OCR
Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4769**: Statistics 4

Advanced GCE

Mark Scheme for June 2016

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

1. **Annotations and abbreviations**

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

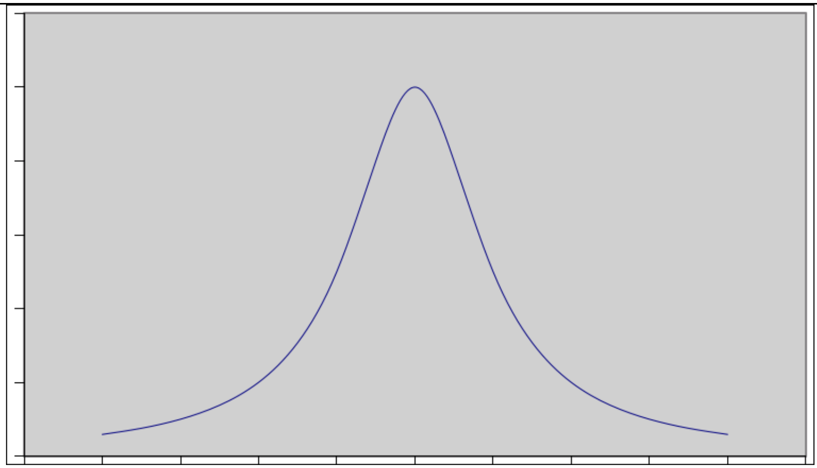
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		Answer	Marks	Guidance
1	(i)	 Centred on $x = m$ Mode: observe that f(x) takes its maximum when $x - m = 0$ (or differentiate and set to zero) Median: integral is $(1/\pi)\arctan(x - m)$. Evaluated between $-\infty$ and m or m and ∞ this evaluates to 0.5.	G2 M1E1 M1A1 **[6]**	Derivative is $-\frac{1}{\pi}\frac{2(x-m)}{(1+(x-m)^2)^2}$ "symmetry" SC B1
	(ii)	Likelihood function is f(x_1) Differentiate wrt m and set to zero: $\frac{1}{\pi}\frac{2(x_1-m)}{(1+(x_1-m)^2)^2} = 0$ Giving $x_1 = m$	B1 M1 A1 **[3]**	 Answer given

Question		Answer	Marks	Guidance
	(iii)	Likelihood function is $f(x_1)\,f(x_2)$	B1	
		The derivative of the log likelihood wrt m is		
		$\dfrac{x_1 - m}{1+(x_1-m)^2} + \dfrac{x_2 - m}{1+(x_2-m)^2}$	M1A1	
		Setting to zero and convincing algebra to given result	M1A1 **[5]**	
	(iv)	$\hat{m} = \frac{1}{2}(x_1 + x_2)$	B1	
		$\hat{m} = \frac{1}{2}(x_1 + x_2) \pm \frac{1}{2}\sqrt{(x_1 - x_2)^2 - 4}$	M1A1	Or equivalent
		Convincing argument that if $\lvert x_1 - x_2 \rvert < 2$ then only 1 root is real	E1	
		Convincing argument that the ML function is positive, continuous and tends to zero at $\pm\infty$. Hence the only turning point is a maximum	E1 **[5]**	
	(v)	The three values of $\hat{m}$ are: 0, $\pm\sqrt{3}$	B1B1	B1 for 0, B1 both
		Likelihood at zero is $\frac{1}{25\pi^2}$ or $\frac{0.04}{\pi^2}$ (approx. 0.0040)	B1	Accept approx. values
		Likelihood at $\pm\sqrt{3}$ is $\frac{1}{16\pi^2}$ or $\frac{0.0625}{\pi^2}$ (approx. 0.0063)	B1	
		So $\pm\sqrt{3}$ are joint MLE, 0 is not MLE.	E1 **[5]**	
			[24]	

Question			Answer	Marks	Guidance	
2	(i)		$M_X(\theta) = \mathrm{E}\left(\mathrm{e}^{X\theta}\right) = \int_0^\infty \mathrm{e}^{x\theta}\lambda\mathrm{e}^{-\lambda x}\mathrm{d}x$	M1	With limits shown	
			$= \int_0^\infty \lambda\mathrm{e}^{x(\theta-\lambda)}\mathrm{d}x$			
			$= \left[\frac{\lambda}{\theta-\lambda}e^{x(\theta-\lambda)}\right]_0^\infty$	A1	Indefinite integral	
			$= \frac{\lambda}{(\lambda-\theta)}$ or $\left(1-\frac{\theta}{\lambda}\right)^{-1}$	A1 **[3]**	Correct www	Condone omission of the fact that upper limit requires $\theta < \lambda$
	(ii)		Differentiate once and substitute $\theta = 0$ $M'_X(\theta) = \lambda(\lambda-\theta)^{-2}$	M1A1	2nd derivative	
			Hence $\mathrm{E}(X) = 1/\lambda$	A1		
			$M''_X(\theta) = 2\lambda(\lambda-\theta)^{-3}$	M1		OR Expanding the mgf and identifying coefficients of θ and θ^2, M1 A1 A1
			$\mathrm{E}(X^2) = 2/\lambda^2$	A1		
			$\mathrm{Var}(X) = 2/\lambda^2 - 1/\lambda^2 = 1/\lambda^2$	A1 **[6]**		
	(iii)		$\mathrm{E}(Y) = n/\lambda$	B1	FT (ii) , both	
			$\mathrm{Var}(Y) = n/\lambda^2$	B1		
			Use the convolution theorem:			
			$\mathrm{M}_Y(\theta) = (\mathrm{M}_X(\theta))^n = \left(\frac{\lambda}{\lambda-\theta}\right)^n$	M1	For a product	
				A1 **[4]**	cao	

Question			Answer	Marks	Guidance	
	(iv)		Use $M_{aX+b}(\theta) = e^{b\theta} M_X(a\theta)$ To obtain $M_Z(\theta)$ as $e^{-\sqrt{n}\theta}\left(\frac{\lambda}{\lambda-\theta\left(\frac{\lambda}{\sqrt{n}}\right)}\right)^n$ Simplifying to $e^{-\sqrt{n}\theta}\left(\frac{1}{1-\frac{\theta}{\sqrt{n}}}\right)^n$	M1 A1 A1 A1 **[4]**	For factor $e^{-\sqrt{n}\theta}$ oe Second factor This version may be seen in (v)	
	(v)		Logarithm is $-\sqrt{n}\theta - n\ln\left(1-\frac{\theta}{\sqrt{n}}\right)$ $= -\sqrt{n}\theta + n\left(\frac{\theta}{\sqrt{n}} + \frac{\theta^2}{2n} + \cdots\right)$ First two terms cancel and $\frac{1}{2}\theta^2$ seen Next term seen or "remaining terms have powers of n in denominator" Hence mgf tends to exp $\left(\frac{1}{2}\theta^2\right)$ Which is mgf of standard Normal	M1 A1 M1 A1 A1 E1 E1 **[7]**	Expression with two terms For series expansion Answer given so working must be convincing	
				[24]		

Question		Answer	Marks	Guidance
3	(i)	H_0: $\mu_1 = \mu_2$	B1	Zero if sample means used.
		H_1: $\mu_1 \neq \mu_2$ where μ_1 and μ_2 are the means in the underlying population	B1	B1 if not clearly population means
		$\bar{x} = \frac{598}{12} = 49.8333$ $\quad \bar{y} = \frac{707}{12} = 58.9167$	B1	
		$\sum(x-\bar{x})^2 = 31196 - \frac{598^2}{12} = 1395.66667$; $[s_x^2 = 126.87\ldots, s_x = 11.264\ldots]$	M1	Accept alternative forms if correctly used later
		$\sum(y-\bar{y})^2 = 43543 - \frac{707^2}{12} = 1888.91667$; $[s_y^2 = 171.719\ldots, s_y = 13.104\ldots]$	A1	
		Pooled variance estimate = $\frac{(1395.666\ldots+1888.916\ldots)}{(11+11)} = 149.299$	M1A1	$\frac{11s_x^2+11s_y^2}{22}$
		Test statistic: $\frac{58.9167-49.8333}{\sqrt{149.299}\sqrt{\frac{1}{12}+\frac{1}{12}}} = 1.8209$	M1 A1	Correct construction, their s, $\bar{x}, \bar{y}$
		5% two-tailed critical value for t_{22} is 2.0739.	B1	2.0772 by interpolation from tables
		Hence no reason to reject H_0, no reason to suppose that standards of literacy and numeracy are different in the underlying population, on average.	M1 A1 **[12]**	no reason to reject H_0 context
	(ii)	Scores in the underlying population distributed Normally	B1	
		With common variance	B1	Accept same median and different medians
		Wilcoxon rank sum test (or Mann-Whitney 2 sample test)	B1	
		H_0: literacy scores and numeracy scores have the same distribution		
		H_1: literacy scores and numeracy scores have the same distribution but for a shift in location	B1	
		The t test will be more powerful because	B1	
		it uses the magnitudes of the data rather than just their ranks.	B1 **[6]**	

Question			Answer	Marks	Guidance	
	(iii)		In a paired sample design, all the students in the sample would do both assessments.	B1	This part of the question is entirely descriptive and candidates may present answers which are correct but expressed differently. Marks should be awarded accordingly.	
			The order in which the students do the assessments should be randomised and/or blocked for balance.	B1		
			The data used in the test would be the differences in their scores.	B1		
			A single sample *t* test (or Wilcoxon if Normality cannot be assumed) would be used	B1		
			This would be better than the two sample design used because the variation between students would be factored out.	B1		
			The design would therefore be more sensitive to differences between literacy and numeracy.	B1 **[6]**		

Question			Answer	Marks	Guidance	
4	(i)		$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$	B1		
			where: Y_{ij} is the jth value in the ith group	B1		
			μ is the global mean in the underlying population	B1		
			α_i is the 'treatment effect' in the ith group	B1	Or $\mu_i - \mu$	
			ε_{ij} is a random error term	B1	Accept " residual"	
			In this context, μ measures the average effect of the exercise regimes, and the α_i represent the differences from the mean for the three regimes	E1	Context explained at least once	'Groups' are exercise regimes
			ε_{ij} iid N(0, σ^2)	B1	Distributional assumption	
			H_0: the three exercise regimes give the same (population) increase in mean pulse rate H_1: the three exercise regimes do not give the same (population) increase in mean pulse rate	B1	Or: $\alpha_1 = \alpha_2 = \alpha_3 (= 0)$ Not all α_i the same	
			$\sum \frac{T_i^2}{n_i} - \frac{T^2}{n} = \frac{244^2}{5} + \frac{368^2}{6} + \frac{266^2}{5} - \frac{878^2}{16} = 448.8167$	M1A1		
			$\sum \sum y_{ij}^2 - \frac{T^2}{n} = 49544 - \frac{878^2}{16} = 1363.75$	M1A1		
			ANOVA table:			
			(see table below)	A1Ft B1 A1Ft B1	Within Groups Sum Sq Df all 3 F ratio Ft their Sum Sqs F critical; 3.81 from tables	Ft their Total SS-BGSS
			Result not significant.	M1		
			Insufficient evidence to suppose that the exercise regimes have different effects on pulse rate on average	A1 **[18]**		

Source of Variation	Sum Sq	df	MS	F ratio	F critical
Between Groups	448.8167	2	224.41	3.1885	3.8056
Within Groups	914.9333	13	70.379		
Total	1363.75	15			

Question			Answer	Marks	Guidance	
	(ii)		The analysis using three tests is not equivalent to ANOVA, and the multiple comparisons procedure is worse than ANOVA The three tests are not independent The significance level of the whole procedure is therefore impossible to assess A comparison with the different result obtained in (i), and why this may be so.	B1 B1 B1 B1 B1 B1 **[6]**	Other points could be made	eg Multiple comparisons are likely to generate more type I errors than the nominal significance level would suggest. However, multiple comparisons are useful post hoc to identify where the largest differences have occurred
				[24]		

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