		M1	moments about A (ok without 70)
	$0.5\times F\sin\theta = 70\times1.4 + 50 \times 2.8$	A1	$0.5\sin\theta = 0.4615$
	$F = 516$ N	A1 **4**	**SR** 1 for 303 (omission of beam)
(ii)	$F \sin\theta = 120 + Y$ (resolving vertically)	M1	M1/A1 for moments
	$Y = 356$ ✓ their F × 12/13 – 120	A1 ✓	(B)$Y\times2.8+1.4\times70=2.3\times516$✓$\times12/13$
	$X = F\cos\theta$ (resolving horizontally)	M1	(C) $0.5\times Y = 0.9\times70 + 2.3\times50$
	$X = 198$ ✓ their F × 5/13	A1 ✓	(D) $1.2X = 1.4\times70 +2.8\times50$
	Force = $\sqrt{(356^2 + 198^2)}$	M1	
	407 or 408 N	A1 **6**	**10**

4 (i)	$T = 0.4 \times 0.6 \times 2^2$	M1	
	$T = 0.96$ N	A1 **2**	
(ii)	$S - T$	B1	may be implied
	$S - T = 0.1 \times 0.3 \times 2^2$	M1	
		A1	
	$S = 1.08$	A1 **4**	
(iii)	$v = r\omega$	M1	
	$v_P = 0.6$	A1	
	$v_B = 1.2$	A1	
	$\frac{1}{2}\times0.1\times0.6^2 + \frac{1}{2}\times0.4\times1.2^2$	M1	(0.018 + 0.288) separate speeds
	0.306	A1 **5**	**11**

5 (i)	$\bar{d} = (2\times 6 \sin\pi/4)/3\pi/4$ $\bar{d} = 3.60$	M1 A1 **2**	must be correct formula with rads AG
(ii)	$\bar{d}\cos45° =$ "2.55" $5\bar{x} = 3 \times 3 + 2 \times$ "2.55" $\bar{x} = 2.82$ $5\ \bar{y} = 3 \times 6 + 2 \times (12 +$ "2.55"$)$ $\bar{y} = 9.42$	B1 M1 A1 A1 M1 A1 A1 **7**	may be implied moments must not have areas 2kg/3kg misread (swap) gives (2.73,11.13) θ = 21.7° (MR – 2) (max 7 for (ii) + (iii)) SR -1 for $\bar{x}$, $\bar{y}$ swap
(iii)	$\tan\theta = 2.82/8.58$ $\theta = 18.2°$ ✓	M1 A1 **2**	M0 for their $\bar{x}$ / $\bar{y}$ ✓ their $\bar{x}/(18 - \bar{y})$ **11**

6 (i)	$I = 0.9 = 6\times0.2 - v \times 0.2$ $v = 1.5$	M1 A1 A1 **3**	needs to be mass 0.2
(ii)	$0.6 = (c - b)/6$ $6 \times 0.2 = 0.2b + 0.1c$ $b = 2.8$ $0.4 \times 5 + 0.2 \times 1.5 = 0.4a + 0.2 \times 6$ or $I = 0.9 = - 0.4a - - 0.4 \times 5$ $a = 2.75$ $2.75 < 2.8$ no further collision	M1 A1 M1 A1 A1 M1 A1 A1 M1 A1 **10**	restitution (allow 1.5 for M1) momentum (allow 1.5 for M1) 1st collision (needs their 1.5 for M1) compare *v*'s of *A* and *B* (calculated) **13**

7(i)	$9 = 17\cos25° \times t$ $t = 0.584$ (or 9/17cos25°) $d = 17\sin25°\times0.584 + ½\times9.8\text{x}\times0.584^2$ (d = *ht* lost (5.87) $h = 2.13$	M1 A1 M1 A1 A1 **5**	B1 $y=x\tan\theta-4.9x^2/v^2\cos^2\theta$ M1/A1 $y=9\tan(-25°)-4.9\times9^2/17^2\cos^2 25°$ A1 y = -5.87 2.13
(ii)	$v_h = 17\cos25°$ (15.4) $v_v = 17\sin25° + 9.8\times0.584$ or $v_v^{\ 2} = (17\sin25°)^2 + 2\times9.8\times5.87$ $v_v = 12.9$ $\tan\theta = 12.9/15.4$ $\theta = 40.0°$ below horizontal	B1 M1 A1 M1 A1 **5**	M1/A1 $dy/dx =$ $\tan\theta - 9.8x/v^2\cos^2\theta$ A1 $dy/dx = -0.838$ M1 $\tan^{-1}(-.838)$ or 50.0° to vertical
(iii)	speed $= \sqrt{(12.9^2 + 15.4^2)}$ $½mv^2 = ½m\times20.1^2 \times 0.7$ $v = 16.8$ m s^{-1}	M1 A1 ✓ M1 A1 **4**	(20.1) NB 0.3 instead of 0.7 gives 11.0 (M0) **14**

4730 Mechanics 3

1 i	Horiz. comp. of vel. after impact is 4ms^{-1} Vert. comp. of vel. after impact is $\sqrt{5^2 - 4^2}$ = 3ms^{-1} Coefficient of restitution is 0.5	B1 B1 B1 [3]	May be implied AG From e = 3/6
ii	Direction is vertically upwards Change of velocity is 3 – (-6) Impulse has magnitude 2.7Ns	B1 M1 A1 [3]	 From $m(\Delta v) = 0.3\times9$
2 i	Horizontal component is 14N $80\times1.5 = 14\times1.5 + 3Y$ or $3(80 – Y) = 80\times1.5 + 14\times1.5$ or $1.5(80 – Y) = 14\times0.75 + 14\times0.75 + 1.5Y$ Vertical component is 33N upwards	B1 M1 A1 A1 [4]	 For taking moments for *AB* about *A* or *B* or the midpoint of *AB* AG
ii	Horizontal component at *C* is 14N [Vertical component at *C* is $(\pm)\sqrt{50^2 - 14^2}$] [$W = (\pm)48 - 33$] Weight is 15N	B1 M1 DM1 A1 [4]	May be implied for using $R^2 = H^2 + V^2$ For resolving forces at *C* vertically
3 i	 $4\times3\cos60^o – 2\times3\cos60^o = 2b$ $b = 1.5$ **j** component of vel. of *B* = (-)3sin60° [$v^2 = b^2 + (-3\sin60^o)^2$] Speed ($3\text{ms}^{-1}$) is unchanged [Angle with l.o.c. = $\tan^{-1}(3\sin60^o/1.5)$] Angle is 60°.	M1 A1 A1 B1ft M1 A1ft M1 A1ft [8]	For using the p.c.mmtm parallel to l.o.c. ft consistent sin/cos mix For using $v^2 = b^2 + v_y^2$ AG ft - allow same answer following consistent sin/cos mix. For using angle = $\tan^{-1}(\pm v_y/v_x)$ ft consistent sin/cos mix
ii	[$e(3\cos60^o + 3\cos60^o) = 1.5$] Coefficient is 0.5	M1 A1ft [2]	For using NEL ft - allow same answer following consistent sin/cos mix throughout.

4 i	$F - 0.25v^2 = 120v(\mathrm{d}v/\mathrm{d}x)$	M1 A1	For using Newton's second law with $a = v(\mathrm{d}v/\mathrm{d}x)$
	$F = 8000/v$	B1	
	$[32000 - v^3 = 480v^2(\mathrm{d}v/\mathrm{d}x)]$	M1	For substituting for F and multiplying throughout by $4v$ (or equivalent)
	$\frac{480v^2}{v^3 - 32000}\frac{\mathrm{d}v}{\mathrm{d}x} = -1$	A1 [5]	AG
ii	$\int \frac{480v^2}{v^3 - 32000}\mathrm{d}v = -\int \mathrm{d}x$	M1	For separating variables and integrating
	$160\ \ln(v^3 - 32000) = -x \quad (+A)$	A1	
	$160\ \ln(v^3 - 32000) = -x + 160\ \ln 32000$ or $160\ \ln(v^3 - 32000) - 160\ \ln 32000 = -500$	M1 A1ft	For using $v(0) = 40$ or $[160\ \ln(v^3 - 32000)]^{v}_{40} = [-x]^{500}_{0}$ ft where factor 160 is incorrect but +ve,
	$(v^3 - 32000)/32000 = e^{-x/160}$ Speed of m/c is $32.2\mathrm{ms}^{-1}$	B1ft B1 [6]	Implied by $(v^3 - 32000)/32000 = e^{-3.125}$ (or = 0.0439 ..). ft where factor 160 is incorrect but +ve, or for an incorrect non-zero value of A
5 i	$x_{\max} = \sqrt{1.5^2 + 2^2} - 1.5\ (= 1)$	B1	
	$[T_{\max} = 18\times1/1.5]$	M1	For using $T = \lambda x/L$
	Maximum tension is 12N	A1 [3]	
	(a)	M1	For using EE = $\lambda x^2/2L$ May be scored with correct EE terms in expressions for total energy on release and total energy at lowest point
	Gain in EE = $2[18(1^2 - 0.2^2)]/(2\times1.5)$ (11.52)	A1	
	Loss in GPE = 2.8mg (27.44m)	B1	May be scored with correct GPE terms in expressions for total energy on release and total energy at lowest point
ii	$[2.8m \times 9.8 = 11.52]$	M1	For using the p.c.energy
	$m = 0.42$	A1 [5]	AG
	(b)	M1	For using the p.c.energy KE, PE & EE must all be represented
	$\frac{1}{2}mv^2 = mg(0.8) + 2\times18\times0.2^2/(2\times1.5)$ or $\frac{1}{2}mv^2 = 2\times18\times1^2/(2\times1.5) - mg(2)$	A1ft	ft only when just one string is considered throughout in evaluating EE
	Speed at M is $4.24\mathrm{ms}^{-1}$	A1ft [3]	ft only for answer 4.10 following consideration of only one string

6 **i**	[$-mg\sin\theta = m\,L(d^2\theta/dt^2)$] $d^2\theta/dt^2 = -(g/L)\sin\theta$	M1 A1 [2]	For using Newton's second law tangentially with $a = Ld^2\theta/dt^2$ AG
ii	[$d^2\theta/dt^2 = -(g/L)\ \theta$] $d^2\theta/dt^2 = -(g/L)\ \theta$ ➔ motion is SH	M1 A1 [2]	For using $\sin\theta \approx \theta$ because θ is small ($\theta_{max} = 0.05$) AG
iii	[$4\pi/7 = 2\pi/\sqrt{9.8/L}$] $L = 0.8$	M1 A1 [2]	For using $T = 2\pi/n$ where $-n^2$ is coefficient of θ
iv	[$\theta = 0.05\cos3.5\times0.7$] $\theta = -0.0385$ $t = 1.10$ (accept 1.1 or 1.09)	M1 A1ft M1 A1ft [4]	For using $\theta = \theta_o\cos nt$ {$\theta = \theta_o\sin nt$ not accepted unless the t is reconciled with the t as defined in the question} ft incorrect L {$\theta = 0.05\cos[4.9/(5L)^{½}]$} For attempting to find 3.5t ($\pi < 3.5t < 1.5\pi$) for which $0.05\cos3.5t$ = answer found for θ or for using $3.5(t_1 + t_2) = 2\pi$ ft incorrect L {$t = [2\pi\,(5L)^{½}]/7 - 0.7$}
v	$\dot{\theta}^2 = 3.5^2(0.05^2 - (-0.0385)^2)$ or $\dot{\theta} = -3.5\times0.05\sin(3.5\times0.7)$ ($\dot{\theta} = -0.1116..$) Speed is 0.0893ms^{-1} (Accept answers correct to 2 s.f.)	M1 A1ft A1ft [3]	For using $\dot{\theta}^2 = n^2(\theta_o^2 - \theta^2)$ or $\dot{\theta} = -n\ \theta_o\sin nt$ {also allow $\dot{\theta} = n\theta_o\cos nt$ if $\theta = \theta_o\sin nt$ has been used previously} ft incorrect θ with or without 3.5 represented by $(g/L)^{½}$ using incorrect L in (iii) or for $\dot{\theta} = 3.5\times0.05\cos(3.5\times0.7)$ following previous use of $\theta = \theta_o\sin nt$ ft incorrect L ($L\times0.089287/0.8$ with $n = 3.5$ used or from $\lvert 0.35\sin\{4.9/[5L]^{½}\}/[5L]^{½}\rvert$ **SR** for candidates who use $\dot{\theta}$ as v. (Max 1/3) For v = ±0.112 B1

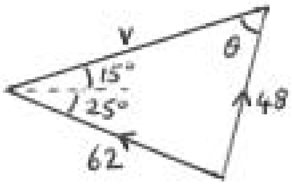
7 i	Gain in PE = $mga(1 - \cos\theta)$ [½ mu^2 – ½ mv^2 = $mga(1 - \cos\theta)$]	B1 M1	For using KE loss = PE gain
	$v^2 = u^2 - 2ga(1 - \cos\theta)$ [$R - mg\cos\theta = m$(accel.)] $R = mv^2/a + mg\cos\theta$ [$R = m\{u^2 - 2ga(1 - \cos\theta)\}/a + mg\cos\theta$] $R = mu^2/a + mg(3\cos\theta - 2)$	A1 M1 A1 M1 A1 [7]	For using Newton's second law radially For substituting for v^2 AG
ii	[$0 = mu^2/a - 5mg$] $u^2 = 5ag$ [$v^2 = 5ag - 4ag$] Least value of v^2 is ag	M1 A1 M1 A1 [4]	For substituting $R = 0$ and $\theta = 180^\circ$ For substituting for u^2 (= $5ag$) and $\theta = 180^\circ$ in v^2 (expression found in (i)) { but M0 if $v = 0$ has been used to find u^2} AG
iii	[$0 = u^2 - 2ga(1 - \sqrt{3}/2)$] $u^2 = ag(2 - \sqrt{3})$	M1 A1 [2]	For substituting $v^2 = 0$ and $\theta = \pi/6$ in v^2 (expression found in (i)) Accept $u^2 = 2ag(1 - \cos\pi/6)$

4731 Mechanics 4

1 (i)	Using $\omega_2^{\,2} = \omega_1^{\,2} + 2\alpha\theta$, $67^2 = 83^2 + 2\alpha \times 1000$	M1	
	$\alpha = -1.2$	A1	
	Angular deceleration is $1.2\ \text{rad}\,\text{s}^{-2}$	[**2**]	
(ii)	Using $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$,	M1	
	$400 = 83t - 0.6t^2$	A1ft	
	$t = 5\ \ or\ \ 133\frac{1}{3}$	M1	Solving to obtain a value of *t*
	Time taken is 5 s	A1 [**4**]	
	Alternative for (ii) $\omega_2^{\,2} = 83^2 - 2 \times 1.2 \times 400$ M1A1 ft $\omega_2 = 77$ $77 = 83 - 1.2t$ M1 $t = 5$ A1		*(M0 if $\omega = 67$ is used in (ii))*

2	Volume $V = \int \pi y^2 \mathrm{d}x = \int_a^{2a} \pi \frac{a^6}{x^4} \mathrm{d}x$	M1	*π may be omitted throughout*
	$= \pi \left[-\frac{a^6}{3x^3} \right]_a^{2a} = \frac{7}{24}\pi a^3$	A1	For integrating x^{-4} to obtain $-\frac{1}{3}x^{-3}$
	$V\bar{x} = \int \pi x y^2 \mathrm{d}x$	M1	for $\int x y^2 \mathrm{d}x$
	$= \int_a^{2a} \pi \frac{a^6}{x^3} \mathrm{d}x$	A1	Correct integral form (including limits)
	$= \pi \left[-\frac{a^6}{2x^2} \right]_a^{2a} = \frac{3}{8}\pi a^4$	A1	For integrating x^{-3} to obtain $-\frac{1}{2}x^{-2}$
	$\bar{x} = \dfrac{\frac{3}{8}\pi a^4}{\frac{7}{24}\pi a^3}$	M1	*Dependent on previous M1M1*
	$= \dfrac{9a}{7}$	A1 [**7**]	

3 (i)	$I = \frac{1}{2}(4m)(2a)^2 + (4m)a^2$ $+ m(3a)^2$ $= 21ma^2$	M1 A1 B1 A1 **[4]**	Applying parallel axes rule
(ii)	From P, $\bar{x} = \frac{(4m)a + m(3a)}{5m} \quad (= \frac{7a}{5})$ Period is $2\pi\sqrt{\frac{21ma^2}{5mg(\frac{7}{5}a)}}$ $= 2\pi\sqrt{\frac{3a}{g}}$	M1 M1 A1 ft A1 **[4]**	Correct formula $2\pi\sqrt{\frac{I}{mgh}}$ seen *or* using $L = I\ddot{\theta}$ and period $2\pi / \omega$
	Alternative for (ii) $-4mga\sin\theta - mg(3a)\sin\theta = (21ma^2)\ddot{\theta}$ M1 M1 Period is $2\pi\sqrt{\frac{21ma^2}{7mga}} = 2\pi\sqrt{\frac{3a}{g}}$ A1 ft A1		Using $L = I\ddot{\theta}$ with three terms Using period $2\pi / \omega$

4 (i)	 $\frac{\sin\theta}{62}=\frac{\sin 40}{48}$ $\theta = 56.1°$ *or* $123.9°$ Bearings are $018.9°$ and $311.1°$	M1 M1 A1 A1A1 **[5]**	Velocity triangle One value sufficient Accept $19°$ and $311°$
(ii)	Shorter time when $\theta = 56.1°$ $\frac{v}{\sin 83.87}=\frac{48}{\sin 40}$ Relative speed is $v = 74.25$ Time to intercept is $\frac{3750}{74.25}$ $= 50.5 \text{ s}$	B1 ft M1 M1 A1 **[4]**	Or $v^2 = 62^2 + 48^2 - 2\times 62\times 48\cos 83.87$ *Dependent on previous M1*
	Alternative for (i) and (ii) $\begin{pmatrix}48\sin\phi\\48\cos\phi\end{pmatrix}t=\begin{pmatrix}3750\sin 75\\3750\cos 75\end{pmatrix}+\begin{pmatrix}62\sin 295\\62\cos 295\end{pmatrix}t$ M1 M1 $3.732\cos\phi - \sin\phi = 3.208$ A1 M1 M1 $\phi = 18.9°$ and $311.1°$ A1A1 B1 ft $t = 50.5$ A1		component eqns (displacement or velocity) obtaining eqn in ϕ *or* t *or* v $(=3750/t)$ correct simplified equation or $t^2 - 231.3t + 9131.5 = 0$ $[t = 50.5,\ 180.8]$ or $v^2 - 94.99v + 1540 = 0\ [v = 74.25, 20.74]$ solving to obtain a value of ϕ solving to obtain a value of t *(max A1 if any extra values given)* appropriate selection for shorter time

5 (i)	Area is $\int_0^2 (8-x^3)\,\mathrm{d}x \;=\left[\;8x-\tfrac{1}{4}x^4\;\right]_0^2=12$	B1	
	Mass per m^2 is $\rho=\frac{63}{12}=5.25$	M1	
	$I_y=\sum(\rho\, y\,\delta x)x^2=\rho\int x^2 y\,\mathrm{d}x$	M1	for $\int x^2 y\,\mathrm{d}x$ or $\int x^3\,\mathrm{d}y$
	$=\rho\int_0^2(8x^2-x^5)\,\mathrm{d}x$	A1	or $\frac{1}{3}\rho\int_0^8(8-y)\,\mathrm{d}y$
	$=\rho\left[\;\tfrac{8}{3}x^3-\tfrac{1}{6}x^6\;\right]_0^2=\frac{32}{3}\rho$	A1	for $\frac{32}{3}$
	$=\frac{32}{3}\times\frac{63}{12}=56\ \mathrm{kg\,m^2}$	A1 AG [6]	
(ii)	Anticlockwise moment is $800-63\times 9.8\times\frac{4}{5}$ $=306.08\ \mathrm{N\,m}\;>0$	M1	
	so it will rotate anticlockwise	A1 [2]	Full explanation is required; (anti)clockwise should be mentioned before the conclusion
(iii)	$I=I_x+I_y=1036.8+56\quad(=1092.8)$	B1	
	WD by couple is $800\times\frac{1}{2}\pi$	B1	
	Change in PE is $63\times 9.8\times\left(\frac{24}{7}-\frac{4}{5}\right)$	B1	
	$800\times\frac{1}{2}\pi=\frac{1}{2}I\omega^2-63\times 9.8\times\left(\frac{24}{7}-\frac{4}{5}\right)$	M1 A1	Equation involving WD, KE and PE *May have an incorrect value for I; other terms and signs are cao*
	$1256.04=546.4\omega^2-1622.88$ $\omega=2.30\ \mathrm{rad\,s^{-1}}$	A1 [6]	

6 (i)	GPE is $mg(a\sin 2\theta)$	B1	Or $mg(2a\cos\theta\sin\theta)$
	$\mathrm{AB} = 2a\cos\theta$ *or* $\mathrm{AB}^2 = a^2 + a^2 - 2a^2\cos(\pi - 2\theta)$ EPE is $\frac{\sqrt{3}mg}{2a}(2a\cos\theta)^2$	B1	Any correct form
	$= \sqrt{3}mga(1+\cos 2\theta)$	M1	Expressing EPE and GPE in terms of $\cos 2\theta$ and $\sin 2\theta$
	Total PE is $V = \sqrt{3}mga(1+\cos 2\theta) + mga\sin 2\theta$ $= mga(\sqrt{3} + \sqrt{3}\cos 2\theta + \sin 2\theta)$	A1 AG [**4**]	
(ii)	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = mga(-2\sqrt{3}\sin 2\theta + 2\cos 2\theta)$	B1	(B0 for $\frac{\mathrm{d}V}{\mathrm{d}\theta} = -2\sqrt{3}\sin 2\theta + 2\cos 2\theta$)
	$= 0$ when $2\sqrt{3}\sin 2\theta = 2\cos 2\theta$ $\tan 2\theta = \frac{1}{\sqrt{3}}$	M1	
	$\theta = \frac{\pi}{12},\ -\frac{5\pi}{12}$	M1 A1A1 [**5**]	Solving to obtain a value of θ Accept $0.262,\ -1.31$ or $15°,\ -75°$
(iii)	$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = mga(-4\sqrt{3}\cos 2\theta - 4\sin 2\theta)$	B1ft	
	When $\theta = \frac{\pi}{12},\ \frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -8mga < 0$	M1	Determining the sign of V'' *or* M2 for alternative method for max / min
	so this position is unstable	A1	
	When $\theta = -\frac{5\pi}{12},\ \frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = 8mga > 0$ so this position is stable	A1 [**4**]	

7 (i)	Initially $\cos\theta = \frac{0.6}{1.5} = 0.4$ $\frac{1}{2}\times 4.9\omega^2 = 6\times 9.8(0.5\times 0.4 - 0.5\cos\theta)$ $\omega^2 = 12(0.4 - \cos\theta)$ $\omega^2 = 4.8 - 12\cos\theta$	M1 A1 A1 AG **[3]**	Equation involving KE and PE
(ii)	$6\times 9.8\times 0.5\sin\theta = 4.9\alpha$ $\alpha = 6\sin\theta \quad (\text{rad s}^{-2})$	M1 A1 **[2]**	or $2\omega\frac{\mathrm{d}\omega}{\mathrm{d}\theta} = 12\sin\theta$ or $2\omega\frac{\mathrm{d}\omega}{\mathrm{d}t} = 12\sin\theta\frac{\mathrm{d}\theta}{\mathrm{d}t}$
(iii)	$6\times 9.8\cos\theta - F = 6\times 0.5\omega^2$ $58.8\cos\theta - F = 14.4 - 36\cos\theta$ $F = 94.8\cos\theta - 14.4$ $6\times 9.8\sin\theta - R = 6\times 0.5\alpha$ $58.8\sin\theta - R = 18\sin\theta$ $R = 40.8\sin\theta$	M1 M1 A1 AG M1 M1 A1 **[6]**	for radial acceleration $r\omega^2$ radial equation of motion *Dependent on previous M1* for transverse acceleration $r\alpha$ transverse equation of motion *Dependent on previous M1*
(iv)	If *B* reaches the ground, $\cos\theta = -0.4$ $F = -52.32$ $\sin\theta = \sqrt{0.84}$ [$\theta = 1.982$ *or* $113.6°$] $R = 37.39$ Since $\frac{52.32}{37.39} = 1.40 > 0.9$, this is not possible	M1 A1 M1 A1 **[4]**	*Allow M1A0 if* $\cos\theta = +0.4$ *is used* Obtaining a value for *R* Or $\mu R = 33.65$, and $52.32 > 33.65$
	Alternative for (iv) Slips when $F = -0.9R$ $94.8\cos\theta - 14.4 = -36.72\sin\theta$ M1 $\theta = 1.798$ [$103.0°$] A1 *B* reaches the ground when $\cos\theta = -0.4$ M1 $\theta = 1.982$ [$113.6°$] so it slips before this A1		*Allow M1A0 if* $F = +0.9R$ *is used* *Allow M1A0 if* $\cos\theta = +0.4$ *is used*

4732 Probability & Statistics 1

1			Q1: if consistent "0.8" incorrect or $^1/_8$, $^7/_8$ or 0.02 allow M marks in ii , iii & 1st M1 in i
i	Binomial stated 0.9437 – 0.7969 or $^8C_3\times0.2^3\times0.8^5$ = 0.147 (3 sfs)	M1 M1 A1 3	or implied by use of tables or $^8C_3$ or $0.2^a\times0.8^b$ $(a+b = 8)$
ii	1– 0.7969 = 0.203 (3 sf)	M1 A1 2	allow 1– 0.9437 or 0.056(3) or equiv using formula
iii	8 × 0.2 oe 1.6	M1 A1 2	8 × 0.2 = 2 M1A0 1.6 ÷ 8 or $^1/_{1.6}$ M0A0
Total		**7**	
2	first two d's = ±1 Σd^2 attempted (= 2) $1-\frac{6\times\text{"2"}}{7(7^2-1)}$ = $^{27}/_{28}$ or 0.964 (3 sfs)	B1 M1 M1dep A1	S_{xx} or S_{yy} = 28 B1 S_{xy} = 27 B1 $S_{xy}/\sqrt{(S_{xx}S_{yy})}$ M1 dep B1 1234567 & 1276543 (ans $^2/_7$): MR, lose A1
Total		**4**	
3 i	x independent or controlled or changed Value of y was measured for each x x not dependent	B1 1	Allow Water affects yield, or yield is dependent or yield not control water supply Not just y is dependent Not x goes up in equal intervals Not x is fixed
ii	(line given by) minimum sum of squs	B1 B1 2	B1 for "minimum" or "least squares" with inadequate or no explanation
iii	S_{xx} = 17.5 or 2.92 S_{yy} = 41.3 or 6.89 S_{xy} = 25 or 4.17 $r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}}$ = 0.930 (3 sf)	B1 M1 A1 3	or $91 - 21^2/_6$ or $394 - 46^2/_6$ B1 for any one or $186 - {}^{21\times46}/_6$ dep B1 0.929 or 0.93 with or without wking B1M1A0 **SC** incorrect n: max B1M1A0
iv	Near 1 or lg, high, strong, good corr'n or relnship oe Close to st line or line good fit	B1ft B1 2	$\lvert r\rvert$ small: allow little (or no) corr'n oe Not line accurate. Not fits trend
Total		**8**	

4			Q4: if consistent "0.7" incorrect or $^1/_3$, $^2/_3$ or 0.03 allow M marks in ii , iii & 1st M1 in i
i	Geo stated $0.7^3 \times 0.3$ alone $^{1029}/_{10000}$ or 0.103 (3 sf)	M1 M1 A1 3	or implied by $q^n \times p$ alone ($n > 1$) $0.7^3 - 0.7^4$
ii	0.7^4 alone = $^{2401}/_{10000}$ or 0.240 (3 sf)	M1 A1 2	$1 - (0.3 + 0.7\times0.3 + 0.7^2\times0.3 + 0.7^3\times0.3)$ NB $1-0.7^4$: M0
iii	$1 - 0.7^5$ = 0.832 (3 sfs)	M2 A1 3	or $0.3 + 0.7\times0.3 + + \ldots. \ +0.7^4\times0.3$ M2 M1 for one term extra or omitted or wrong or for 1– (above) M1 for $1-0.7^6$ or 0.7^5 NB Beware: $1 - 0.7^6 = 0.882$
		8	
5i	$^{25}/_{10}$ = 2.5	M1 A1 2	Allow $^{25}/_{(9\text{to}10)}$ or 2.78: M1
ii	(19.5, 25) (9.5, 0)	B1 B1 2	Allow (24.5, 47) Both reversed: SC B1 If three given, ignore (24.5, 47)
iii	Don't know exact or specific values of x (or min or max or quartiles or median or whiskers). oe Can only estimate (min or max or quartiles or median or whiskers) oe Can't work out (…..) oe Data is grouped oe	B1 1	Exact data not known Allow because data is rounded
Total		**5**	

6i	$\Sigma x \div 11$ 70 Σx^2 attempted $\sqrt{\frac{\sum x^2}{11} - \bar{x}^2}$ $=\sqrt{(^{54210}/_{11} - 70^2)}$ or $\sqrt{28.18}$ or 5.309 (= 5.31) **AG**	M1 A1 M1 A1 4	≥ 5 terms, or $\sum(x - \bar{x})^2$ or $\sqrt{\frac{\sum(x-\bar{x})^2}{11}}$ $= \sqrt{^{310}/_{11}}$ or $\sqrt{28.18}$ ie correct substn or result If × $^{11}/_{10}$: M1A1M1A0
ii	Attempt arrange in order med = 67 74 and 66 IQR = 8	M1 A1 M1 A1 4	or (72.5 – 76.5) – (65.5 – 66.5) incl must be from 74 – 66
			iii, iv & v: ignore extras
iii	no (or fewer) extremes this year oe sd takes account of all values sd affected by extremes less spread tho' middle 50% same less spread tho' 3rd & 9th same or same gap	B1 1	fewer high &/or low scores highest score(s) less than last year Not less spread or more consistent Not range less
iv	sd measures spread or variation or consistency oe	B1 1	sd less means spread is less oe or marks are closer together oe
v	more consistent, more similar, closer together, nearer to mean less spread	B1 1	allow less variance Not range less Not highest & lowest closer
Total		**11**	
7i	$^8C_3$ = 56	M1 A1 2	
ii	$^7C_2$ or or $^7P_2$ / $^8P_3$ ÷($^8C_3$ or "56") only = $^3/_8$ $^1/_8$ not from incorrect × 3 only or $^1/_8+^7/_8\times^1/_7+^7/_8\times^6/_7\times^1/_6$	M1 M1 A1 3	$^8C_1+^7C_1+^6C_1$ or 21 or 8×7×6 or $^{..}/_8\times^{..}/_7\times^{..}/_6$ indep, dep ans < 1 $^7/_8\times^6/_7\times^5/_6$ 1 – prod 3 probs
iii	$^8P_3$ or 8×7×6 or $^8C_1\times^7C_1\times^6C_1$ or 336 1 ÷ $^8P_3$ only = $^1/_{336}$ or 0.00298 (3 sf)	M1 M1 A1 3	$^1/_8\times^1/_7\times^1/_6$ only M2 If × or ÷: M1 $(^1/_8)^3$ M1
Total		**8**	

8ia	${}^{18}/_{19}$ or ${}^{1}/_{19}$ seen ${}^{17}/_{18}$ or ${}^{1/}{}_{18}$ seen structure correct ie 6 branches all correct incl. probs and W & R	B1 B1 B1 B1 4	 regardless of probs & labels (or 14 branches with correct 0s & 1s)
b	${}^{1}/_{20} + {}^{19}/_{20} \times {}^{1}/_{19} + {}^{19}/_{20} \times {}^{18}/_{19} \times {}^{1}/_{18}$ $= {}^{3}/_{20}$	M2 A1 3	M1 any 2 correct terms added ${}^{19}/_{20}\times{}^{18}/_{19}\times{}^{17}/_{18}$ $1 - {}^{19}/_{20}\times{}^{18}/_{19}\times{}^{17}/_{18}$
iia	${}^{19}/_{20} \times {}^{18}/_{19}$ $= {}^{9}/_{10}$ oe	M1 A1 2	${}^{19}/_{20}\times{}^{18}/_{19}\times{}^{1}/_{18}+{}^{19}/_{20}\times{}^{18}/_{19}\times{}^{17}/_{18}$ or ${}^{1}/_{20}+{}^{17}/_{20}$
b	$(P(X = 1) = {}^{1}/_{20})$ ${}^{19}/_{20} \times {}^{1}/_{19}$ $= {}^{1}/_{20}$ Σxp $= {}^{57}/_{20}$ or 2.85	 M1 A1 M1 A1 4	 or $1 - ({}^{1}/_{20} + {}^{9}/_{10})$ or 2 probs of ${}^{1}/_{20}$ M1A1 $\geq$ 2 terms, ft their p's if $\Sigma p = 1$ NB: ${}^{19}/_{20}\times 3 = 2.85$ no mks
			With replacement:
ia			Original scheme
ib			${}^{1}/_{20} + {}^{19}/_{20} \times {}^{1}/_{20} + ({}^{19}/_{20})^2 \times {}^{1}/_{20}$ or $1 - ({}^{19}/_{20})^2$ M1
iia			$({}^{19}/_{20})^2$ or $({}^{19}/_{20})^2\times{}^{1}/_{20} + ({}^{19}/_{20})^2\times{}^{19}/_{20}$ M1
b			Original scheme But NB ans 2.85(25…) M1A0M1A0
Total		**13**	

9i	$(1-0.12)^n$		M1	Can be implied by 2nd M1 allow $n-1$
	$\frac{\log 0.05}{\log 0.88}$	or $0.88^{23} = 0.052...$ or $0.88^{24} = 0.046...$	M1	or $\log_{0.88}0.05$ or 23.4(…)
	$n = 24$		A1 3	Ignore incorrect inequ or equals signs
ii	$^6C_2 \times 0.88^4 \times 0.12^2$ (= 0.1295…)		M3	or $0.88^4 \times 0.12^2$ M2 or $^6C_2 \times 0.88^4 \times 0.12^2$ + extra M2 or 2 successes in 6 trials implied or $^6C_2$ M1
	× 0.12		M1	dep ≥ M1
	= 0.0155		A1 5	$0.88^4 \times 0.12^2 \times 0.12$: M2M1 $0.88^4 \times 0.12^3$ M0M0A0 unless clear P(2 success in 6 trials) × 0.12 in which case M2M1A0
Total			**8**	

Total 72 marks

4733 Probability & Statistics 2

1	$\frac{105.0-\mu}{\sigma} = -0.7$; $\frac{110.0-\mu}{\sigma} = -0.5$ Solve: $\sigma = 25$ $\mu = 122.5$	M1 A1 B1 M1 A1 A1 **6**	Standardise once, equate to Φ^{-1}, allow σ^2 Both correct including signs & σ, no cc (continuity correction), allow wrong z Both correct z-values. "1 –" errors: M1A0B1 Get either μ or σ by solving simultaneously σ a.r.t. 25.0 $\mu = 122.5 \pm 0.3$ or 123 if clearly correct, allow from σ^2 but *not* from $\sigma = -25$.
2	Po(20) ≈ N(20, 20) Normal approx. valid as $\lambda > 15$ $1-\Phi\left(\frac{24.5-20}{\sqrt{20}}\right) = 1 - \Phi(1.006)$ = 1 – 0.8427 **= 0.1573**	M1 A1 B1 M1 A1 A1 **6**	Normal stated or implied (20, 20) or (20, √20) or (20, 20^2), can be implied "Valid as $\lambda > 15$", *or* "valid as λ large" Standardise 25, allow wrong or no cc, √20 errors $1.0 < z \le 1.01$ Final answer, art 0.157
3	$H_0 : p = 0.6$, $H_1 : p < 0.6$ where p is proportion in population who believe it's good value $R \sim B(12, 0.6)$ α: $P(R \le 4) = 0.0573$ > 0.05	B2 M1 A1 B1	Both, B2. Allow π, % One error, B1, except x or $\bar{x}$ or r or R: 0 B(12, 0.6) stated or implied, e.g. N(7.2, 2.88) *Not* P(< 4) or P(≥ 4) or P(= 4) Must be using P(≤ 4), or P(> 4) < 0.95 and binomial
	β: CR is ≤ 3 and 4 > 3 $p = 0.0153$	B1 A1	Must be using CR; explicit comparison needed
	Do not reject H_0. Insufficient evidence that the proportion who believe it's good value for money is less than 0.6	M1 A1 **7**	Correct conclusion, needs B(12,0.6) and ≤ 4 Contextualised, some indication of uncertainty [**SR**: N(7.2, …) or Po(7.2): poss B2 M1A0] [**SR**: P(< 4) or P(= 4) or P(≥ 4): B2 M1A0]
4 (i)	Eg "not all are residents"; "only those in street asked"	B1 B1 **2**	One valid relevant reason A definitely different valid relevant reason *Not* "not a random sample", *not* "takes too long"
(ii)	Obtain list of whole population Number it sequentially Select using random numbers [Ignore method of making contact]	B1 B1 B1 **3**	"Everyone" or "all houses" must be implied *Not* "number it with random numbers" unless then "arrange in order of random numbers" **SR**: "Take a random sample": B1 **SR**: Systematic: B1 B0, B1 if start randomly chosen
(iii)	Two of: α: Members of population equally likely to be chosen β: Chosen independently/randomly γ: Large sample (e.g. > 30)	B1 B1 **2**	One reason. NB : If "independent", must be "chosen" independently, not "views are independent" Another reason. Allow "fixed sample size" but not both that and "large sample". Allow "houses"

5	(i)	Bricks scattered at constant average rate & independently of one another	B1 B1	 2	B1 for each of 2 different reasons, in context. (Treat "randomly" ≡ "singly" ≡ "independently")
	(ii)	Po(12) P(≤ 14) – P(≤ 7) [= .7720 – .0895] [or P(8) + P(9) + … + P(14)] **= 0.6825**	B1 M1 A1	 3	Po(12) stated or implied Allow one out at either end or both, eg 0.617, or wrong column, but *not* from Po(3) nor, eg, .9105 – .7720 Answer in range [0.682, 0.683]
	(iii)	$e^{-\lambda} = 0.4$ $\lambda = -\ln(0.4)$ = 0.9163 Volume = 0.9163 ÷ 3 = **0.305**	B1 M1 A1 M1	 4	This equation, aef, can be implied by, eg 0.9 Take ln, or 0.91 by T & I λ art 0.916 or 0.92, can be implied Divide their λ value by 3 [**SR**: Tables, eg 0.9÷3: B1 M0 A0 M1]
6	(i)	33.6 $\frac{115782.84}{100} - 33.6^2$ [= 28.8684] $\times\frac{100}{99}$ **= 29.16**	B1 M1 M1 A1	 4	33.6 clearly stated [not recoverable later] Correct formula used for biased estimate $\times\frac{100}{99}$, M's independent. Eg $\frac{\Sigma r^2}{99}[-33.6^2]$ **SR** B1 variance in range [29.1, 29.2]
	(ii)	$\bar{R} \sim \text{N}(33.6, 29.16/9)$ $= \text{N}(33.6, 1.8^2)$ $1-\Phi\left(\frac{32-33.6}{\sqrt{3.24}}\right)$ [= Φ(0.8889)] **= 0.8130**	M1 A1 M1 A1	 4	Normal, their μ, stated or implied Variance [their (i)]÷9 [*not* ÷100] Standardise & use Φ, 9 used, answer > 0.5, allow √ errors, allow cc 0.05 but *not* 0.5 Answer, art 0.813
	(iii)	No, distribution of R is normal so that of $\bar{R}$ is normal	B2	2	Must be saying this. Eg "9 is not large enough": B0. Both: B1 max, unless saying that n is irrelevant.
7	(i)	$\frac{2}{9}\int_0^3 x^3(3-x)dx = \frac{2}{9}\left[\frac{3x^4}{4}-\frac{x^5}{5}\right]_0^3$ [= 2.7] – $(1\frac{1}{2})^2$ $= \frac{9}{20}$ or **0.45**	M1 A1 B1 M1 A1	 5	Integrate x^2f(x) from 0 to 3 [*not* for μ] Correct indefinite integral Mean is 1½, soi [not recoverable later] Subtract their μ^2 Answer art 0.450
	(ii)	$\frac{2}{9}\int_0^{0.5} x(3-x)dx = \frac{2}{9}\left[\frac{3x^2}{2}-\frac{x^3}{3}\right]_0^{0.5}$ $= \frac{2}{27}$ AG	M1 A1	 2	Integrate f(x) between 0, 0.5, must be seen somewhere Correctly obtain given answer $\frac{2}{27}$, decimals other than 0.5 not allowed, 1 more line needed (eg [ ] = ⅓)
	(iii)	B(108, $\frac{2}{27}$) ≈ N(8, 7.4074) $1-\Phi\left(\frac{9.5-8}{\sqrt{7.4074}}\right)$ = 1 – Φ(0.5511) **= 0.291**	B1 M1 A1 M1 A1 A1	 6	B(108, $\frac{2}{27}$) seen or implied, eg Po(8) Normal, mean 8 … … variance (or SD) 200/27 or art 7.41 Standardise 10, allow √ errors, wrong or no cc, needs to be using B(108,…) Correct √ and cc Final answer, art 0.291

	(iv)	$\overline{X} \sim N(1.5, \frac{1}{240})$	B1 B1√ B1√ **3**	Normal NB: *not* part (iii) Mean their μ Variance or SD (their 0.45)/108 [not (8, 50/729)]
8	(i)	$H_0 : \mu = 78.0$ $H_1 : \mu \neq 78.0$ $z = \frac{76.4 - 78.0}{\sqrt{68.9/120}} = -2.1115$ > -2.576 or $0.0173 > 0.005$	B1 B1 M1 A1 B1	Both correct, B2. One error, B1, but *x* or $\bar{x}$: B0. Needs ±(76.4 – 78)/√(*σ*÷120), allow √ errors art –2.11, or *p* = 0.0173 ± 0.0002 Compare *z* with (–)2.576, or *p* with 0.005
		$78 \pm z\sqrt{(68.9/120)}$ $= 76.048$ $76.4 > 76.048$	M1 A1√ B1	Needs 78 and 120, can be – only Correct CV to 3 sf, √ on *z* *z* = 2.576 and compare 76.4, allow from 78 ↔ 76.4
		Do not reject H_0. Insufficient evidence that the mean time has changed	M1 A1√ **7**	Correct comparison & conclusion, needs 120, "like with like", correct tail, $\bar{x}$ and μ right way round Contextualised, some indication of uncertainty
	(ii)	$\frac{1}{\sqrt{68.9/n}} > 2.576$ $\sqrt{n} > 21.38$, $n_{\min} = \mathbf{458}$ Variance is estimated	M1 M1 A1 B1 **4**	IGNORE INEQUALITIES THROUGHOUT Standardise 1 with *n* and 2.576, allow √ errors, cc etc but *not* 2.326 Correct method to solve for √*n* (*not* from *n*) 458 only (*not* 457), *or* 373 from 2.326, signs correct Equivalent statement, allow "should use *t*". In principle nothing superfluous, but "variance stays same" B1 bod

Specimen Answers

Question 4: Part (i)

α	Takes too long/too slow	B0
β	Interviewing people in the street isn't a random sample	B0
γ	Many tourists so not representative	B1
δ	Those who don't shop won't have their views considered	B1
ε	Interviewers biased as to who they ask	B1
ζ	Views influenced by views of others	B1

Part (ii)

α	Choose a random sample of the town and ask their opinion	B1
β	Choose names at random from the town's phone book	B1
γ	A random number machine determines which house numbers should be used, and every street should have the same proportion of residents interviewed	B0B0B1
δ	Visit everyone door to door and give them a questionnaire	B1B0B0
ε	Assign everyone a number and select randomly	B1B0B0
ζ	Assign everyone a number and select using random numbers	B1B0B1
η	Ditto + "ignoring numbers that don't correspond to a resident"	B1B1B1
θ	Assign each eligible person a number and pick numbers from a hat	B1B1B0
ι	Put names of all residents into a hat and pick them out	B1B1B0

[NB: postal survey is biased]

Part (iii)

α	One person's view should not affect another's	B0
β	It is without bias	B0
γ	Results occur randomly	B0
δ	Should be asked if they are for or against (binomial testing)	B0
ε	It will survey a diverse group from different areas so should be representative	B0
ζ	Everyone's should be chose independently of everyone else	B1
η	The sample size must be large	B1
θ	Participants are chosen at random and independently from one another	B1 only

[though η & θ together would get B2]

Question 5 (i)

α	Number of bricks must always be the same		B0
β	Results occur randomly		B0
γ	The chance of a brick being in one place is always the same		B0
δ	Events must occur independently and at constant average rate		B0
ε	They must occur independently and at constant average rate		B1 only
ζ	Bricks' locations must be random and independent	*[effectively the same]*	B1 only
η	Only one brick in any one place; bricks independent	*[effectively the same]*	B1 only

4734 Probability & Statistics 3

Penalise 2 sf instead of 3 once only. Penalise final answer ≥ 6 sf once only.

1	(i)	$\int_0^1 \frac{2}{5}x^2 dx + \int_1^4 \frac{2}{5}\sqrt{x}\, dx$ $= \left[\frac{2x^3}{15}\right]_0^1 + \left[\frac{4x^{3/2}}{15}\right]_1^4 = 2$	M1 A1 A1	 3	Attempt to integrate xf(x), both parts added, limits Correct indefinite integrals Correct answer
	(ii)	$\int_2^4 \frac{2}{5\sqrt{x}} dx = \left[\frac{4\sqrt{x}}{5}\right]_2^4 = \frac{4}{5}(2-\sqrt{2})$ or 0.4686	M1 A1 A1	 3	Attempt correct integral, limits; needs "1 –" if $\mu < 1$ Correct indefinite integral, √ on their μ Exact aef, or in range [0.468, 0.469]
2	(i)	Po(0.5), Po(0.75) Po(0.7) and Po(0.9) $A + B \sim$ Po(1.6) P($A + B \geq 5$) = 0.0237 B(20, 0.0237) $0.9763^{20} + 20\times0.9763^{19}\times0.0237$ **= 0.9195**	M1 A1 M1 A1 M1 A1√ A1	 7	0.5, 0.75 scaled These Sum of Poissons used, can have wrong parameters 0.0237 from tables or calculator Binomial (20, their p), soi Correct expression, their p Answer in range [0.919, 0.92]
	(ii)	Bacteria should be independent in drugs; *or* sample should be random	B1	1	Any valid relevant comment, must be contextualised
3	(i)	Sample mean = 6.486 $s^2 = 0.00073$ $6.486 \pm 2.776 \times \sqrt{\frac{0.00073}{5}}$ **(6.45, 6.52)**	B1 B1 M1 B1 A1A1	 6	 0.000584 if divided by 5 Calculate sample mean ± $ts/\sqrt{5}$, allow 1.96, s^2 etc $t = 2.776$ seen Each answer, cwo (6.45246, 6.5195)
	(ii)	2π × above [= (40.5, 41.0)]	M1	1	
4	(i)	H$_0$: $p_1 = p_2$; H$_1$: $p_1 \neq p_2$, where p_i is the proportion of all solvers of puzzle i Common proportion 39/80 $s^2 = 0.4875 \times 0.5125 / 20$ $(\pm)\frac{0.6-0.375}{0.1117} = (\pm)2.013$ 2.013 > 1.96, or 0.022 < 0.025 Reject H$_0$. Significant evidence that there is a difference in standard of difficulty	B1 M1A1 B1 M1 A1√ M1 A1√	 8	Both hypotheses correctly stated, allow eg $\hat{p}$ [= 0.4875] [= 0.01249, σ = 0.11176] (0.6 – 0.375)/s Allow 2.066√ from unpooled variance, p = 0.0195 Correct method and comparison with 1.96 or 0.025, allow unpooled, 1.645 from 1-tailed *only* Conclusion, contextualised, not too assertive
	(ii)	One-tail test used Smallest significance level 2.2(1)%	M1 A1	 2	One-tailed test stated or implied by Φ("2.013"), OK if off-scale; allow 0.022(1)

5 (i)	Numbers of men and women should have normal dists;	B1	Context & 3 points: 2 of these, B1; 3, B2; 4, B3.
	with equal variance;	B1	[Summary data: 14.73 49.06 52.57
	distributions should be independent	B1 **3**	16.24 62.18 66.07]
(ii)	H$_0$: $\mu_M = \mu_W$; H$_1$: $\mu_M \neq \mu_W$	B1	Both hypotheses correctly stated
	$3992-\frac{221^2}{15}+5538-\frac{276^2}{17}$ [≈ 1793]	M1 A1	Attempt at this expression (see above) Either 1793 or 30
	1793/(14 + 16) = 59.766	A1	Variance estimate in range [59.7, 59.8] (or √ = 7.73)
	$(\pm)\frac{221/15-276/17}{\sqrt{59.766(\frac{1}{15}+\frac{1}{17})}}$ = (–)0.548	M1	Standardise, allow wrong (but not missing) 1/n
		A1√	Correct formula, allow $s^2(\frac{1}{15}+\frac{1}{17})$ or $(\frac{s_1^2}{15}+\frac{s_2^2}{17})$,
		A1	allow 14 & 16 in place of 15, 17; 0.548 or –0.548
	Critical region: $\| t \| \geq 2.042$	B1	2.042 seen
	Do not reject H$_0$. Insufficient evidence of a difference in mean number of days	M1 A1√ **10**	Correct method and comparison type, must be t, allow 1-tail; conclusion, in context, not too assertive
(iii)	Eg Samples not indep't so test invalid	B1 **1**	Any relevant valid comment, eg "not representative"

6 (i)	F(0) = 0, F(π/2) = 1 Increasing	B1 B1 **2**	Consider both end-points Consider F between end-points, can be asserted
(ii)	$\sin^4(Q_1) = ¼$ $\sin(Q_1) = 1/\sqrt{2}$ $Q_1 = \pi/4$	M1 A1 A1 **3**	Can be implied. Allow decimal approximations Or 0.785(4)
(iii)	$\text{G}(y) = \text{P}(Y \le y) = \text{P}(T \le \sin^{-1} y)$ $= \text{F}(\sin^{-1} y)$ $= y^4$ $g(y) = \begin{cases} 4y^3 & 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$	M1 A1 A1 M1 A1 **5**	Ignore other ranges Differentiate G(y) Function and range stated, allow if range given in G
(iv)	$\int_0^1 \frac{4}{1+2y}\,\text{d}y = \left[2\ln(1+2y)\right]_0^1$ $= \mathbf{2 \ln 3}$	M1 A1 A1 **3**	Attempt $\int \frac{g(y)}{y^3+2y^4}\text{d}y$; $\int_0^1 \frac{4}{1+2y}\text{d}y$ Or 2.2, 2.197 or better
7 (i) α	$\Phi\left(\frac{8.084-8.592}{0.7534}\right) = \Phi(-0.674) = 0.25$ $\Phi(0) - \Phi(\text{above}) = 0.25$ P(8.592 ≤ X ≤ 9.1) = same by symmetry	M1 A1 A1 A1 **4**	Standardise once, allow √ confusions, ignore sign Obtain 0.25 for one interval For a second interval, justified, eg using $\Phi(0) = 0.5$ For a third, justified, eg "by symmetry"
or β	$\frac{x-8.592}{0.7534} = 0.674$ $x = 8.592 \pm 0.674 \times 0.7534$ $= (8.084, 9.100)$	M1A1 A1A1	[from probabilities to ranges] A1 for art 0.674
(ii)	H_0: normal distribution fits data All E values 50/4 = 12.5 $X^2 = \frac{4.5^2+9.5^2+1.5^2+3.5^2}{12.5} = 10$ 10 > 7.8794 Reject H_0. Significant evidence that normal distribution is not a good fit.	B1 B1 M1 A1 B1 M1 A1√ **7**	*Not* N(8.592, 0.7534). Allow "it's normally distributed" [Yates: 8.56: A0] CV 7.8794 seen Correct method, incl. formula for χ^2 and comparison, allow wrong ν Conclusion, in context, not too assertive
(iv)	$8.592 \pm 2.576 \times \frac{0.7534}{\sqrt{49}}$ (8.315, 8.869)	M1 A1 A1 **3**	Allow √ errors, wrong σ or z, allow 50 Correct, including $z = 2.576$ or $t_{49} = 2.680$, *not* 50 In range [8.31, 8.32] and in range (8.86, 8.87], even from 50, or (8.306, 8.878) from t_{49}

4735 Probability & Statistics 4

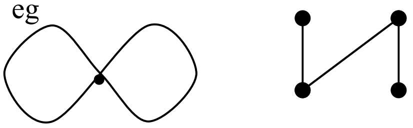
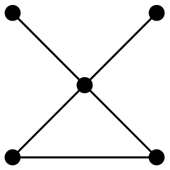
1	$\mathrm{M}_{X_1+X_2}(t) = (\mathrm{e}^{\mu_1 t+\frac{1}{2}\sigma_1{}^2t^2})(\mathrm{e}^{\mu_2 t+\frac{1}{2}\sigma_2{}^2})$ $= \mathrm{e}^{(\mu_1+\mu_2)t+\frac{1}{2}(\sigma_1{}^2+\sigma_2{}^2)t^2}$ *oe* $X_1 + X_2 \sim$ Normal distribution with mean $\mu_1 + \mu_2$, variance $\sigma_1{}^2 + \sigma_2{}^2$	M1 A1 A1 A1A1 **5** **{5}**	MGF of sum of independent RVs No suffices:- Allow M1A0A1A0A0
2 (i)	Non-parametric test used when the distribution of the variable in question is unknown	B1 **1**	
(ii)	H_0: $m_{V-A} = 0$, H_1: $m_{V-A} \neq 0$ where m_{V-A} is the median of the population differences Difference and rank, bottom up $P = 65$ $Q = 13$ $T = 13$ Critical region: $T \le 13$ 13 is inside the CR so reject H_0 and accept that there is sufficient evidence at the 5% significance level that the medians differ Use B(12, 0.5) P(≤ 4) = 0.1938 or CR = {0,1,2,10,11,12} > 0.025, accept that there is insufficient evidence, etc CWO	B1 M1 A1 B1 M1 A1 M1 A1 A1 **9**	Allow $m_V = m_A$ etc Allow $P > Q$ stated Penalise over-assertive conclusions once only. Or 4 not in CR
(iii)	Wilcoxon test is more powerful than the sign test	B1 **1** **{11}**	Use more information, more likely to reject NH
3(i)	$A + B$ $= \int_{-\infty}^{0} \mathrm{e}^{2x}\mathrm{e}^{xt}\,\mathrm{d}x + \int_{0}^{\infty} \mathrm{e}^{-2x}\mathrm{e}^{xt}\,\mathrm{d}x$ $= \left[\frac{1}{2+t}\mathrm{e}^{(2+t)x}\right]_{-\infty}^{0} + \left[-\frac{1}{2-t}\mathrm{e}^{\text{-(2-t)x}}\right]_{0}^{\infty}$ = 1/(2 + t) + 1/(2 – t) = 4/(4 – t^2) AG t < -2, A infinite; t > 2, B infinite	 M1 B1 B1 A1 B1 **5**	 Added, correct limits Correct integrals Allow sensible comments about denom of M(t)
(ii)	**Either**: 4/(4 – t^2) = (1 – ¼t^2)$^{-1}$ = 1 + ¼t^2 + …	M1 A1	Expand
	Or: M′ (t) = 8t/(4 – t^2)^2 M″(t) = 8/(4 – t^2)^2 + t×…		M1 A1
	E(X) = 0 Var(X) = 2×¼ – 0 = ½	M1 A1 **4** **{9}**	 For M″(0) – [M′(0)]^2 or equivalent 0.5 – 0 = 0.5

4 (i)	G(1)=1 [$a+b=1$] G′(1) = – 0.7 [$-a + 2b = -0.7$] Solve to obtain $\boldsymbol{a} = \mathbf{0.9}$, $\boldsymbol{b} = \mathbf{0.1}$	M1 M1 M1 A1 **4**	
(ii)	$G''(t)$ [$=1.8/t^3 + 0.2$] and $G''(1) + G'(1) - [G'(1)^2]$ used Var = $2 - 0.7 - 0.7^2$ = **0.81**	M1 A1 **2**	
(iii)	$[(0.9 + 0.1t^3)/t]^{10}$ Method to obtain coefficient of t^{-7} $10 \times 0.9^9 \times 0.1$ = **0.387** to 3SF	M1 M1 A1 ft A1 **4** **{10}**	$[(a + bt^3)/t]^{10}$ For both Use of MGF. $10a^9b$
5 (i)	Marginal dist of X_A: 0.30 0.45 0.15 0.10 E = 0.45 + 0.3 + 0.3 = 1.05 Var = $0.45 + 0.6 + 0.9 - 1.05^2$ = **0.8475**	B1 B1 B1 **3**	
(ii)	Consider a particular case to show $P(X_A \text{ and } X_B) \neq P(X_A)P(X_B)$ So X_A and X_B are not independent	M1 A1 **2**	Or $E(X_A)$, $E(X_B)$ and $E(X_AX_B)$ 1.05, 1.15, 1.09; $E(X_A)E(X_B)$ = 1.0275, ft on wrong $E(X_A)$
(iii)	Cov = $E(X_AX_B) - E(X_A)E(X_B)$ = 1.09 – 1.15×1.05 = –0.1175 $\text{Var}(X_A - X_B) = \text{Var}(X_A) + \text{Var}(X_B) - 2\text{Cov}(X_A, X_B)$ = **1.91**	M1 A1ft M1 A1 **4**	Or from distribution of X_A - X_B Wrong $E(X_A)$
(iv)	Requires $P(X_A, X_B)/P(X_A+X_B = 1)$ = 0.13/(0.16 + 0.13) = 13/29 = **0.448**	 M1 A1A1 A1 **4** **{13}**	

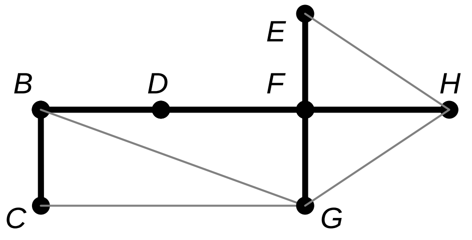
6 (i)	$\int_a^\infty x\mathrm{e}^{-(x-a)}\mathrm{d}x = \left[-x\mathrm{e}^{-(x-a)}\right]_a^\infty + \int_a^\infty \mathrm{e}^{-(x-a)}\mathrm{d}x$ $= a + [-\ \mathrm{e}^{-(x-a)}]$ $= a + 1$ AG	M1B1 A1 **3**	Correct limits needed for M1; no, or incorrect, limits allowed for B1
(ii)	$\mathrm{E}(T_1) = (a+1) + 2(a+1) - 2\,(a+1) - 1$ $= a$ $\mathrm{E}(T_2) = \frac{1}{4}(a+1+a+1) + (n-2)(a+1)/[2(n-2)] - 1$ $= a$ (So both are unbiased estimators of a)	M1 A1 M1 A1 **4**	
(iii)	$\sigma^2 = \mathrm{Var}(X)$ $\mathrm{Var}(T_1) = (1 + 4 + 1 + 1)\sigma^2 = 7\sigma^2$ $\mathrm{Var}(T_2) = 2\sigma^2/16 + (n-2)\sigma^2/[2(n-2)^2]$ $= n\sigma^2/[8(n-2)]$ oe This is clearly $< 7\sigma^2$, so T_2 is more efficient	M1 A1 B1 A1 **4**	
(iv)	eg $^1/_n(X_1 + X_2 + \ldots.+X_n) - 1$	B2 **2** **{13}**	B1 for sample mean
7 (i)	D denotes "The person has the disease" (a) $\mathrm{P}(D) = p$, $\mathrm{P}(D') = 1 - p$, $\mathrm{P}(+\|D) = 0.98$, $\mathrm{P}(+\|D') = 0.08$ $\mathrm{P}(+) = p\times0.98 + 0.08\times(1 - p)$ $= 0.08 + 0.9p$ $\mathrm{P}(D\,\|+) = \mathrm{P}(+\|D)(\mathrm{P}(D)/\mathrm{P}(+)$ $= 0.98p/(0.08 + 0.9p)$ (b) $\mathrm{P}(D')\times\mathrm{P}(+\|D') + \mathrm{P}(D)\times\mathrm{P}(-\|D)$ $= 0.08 - 0.06p$	 M1 M1 A1 M1 A1 **5**	 Use conditional probability
(ii)	$\mathrm{P}(++) = 0.98^2\times p + 0.08^2\times(1 - p)$ $\mathrm{P}(\mathrm{D}\|++) = 0.9604p/(0.954p + 0.0064)$	M1 A1 **2**	
(iii)	Expected number with 2 tests: $24000\times 0.0809 = a$ Expected number with 1 test: $24000 \times 0.9191 = b$ Expected total cost = £$(10a + 5b)$ = **£129 708**	 M1 M1 M1 A1 **4** **{11}**	 Or: $0.08 + 0.9\times0.001$ oe $\times5\times24000$ $+5\times24000$ (dep 1st M1) Or £130 000

4736 Decision Mathematics 1

1 (i)	[43 172 536 17 314 462 220 231]			
	43 172 536 17 220	M1	First folder correct	
	314 462	M1	Second folder correct	
	231	A1	All correct (cao)	[3]
(ii)	536 462 314 231 220 172 43 17	B1	List sorted into decreasing order seen (cao) [Follow through from a decreasing list with no more than 1 error or omission]	
	536 462	M1	First folder correct	[3]
	314 231 220 172 43 17	A1	All correct	
(iii)	$(5000 \div 500)^2 \times 1.3$	M1	$10^2 \times 1.3$ or any equivalent calculation	
	= 130 seconds	A1	Correct answer, with units	[2]
			Total =	**8**

2 (i)	The sum of the orders must be even, (but 1+2+3+3 = 9 which is odd).	B1	There must be an even number of odd nodes.	[1]
(ii) a	eg 	M1	A graph with five vertices that is neither connected nor simple	
		A1	Vertex orders 1, 1, 2, 2, 4	[2]
b	Because it is not connected	B1	You cannot get from one part of the graph to the other part.	[1]
c	eg 	B1	A connected graph with vertex orders 1, 1, 2, 2, 4 (Need not be simple)	[1]
(iii) a	There are five arcs joined to *A*.	M1	A reasonable attempt (for example, identifying that there are five arcs joined to *A*)	
	Either Ann has met (at least) three of the others or she has met two or fewer, in which case there are at least three that she has not met. In the first case at least three of the arcs joined to *A* are blue, in the second case at least three of the arcs joined to *A* are red.	A1	A convincing explanation (this could be a list of the possibilities or a well reasoned explanation)	[2]
b	If any two of Bob, Caz and Del have met one another then *B*, *C* and *D* form a blue triangle with *A*. Otherwise *B*, *C* and *D* form a red triangle.	M1 A1	A reasonable or partial attempt (using *A* with *B*, *C*, *D*) A convincing explanation (explaining both cases fully)	[2]
			Total =	**9**

3 **(i)**	$y \geq x$ $x + y \leq 8$ $x \geq 1$	M1 M1 M1 A1	Line $y = x$ in any form Line $x + y = 8$ in any form Line $x = 1$ in any form All inequalities correct [Ignore extra inequalities that do not affect the feasible region]	[4]
(ii)	(1, 1), (1, 7), (4, 4)	M1 A1	Any two correct coordinates All three correct [Extra coordinates given ⇒ M1, A0]	[2]
(iii)	(1, 7) □ 23 (4, 4) □ 20 At optimum, $x = 1$ and $y = 7$ Maximum value = 23	M1 A1 A1	Follow through if possible Testing vertices or using a line of constant profit (may be implied) Accept (1, 7) identified 23 identified	[3]
(iv)	$2\times1 + k\times7 \geq 2\times4 + k\times4$ □ $k \geq 2$	M1 A1	$2 + 7k$ or implied, or using line of gradient $-\frac{2}{k}$ Greater than or equal to 2 (cao) [$k > 2$ ⇒ M1, A0]	[2]
			Total =	**11**

4 **(i)**	*A*: 1 | 0 | *E*: 6 | 8 | 8 *B*: 2 | 2 | 2 *D*: 4 | 5 | 6 5 *F*: 5 | 6.5 | 6.5 *H*: 7 | 9.5 | 9.5 *C*: 3 | 4.5 | 4.5 *G*: | | 14 13.5 10.5	M1 M1 A1	Both 6 and 5 shown at *D* [5 may appear as perm label only] 14, 13.5 and 10.5 shown at *G* No extra temporary labels All temporary labels correct [condone perm values only appearing as perm labels] [Dep on both M marks]	
		B1	All permanent labels correct [may omit *G*, but if given it must be correct]	
		B1	Order of labelling correct [may omit *G* but if given it must be correct]	
	Route = *A* –*B* – *D* – *F* – *H*	B1	cao	
	Length = 9.5 miles	B1	cao	[7]
(ii)	Route Inspection problem	B1	Accept Chinese Postman	[1]
(iii)	Odd nodes: *A*, *D*, *E* and *H*	B1	Identifying or using *A,D,E,H*	
	AD = 5 *AE* = 8 *AH* = 9.5 *EH* = 5 *DH* = 4.5 *DE* = 3.5 10 12.5 13.0	M1 A1	Attempting at least one pairing At least one correct pairing or correct total	
	Repeat *AD* (*A-B-D*) and *EH* (*E-F-H*) Length = 67.5 +10	M1	Adding their 10 to 67.5	
	= 77.5 miles	A1	77.5 (cao)	[5]
(iv)	Repeat arcs *EF* and *FD*	B1	cao [NOT *DE* or *D-F-E*]	
	3.5 + 67.5 = 71 miles	B1	cao	[2]
(v)	*A* – *B* – *C* – *G* –*F* – *D* then method stalls *E* and *H* are missed out	B1	Showing route as far as *D* and then explaining the problem	[1]
(vi)	*C* – *B* – *A* – *D* – *F* – *E* – *H* – *G* – *C*	M1 A1	[If final *C* is missing ⇒ M1, A0] [A diagram needs arrows for A1]	
	37.5 miles	B1	37.5 (cao)	[3]
(vii)		M1 A1	A spanning tree on reduced network (may show *AB*, *AD*) Correct minimum spanning tree marked, with no extra arcs	
	Nodes: *B C D F E H G* Weight = 16 miles	B1 B1	cao cao	
	[Two shortest arcs from *A* are *AB* and *AD*] 2 + 6 + 16 Lower bound = 24 miles	M1 A1	8 + their 16 (or implied) cao	[6]
			Total =	**25**

<table>
<tr><td>5
(i)</td><td>$15x+15y+30z \le 9000$
[divide through by 15 to get $x+y+2z \le 600$ as given]
Stamping out: $5x+8y+10z \le 3600$
Fixing pin: $50x+50y+50z \le 25000$
⇒ $x + y + z \le 500$
Checking: $100x+50y+20z \le 10000$
⇒ $10x+5y+2z \le 1000$</td><td>B1

B1

B1

B1</td><td>$15x+15y+30z \le 9000$

$5x+8y+10z \le 3600$

$x + y + z \le 500$

$10x+5y+2z \le 1000$</td><td>[4]</td></tr>
<tr><td>(ii)</td><td>x, y and z are non-negative</td><td>B1</td><td>$x \ge 0$, $y \ge 0$ and $z \ge 0$</td><td>[1]</td></tr>
<tr><td>(iii)</td><td>(P =) $4x + 3y + z$</td><td>B1</td><td>cao</td><td>[1]</td></tr>
<tr><td>(iv)</td><td>
<table>
<tr><th>P</th><th>x</th><th>y</th><th>z</th><th>s</th><th>t</th><th>u</th><th>v</th><th>RHS</th></tr>
<tr><td>1</td><td>-4</td><td>-3</td><td>-1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>1</td><td>1</td><td>2</td><td>1</td><td>0</td><td>0</td><td>0</td><td>600</td></tr>
<tr><td>0</td><td>5</td><td>8</td><td>10</td><td>0</td><td>1</td><td>0</td><td>0</td><td>3600</td></tr>
<tr><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>500</td></tr>
<tr><td>0</td><td>10</td><td>5</td><td>2</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1000</td></tr>
</table>
</td><td>B1
B1
M1
A1</td><td>Follow through if reasonable
-4 -3 -1 in objective row
Correct use of slack variables
1 1 2 and 600 correct
All constraint rows correct
Accept variations in order of rows and columns</td><td>[4]</td></tr>
<tr><td>(v)</td><td>Pivot on the 10 in the x-column
<table>
<tr><td>1</td><td>0</td><td>-1</td><td>-0.2</td><td>0</td><td>0</td><td>0</td><td>0.4</td><td>400</td></tr>
<tr><td>0</td><td>0</td><td>0.5</td><td>1.8</td><td>1</td><td>0</td><td>0</td><td>-0.1</td><td>500</td></tr>
<tr><td>0</td><td>0</td><td>5.5</td><td>9</td><td>0</td><td>1</td><td>0</td><td>-0.5</td><td>3100</td></tr>
<tr><td>0</td><td>0</td><td>0.5</td><td>0.8</td><td>0</td><td>0</td><td>1</td><td>-0.1</td><td>400</td></tr>
<tr><td>0</td><td>1</td><td>0.5</td><td>0.2</td><td>0</td><td>0</td><td>0</td><td>0.1</td><td>100</td></tr>
</table>
</td><td>B1

M1
A1</td><td>Correct choice of pivot from x- column
[Follow through their tableau and valid pivot if possible: no negative values in RHS column and P value has not decreased]
Pivot row correct
Other rows correct</td><td>[3]</td></tr>
<tr><td></td><td>Pivot on 0.5 in the last row of y-column
<table>
<tr><td>1</td><td>2</td><td>0</td><td>0.2</td><td>0</td><td>0</td><td>0</td><td>0.6</td><td>600</td></tr>
<tr><td>0</td><td>-1</td><td>0</td><td>1.6</td><td>1</td><td>0</td><td>0</td><td>-0.2</td><td>400</td></tr>
<tr><td>0</td><td>-11</td><td>0</td><td>6.8</td><td>0</td><td>1</td><td>0</td><td>-1.6</td><td>2000</td></tr>
<tr><td>0</td><td>-1</td><td>0</td><td>0.6</td><td>0</td><td>0</td><td>1</td><td>-0.2</td><td>300</td></tr>
<tr><td>0</td><td>2</td><td>1</td><td>0.4</td><td>0</td><td>0</td><td>0</td><td>0.2</td><td>200</td></tr>
</table>
</td><td>B1

M1
A1</td><td>Correct choice of pivot from y-column
[Follow through their tableau and valid pivot if possible]
Pivot row correct
Other rows correct</td><td>[3]</td></tr>
<tr><td></td><td>$x = 0$, $y = 200$, $z = 0$, $P = 600$
Make 20 000 metallic badges (and no laminated badges or plastic badges)

To give a profit of £600

6000 seconds (100 min) of printing time not used, 2000 seconds (33 min 20 sec) of stamping out time not used, 15000 seconds (250 min) of fixing pin time not used.
All the checking time is used</td><td>B1

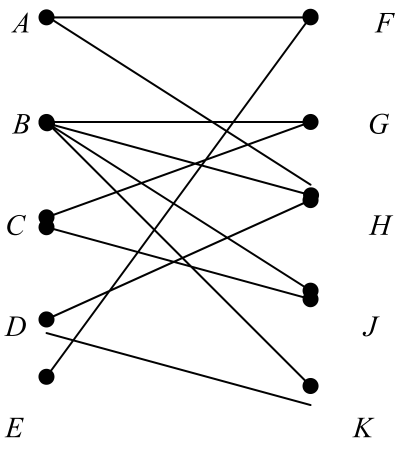
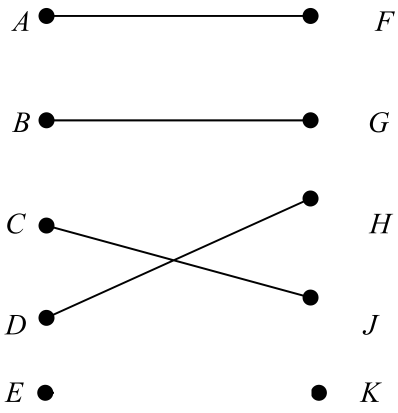
B1

B1</td><td>Interpretation of their x, y and z values in context (may imply zero entries)

Interpretation of their P value in context

Interpretation of their slack variable values</td><td>[3]</td></tr>
<tr><td colspan="4" align="right">Total =</td><td>19</td></tr>
</table>

4737 Decision Mathematics 2

1(a) (i)		B1	A correct bipartite graph	[1]
(ii)		B1	A second bipartite graph showing the incomplete matching correctly	[1]
(iii)	$E = F - A = H - D = K$	B1	This path in any reasonable form	
	Fiona = Egg and tomato $F = E$ Gwen = Beef and horseradish $G = B$ Helen = Avocado and bacon $H = A$ Jack = Chicken and stuffing $J = C$ Mr King = Duck and plum sauce $K = D$	B1	This complete matching	[2]
(iv)	Interchange Gwen and Jack $F = E \quad G = C \quad H = A \quad J = B \quad K = D$	B1	This complete matching	[1]

(b)

Reduce rows

	F	*G*	*H*	*J*	*K*
L	7	7	7	7	0
M	2	6	4	2	0
N	8	8	8	6	0
O	1	3	2	1	0
P	6	9	7	5	0

M1 Substantially correct attempt to reduce rows

Reduce columns

	F	*G*	*H*	*J*	*K*
L	6	4	5	6	0
M	1	3	2	1	0
N	7	5	6	5	0
O	0	0	0	0	0
P	5	6	5	4	0

M1 Substantially correct attempt to reduce columns

A1 cao

[3]

Cross out 0's using two (minimum no. of) lines

	F	*G*	*H*	*J*	*K*
L	6	4	5	6	**0**
M	1	3	2	1	**0**
N	7	5	6	5	**0**
O	**0**	**0**	**0**	**0**	**0**
P	5	6	5	4	**0**

Augment by 1

	F	*G*	*H*	*J*	*K*
L	5	3	4	5	0
M	0	2	1	0	0
N	6	4	5	4	0
O	0	0	0	0	1
P	4	5	4	3	0

M1 Substantially correct attempt at augmenting

A1 Augmenting correctly

[2]

Cross out 0's using three (minimum no. of) lines

	F	*G*	*H*	*J*	*K*
L	5	3	4	5	**0**
M	**0**	**2**	**1**	**0**	**0**
N	6	4	5	4	**0**
O	**0**	**0**	**0**	**0**	**1**
P	4	5	4	3	**0**

Augment by 3

	F	*G*	*H*	*J*	*K*
L	2	**0**	1	2	0
M	**0**	2	1	0	3
N	3	1	2	1	**0**
O	0	0	**0**	0	4
P	1	2	1	**0**	0

M1 Substantially correct attempt at augmenting (by more than 1 in a single step)

A1 Augmenting correctly

Lemon = Gwen
Mandarin = Fiona
Nectarine = Mr King
Orange = Helen
Peach = Jack

B1 Correct allocation

[3]

Total = 13

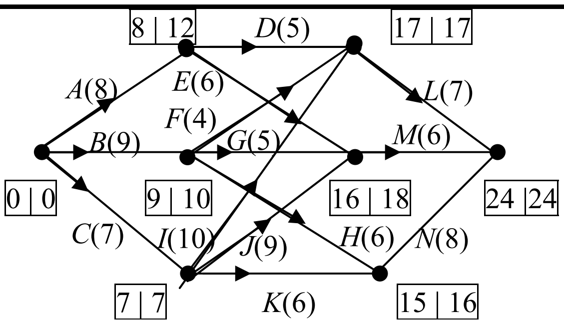
<table>
<tr><td>2 (i)</td><td>
<table>
<tr><th>Stage</th><th>State</th><th>Action</th><th>Working</th><th>Suboptimal maxima</th></tr>
<tr><td>2</td><td>0</td><td>0</td><td>7</td><td>7</td></tr>
<tr><td></td><td>1</td><td>0</td><td>6</td><td>6</td></tr>
<tr><td></td><td>2</td><td>0</td><td>8</td><td>8</td></tr>
<tr><td>1</td><td>0</td><td>0</td><td>5 +7 = 12</td><td></td></tr>
<tr><td></td><td></td><td>1</td><td>6 + 6 = 12</td><td>12</td></tr>
<tr><td></td><td>1</td><td>0</td><td>4 +7 = 11</td><td></td></tr>
<tr><td></td><td></td><td>1</td><td>5 + 6 = 11</td><td></td></tr>
<tr><td></td><td></td><td>2</td><td>6 + 8 = 14</td><td>14</td></tr>
<tr><td></td><td>2</td><td>0</td><td>10 + 7 = 17</td><td>17</td></tr>
<tr><td></td><td></td><td>1</td><td>9 + 6 = 15</td><td></td></tr>
<tr><td></td><td></td><td>2</td><td>6 + 8 = 14</td><td></td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>8 + 12 = 20</td><td></td></tr>
<tr><td></td><td></td><td>1</td><td>9 + 14 = 23</td><td></td></tr>
<tr><td></td><td></td><td>2</td><td>7 + 17 = 24</td><td>24</td></tr>
</table>

Maximum route = (0;0) - (1;2) - (2;0) - (3;0)
Weight = 24</td>
<td>B1
M1
A1
B1

M1

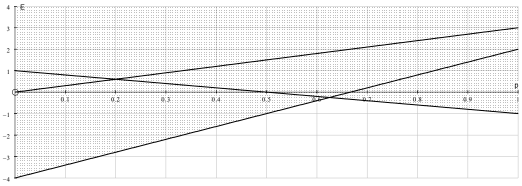
A1
B1

M1

A1
B1
B1</td>
<td>Structure of table correct
Stage and state values correct
Action values correct
Working backwards from stage 2
7, 6, 8 correct in suboptimal
maxima column for stage 2
Working column substantially
correct for stage 1
Sums correct for stage 1
Suboptima maxima values correct
for stage 1
Working column substantially
correct for stage 0
Sums correct for stage 0
Correct route from (0; 0) to (3; 0)
24 cao</td>
<td>[3]
[3]
[3]
[2]</td></tr>
<tr><td>(ii)</td><td>

Minimum completion time = 24
Critical activities: <i>C</i>, <i>I</i>, <i>L</i></td>
<td>B1
M1
A1
M1

A1
B1
B1</td>
<td>Assigning <i>A</i> to <i>N</i> appropriately
Substantially correct forward pass
Forward pass correct
Substantially correct backward pass
Backward pass correct
24 (cao)
<i>C</i>, <i>I</i>, <i>L</i> (cao)</td>
<td>[7]</td></tr>
<tr><td>(iii)</td><td>The critical path is the maximum path
The critical activities form a continuous path with no slack, ie the longest path</td>
<td>M1
A1</td>
<td>Same path is found in both
Recognition of why the solutions are the same, in general</td>
<td>[2]</td></tr>
<tr><td colspan="4">Total =</td><td>20</td></tr>
</table>

3 (i)	For each pairing, the total of the points is 10. Subtracting 5 from each makes the total 0. Eg 3 points and 7 points ⇒ scores of -2 and +2	M1 A1	Sum of points is 10 So sum of scores is zero A specific example earns M1 only	[2]
(ii)	*W* scores -1 *P* has 6 points and *W* has 4 points	B1 B1	-1 6 and 4	[2]
(iii)	*W* is dominated by *Y* $-1 < 1$, $-3 < -2$ and $1 < 2$	B1 B1	*Y* These three comparisons in any form	[2]
(iv)	Collies $\begin{array}{lccccc} & & X & Y & Z & \text{row min} \\ \hline \text{Rovers} & P & 2 & -1 & 3 & -1 \\ & Q & 1 & -3 & -1 & -3 \\ & R & -4 & 1 & 0 & -4 \\ \text{col max} & & 2 & 1 & 3 & \end{array}$ Play-safe for Rovers is *P* Play-safes for Collies is *Y*	M1 A1 A1	Determining row minima and column maxima, or equivalent *P* *Y*	[3]
(v)	$2p - 4(1-p) = 6p - 4$ *Y* gives $1 - 2p$ *Z* gives $3p$	B1 B1	$6p - 4$ in simplified form Both $1 - 2p$ and $3p$ in any form	[2]
(vi)	 $6p - 4 = 1 - 2p \Rightarrow p = \frac{5}{8}$	B1 M1 A1	Their lines drawn correctly on a reasonable scale Solving the correct pair of equations or using graph correctly $\frac{5}{8}$, 0.625, cao	[3]
(vii)	Add 4 throughout matrix to make all values non-negative On this augmented matrix, if Collies play *X* Rovers expect $6p_1 + 5p_2$; if Collies play *Y* Rovers expect $3p_1 + p_2 + 5p_3$; and if Collies play *Z* Rovers expect $7p_1 + 3p_2 + 4p_3$ We want to maximise *M* where *M* only differs by a constant from *m* and, for each value of *p*, *m* is the minimum expected value.	B1 B1 B1	'Add 4', or new matrix written out or equivalent Relating to columns *X*, *Y* and *Z* respectively. Note: expressions are given in the question. For each value of *p* we look at the minimum output, then we maximise these minima.	[3]
(viii)	$p_3 = \frac{3}{8}$ $M = -\frac{1}{4}$	B1 B1	cao cao	[2]
			Total =	**19**

4 (i)	8+0+6+5+4 = 23 gallons per minute	M1 A1	8+0+6+5+4 or 23 23 with units	[2]
(ii)	At most 6 gallons per minute can enter *A* so there cannot be 7 gallons per minute leaving it At most 7 gallons per minute can leave *F* so there cannot be 10 gallons per minute entering it.	B1 B1	Maximum into *A* = 6 Maximum out of *F* = 7	[2]
(iii)	A diagram showing a flow with 12 through *E* Flow is feasible (upper capacities not exceeded) Nothing flows through *A* and *D* Maximum flow through *E* = 12 gallons per minute	M1 M1 A1 B1	Assume that blanks mean 0 12	[4]
(iv) a	If flows through *A* but not *D* its route must be *S* – *A* – *C* – *E*, but the flow through *E* is already a maximum	B1	A correct explanation	[1]
b	*S* – (*B*) – *C* – *D* – *F* – *T* 1 gallon per minute	M1 A1	Follow through their part (iii) 1	[2]
(v)	Flow = 12 + 1 = 13 gallons per minute Cut through *ET* and *FT* or {*S*,*A*,*B*,*C*,*D*,*E*,*F*}, {*T*} = 13 gallons per minute Every cut forms a restriction Every cut ≥ every flow □ min cut ≥ max flow This cut = this flow so must be min cut and max flow	B1 M1 A1 B1	Identifying this cut in any way Use of max flow – min cut theorem min cut ≥ max flow This cut = this flow (or having shown that both are 13)	[4]
(vi)	3 gallons per minute Must flow 6 along *ET* and 7 along *FT*. Can send 4 into *F* from *D* so only need to send 9 through *E*	B1 B1 B1	3 A correct explanation	[3]
(vii)	A diagram showing a flow of 13 without using *BE* Flow is feasible and only sends 9 through *E*	M1 A1	May imply directions and assume that blanks mean 0	[2]
			Total =	**20**

Grade Thresholds

Advanced GCE Mathematics (3890-2, 7890-2)
June 2009 Examination Series

Unit Threshold Marks

7892		Maximum Mark	A	B	C	D	E	U
4721	Raw	72	58	51	44	38	32	0
	UMS	100	80	70	60	50	40	0
4722	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4723	Raw	72	53	46	39	33	27	0
	UMS	100	80	70	60	50	40	0
4724	Raw	72	53	46	39	33	27	0
	UMS	100	80	70	60	50	40	0
4725	Raw	72	49	43	37	32	27	0
	UMS	100	80	70	60	50	40	0
4726	Raw	72	53	46	40	34	28	0
	UMS	100	80	70	60	50	40	0
4727	Raw	72	55	49	43	38	33	0
	UMS	100	80	70	60	50	40	0
4728	Raw	72	62	52	42	33	24	0
	UMS	100	80	70	60	50	40	0
4729	Raw	72	57	48	39	31	23	0
	UMS	100	80	70	60	50	40	0
4730	Raw	72	61	51	41	32	23	0
	UMS	100	80	70	60	50	40	0
4731	Raw	72	55	46	38	30	22	0
	UMS	100	80	70	60	50	40	0
4732	Raw	72	54	47	40	33	27	0
	UMS	100	80	70	60	50	40	0
4733	Raw	72	57	49	41	33	26	0
	UMS	100	80	70	60	50	40	0
4734	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
4735	Raw	72	52	45	38	32	26	0
	UMS	100	80	70	60	50	40	0
4736	Raw	72	57	50	44	38	32	0
	UMS	100	80	70	60	50	40	0
4737	Raw	72	52	46	40	34	29	0
	UMS	100	80	70	60	50	40	0

Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
3890	300	240	210	180	150	120	0
3891	300	240	210	180	150	120	0
3892	300	240	210	180	150	120	0
7890	600	480	420	360	300	240	0
7891	600	480	420	360	300	240	0
7892	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
3890	37.64	54.75	68.85	80.19	88.46	100	18954
3892	58.92	74.42	85.06	91.87	96.04	100	2560
7890	47.57	68.42	83.78	93.17	98.15	100	11794
7892	60.58	80.66	90.76	95.89	98.72	100	2006

For a description of how UMS marks are calculated see:
http://www.ocr.org.uk/learners/ums_results.html

Statistics are correct at the time of publication.

List of abbreviations

Below is a list of commonly used mark scheme abbreviations. The list is not exhaustive.

AEF	Any equivalent form of answer or result is equally acceptable
AG	Answer given (working leading to the result must be valid)
CAO	Correct answer only
ISW	Ignore subsequent working
MR	Misread
SR	Special ruling
SC	Special case
ART	Allow rounding or truncating
CWO	Correct working only
SOI	Seen or implied
WWW	Without wrong working
Ft or √	Follow through (allow the A or B mark for work correctly following on from previous incorrect result.)

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553