Question			Answer	Marks	Guidance
1	(i)		$$\mathrm{P}(X=x)=\frac{\mathrm{e}^{-\theta}\theta^{x}}{x!}$$		
			$\mathrm{L}=\frac{\mathrm{e}^{-\theta}\theta^{x_1}}{x_1!}\cdot\ldots\cdot\frac{\mathrm{e}^{-\theta}\theta^{x_n}}{x_n!}=\frac{\mathrm{e}^{-n\theta}\theta^{\Sigma x_i}}{x_1!x_2!\ldots x_n!}$	M1 A1	M1 for general product form. A1 (a.e.f.) for answer.
			$\ln \mathrm{L} = -n\theta + \Sigma x_i \ln \theta - \Sigma \ln x_i!$	M1 A1	M1 is for taking logs (base e). Allow (±) constant instead of last term.
			$\frac{\mathrm{d}\ln \mathrm{L}}{\mathrm{d}\theta}=-n+\frac{\Sigma x_i}{\theta}$	M1 A1	M1 for differentiating, A1 for answer.
			$= 0$ for ML Est $\hat{\theta}$.	M1	Any or all of the four M1 marks down to here can be awarded in part (iv) if not awarded here.
			$\therefore n\hat{\theta}=\Sigma x_i$, i.e. $\hat{\theta}=\bar{x}$.	A1	
			Confirmation that this is a maximum:	M1	
			$\frac{\mathrm{d}^2\ln \mathrm{L}}{\mathrm{d}\theta^2}=-\frac{\Sigma x_i}{\theta^2}<0$.	A1	
				[10]	
1	(ii)		$\mathrm{P}(X=0)=\mathrm{e}^{-\theta}$.	M1	
			ML Est of $\mathrm{e}^{-\theta}=\mathrm{e}^{-\hat{\theta}}$, i.e. the estimate is $\mathrm{e}^{-\bar{x}}$.	M1 A1	M1 for "invariance property", not necessarily named.
				[3]	
1	(iii)		We have estimate of $\mathrm{P}(X=0)=\mathrm{e}^{-5}=0.0067$,	M1	
			so we might reasonably expect around $1000e^{-5}\approx 6.7$ cases of zero in a sample of size 1000	E1	Sensible use of $n=1000$ and $\bar{x}=5$.
			– finding no such cases seems suspicious.	E1	
				[3]	

Question			Answer	Marks	Guidance
1	(iv)		X has Poisson distribution "scaled up" so that , therefore , where $1 = k\sum_{x=1}^{\infty}\frac{e^{-\theta}\theta^{x}}{x!} = k\left\{\sum_{x=0}^{\infty}\frac{e^{-\theta}\theta^{x}}{x!} - e^{-\theta}\right\} = k\{1 - e^{-\theta}\}$.	M1 M1	M1 for this idea, however expressed. M1 for sum from 0 to ∞ minus value for $x = 0$.
			$\therefore k = \frac{1}{1 - e^{-\theta}}$.	M1	
			$\therefore P(X = x) = \frac{1}{1 - e^{-\theta}}.\frac{e^{-\theta}\theta^{x}}{x!} = \frac{\theta^{x}}{(e^{\theta} - 1)x!}$ [for x = 1, 2, ...].	A1	**Beware printed answer.**
			$\therefore$ L = $\frac{\theta^{\Sigma x_i}}{(e^{\theta} - 1)^{n} x_1!x_2!\ldots x_n!}$	A1	
			$\therefore \ln L = \Sigma x_i \ln\theta - n\ln(e^{\theta} - 1) - \Sigma \ln x_i!$	A1	
			$\therefore \frac{d\ln L}{d\theta} = \frac{\Sigma x_i}{\theta} - \frac{ne^{\theta}}{e^{\theta} - 1}$,	A1	
			and on setting this equal to zero we get that $\hat{\theta}$ satisfies $\frac{\theta e^{\theta}}{e^{\theta} - 1} = \frac{\Sigma x_i}{n} = \bar{x}$.	A1 **[8]**	**Beware printed answer.**
2	(i)	(*A*) (*B*) (*C*)	$\mu = 0$. E(X^2) = 8/3. Var(X) = 8/3.	B1 B1 B1 **[3]**	
2	(ii)		Mgf of X is $M_X(\theta) = E(e^{\theta X})$ $= \left(e^{-2\theta}.\frac{1}{3}\right) + \left(e^{0}.\frac{1}{3}\right) + \left(e^{2\theta}.\frac{1}{3}\right)$ $= \frac{1}{3}\left(1 + e^{2\theta} + e^{-2\theta}\right)$.	M1 A1 **[2]**	Any equivalent form.

Question			Answer	Marks	Guidance
2	**(iii)**		General results:		
			[Convolution theorem] Mgf of sum of independent random variables = product of their mgfs.	B1 M1	B1 for explicit mention of "independent". Note M1 mark required for f.t. to A marks below.
			[Linear transformation result] $\mathrm{M}_{aX+b}(\theta)=\mathrm{e}^{b\theta}\mathrm{M}_X(a\theta)$.	M1	Allow implicit $b = 0$. Note M1 mark required for f.t. to A marks below.
			$\mathrm{M}_{\Sigma X}(\theta)=\left\{\frac{1}{3}\left(1+\mathrm{e}^{2\theta}+\mathrm{e}^{-2\theta}\right)\right\}^n$	A1	
			$\mathrm{M}_{\bar{X}}(\theta)=\left\{\frac{1}{3}\left(1+\mathrm{e}^{2\frac{\theta}{n}}+\mathrm{e}^{-2\frac{\theta}{n}}\right)\right\}^n$	A1	
			$Z=\frac{\bar{X}\sqrt{n}\ n}{\sigma}-\frac{\sqrt{\ }}{\sigma}=\frac{n\bar{X}\sqrt{3}}{2\sqrt{2}}$	B1	Might be implicit in what follows.
			$\mathrm{M}_Z(\theta)=\left\{\frac{1}{3}\left(1+\mathrm{e}^{\frac{2}{n}\frac{\sqrt{3n}}{2\sqrt{2}}\theta}+\mathrm{e}^{-\frac{2}{n}\frac{\sqrt{3n}}{2\sqrt{2}}\theta}\right)\right\}^n$	A1	
			$=\left\{\frac{1}{3}\left(1+\mathrm{e}^{\frac{\theta\sqrt{3}}{\sqrt{2n}}}+\mathrm{e}^{-\frac{\theta\sqrt{3}}{\sqrt{2n}}}\right)\right\}^n$.	A1	**Beware printed answer.**
				[8]	

Question			Answer	Marks	Guidance
2	(iv)		$\mathrm{M}_Z(\theta) = \{\ \frac{1}{3}(1$	M1	M1 for reasonable attempt to expand exponentials.
			$+1+\frac{\theta\sqrt{3}}{\sqrt{2n}}+\frac{3\theta^2}{\sqrt{2}n.2!}+\text{terms in } n^{-3/2}, n^{-2}, \ldots$	A1	A1 for this line.
			$+1-\frac{\theta\sqrt{3}}{\sqrt{2n}}\sqrt{\quad}+\frac{3\theta^2}{\sqrt{2}n.2!}+\text{terms in } n^{-3/2}, n^{-2}, \ldots)\}^n$	A1	A1 for this line.
			cancel $\sqrt{\quad}$	M1	M1 for cancelling first order terms, may be implicit.
			neglect	M1	M1 for neglecting higher order terms in n^{-1} , MUST be explicit.
			$\approx\left\{\frac{1}{3}\left(3+\frac{3\theta^2}{2n}\right)\right\}^n$	A1	
			$=\left(1+\frac{\theta^2}{2n}\right)^n$.	A1	**Beware printed answer.**
				[7]	
2	(v)		Limit is $\mathrm{e}^{\theta^2/2}$. This is mgf of N(0, 1).	M1 M1	M1 for limit, M1 for recognising mgf.
			Mgfs are unique (even in this limiting process).	B1	Bracketed phrase is not needed to earn the mark.
			So (approximately) distribution of Z is N(0, 1).	A1	Bracketed word is not needed to earn the mark.
				[4]	
3	(i)		Type I error: rejecting null hypothesis	B1	Allow B1 out of 2 for P(...).
			when it is true	B1	
			Type II error: accepting null hypothesis	B1	Allow B1 out of 2 for P(...).
			when it is false	B1	
			OC: P(accepting null hypothesis	B1	P(Type II error | the true value of the parameter) scores B1+B1.
			as a function of the parameter under investigation)	B1	
			Power: P(rejecting null hypothesis	B1	P(Type I error | the true value of the parameter) scores B1+B1.
			as a function of the parameter under investigation)	B1	
				[8]	

Question			Answer	Marks	Guidance
3	(ii)		Correct "spike" shape, or point shown. Location of spike or point at θ_0 correct and correctly labelled. Height of spike correct and correctly labelled as 1.	G1 G1 G1 **[3]**	
3	(iii)		$X \sim \mathrm{N}(\mu, 9)$. $n = 25$. H_0: $\mu = 0$. H_1: $\mu \neq 0$. Accept H_0 if $-1 < \bar{x} < 1$.		
			P(Type I error) = $\mathrm{P}\left(\left\|\bar{X}\right\| > 1 \;\middle\|\; \bar{X} \sim \mathrm{N}\left(0, \frac{9}{25}\right)\right)$	M1 M1	M1 for use of $\left\|\bar{X}\right\| > 1$, M1 for distribution of $\bar{X}$. Either or both marks might be implicit in what follows.
			$= \mathrm{P}\left(\left\|\mathrm{N}(0,1)\right\| > \frac{1-0}{3/5}\right) = 2 \times 0.0478 = 0.0956.$	A1	Accept a.w.r.t. 0.096.
			P(Type II error when $\mu = 0.5$) $= \mathrm{P}\left(-1 < \bar{X} < 1 \;\middle\|\; \bar{X} \sim \mathrm{N}\left(0.5, \frac{9}{25}\right)\right)$	M1 M1	As for the two M1 marks for the Type I error.
			$= \mathrm{P}\left(\frac{-1.5}{3/5} < \mathrm{N}(0,1) < \frac{0.5}{3/5}\right) = \mathrm{P}\left(-2.5 < \mathrm{N}(0,1) < 0.8333\right)$	M1	Standardising with correct end-points.
			$= 0.7976 - 0.0062 = 0.7914.$	A1	
			This is high. We are trying to detect only a small departure from H_0, the "error" (σ^2) being comparatively large.	E1 E1 **[9]**	
3	(iv)		Correct shape – must be symmetrical, and reasonable approximation to Normal pdf. "Centre" at 0 and clearly labelled. Height at 0 distinctly less than 1 by about 10% (ft candidate's P(Type I error)). Some indication that height at 0.5 is about 0.8 (ft candidate's P(Type II error)).	G1 G1 G1 G1 **[4]**	

Question			Answer	Marks	Guidance
4	(i)		Randomisation is mainly to guard against possible sources of bias, which may be due to subjective allocation of treatments to units or unsuspected. Replication enables an estimate of experimental error to be made.	B1 B1 B1 B1 B1 B1 **[6]**	Award up to B3 marks. These include B1 for idea of "sources of bias" and another B1 for either idea of " unsuspected" or "subjective". Award up to B3 marks. These include B1 for *some* concept of replication addressing experimental error and another B1 for the explicit idea that it enables this error to be estimated. The third B1 should be awarded for the general quality of the response, especially the lack of extraneous material in it Alternative suggestions offered by candidates may be rewarded *provided they are statistically sound*. Suggestions that are limited to particular designs (eg for the advantages of randomisation within a randomised blocks design) may be rewarded if statistically sound, but for at most 2 of the B3 marks in each set.
4	(ii)		μ is the population mean for the entire experiment. α_i is the population amount by which the mean for the ith treatment differs from μ. e_{ij} ~ ind N (0, σ^2)	B1 B1 B1 B1 B1 B1 B1 **[7]**	B1 for explicit mention of "population", B1 for idea of mean for whole experiment. B1 for explicit mention of "population", B1 for idea of difference of means. For "ind N"; allow "uncorrelated". For mean 0. For variance σ^2 [i.e. that the variance is constant].

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="3">4</td><td rowspan="3">(iii)</td><td rowspan="3"></td><td>Null hypothesis: $\alpha_1 = \alpha_2 = ... = \alpha_k$ (= 0)
Alternative hypothesis: not all α_i are equal
Note that alternative hypothesis is NOT that all the α_i are different – B0 for alternative hypothesis if this is stated.</td><td>B1
B1</td><td>No need for definition of k as the number of treatments, and accept simply $\alpha_1 = \alpha_2 =$... with no explicit upper end to the sequence.
Accept hypotheses stated verbally provided it is clear that *population* parameters (means) are being referred to.</td></tr>
<tr><td><table>
<tr><th>Source of variation</th><th>Sum of squares</th><th>d.f.</th><th>Mean squares</th><th>MS ratio</th></tr>
<tr><td>Between fertilisers</td><td>219.2</td><td>4</td><td>54.8</td><td>2.7</td></tr>
<tr><td>Residual</td><td>304.5</td><td>15</td><td>20.3</td><td></td></tr>
<tr><td>Total</td><td>523.7</td><td>19</td><td></td><td></td></tr>
</table></td><td>B1 B1
M1
M1
A1</td><td>B1 for *each* d.f. (4 and 15).
For *method* of mean squares.
For *method* of mean square ratio.
A1 c.a.o. for 2.7 (2.6695).
f.t. from here provided all M marks earned.</td></tr>
<tr><td>Refer 2.7 to $F_{4,15}$.
Upper 5% point is 3.06.
Not significant.
Seems mean effects of fertilisers are all the same.</td><td>M1
A1
A1
E1
[11]</td><td>No f.t. if wrong but allow M1 for F with candidate’s df if both positive and totalling 19.
cao. No f.t. if wrong (or if not quoted).
Verbal conclusion in context, and not "too assertive".</td></tr>
</table>