4769 June 2011 Qu 1

$\mathrm{f}(x) = \frac{1}{\sqrt{2\pi\theta}} e^{-x^2/2\theta}$ [N(0, θ)]	
(i) $L = \frac{1}{\sqrt{2\pi\theta}} e^{-x_1^2/2\theta} \cdot \frac{1}{\sqrt{2\pi\theta}} e^{-x_2^2/2\theta} \cdot \ldots \cdot \frac{1}{\sqrt{2\pi\theta}} e^{-x_n^2/2\theta}$	M1 product form A1 fully correct
$\left[= (2\pi\theta)^{-n/2} e^{-\Sigma x_i^2/2\theta} \right]$	Note. This A1 mark and the next five A1 marks depend on *all* preceding M marks having been earned.
$\ln L = -\frac{n}{2}\ln(2\pi\theta) - \frac{1}{2\theta}\sum x_i^2$	M1 for ln *L* A1 fully correct
$\frac{d\ln L}{d\theta} = -\frac{n}{2}\cdot\frac{1}{\theta} + \frac{1}{2\theta^2}\sum x_i^2$	M1 for differentiating A1, A1 for each term
$\frac{d\ln L}{d\theta} = 0$ gives $\frac{n}{2\hat{\theta}} = \frac{1}{2\hat{\theta}^2}\sum x_i^2$	M1 A1
i.e. $\hat{\theta} = \frac{1}{n}\sum x_i^2$	A1
Check this is a maximum. Eg:	M1
$\frac{d^2\ln L}{d\theta^2} = \frac{n}{2}\cdot\frac{1}{\theta^2} - \frac{1}{\theta^3}\sum x_i^2$	A1
which, for $\theta = \hat{\theta}$, is $\frac{n}{2\hat{\theta}^2} - \frac{n}{\hat{\theta}^2} = -\frac{n}{2\hat{\theta}^2} < 0$.	A1 for expression involving $\hat{\theta}$ A1 for showing < 0 **[14]**
(ii) First consider E(X^2) = Var(X) + {E(X)}^{2} = θ + 0	M1 A1
$\therefore \mathrm{E}(\hat{\theta}) = \frac{1}{n}(\theta + \theta + \ldots + \theta) = \theta$	A1
i.e. $\hat{\theta}$ is unbiased.	A1 **[4]**
(iii) Here $\hat{\theta} = 10$ and Est Var$(\hat{\theta})$ = 2 × 10^2/100 = 2	B1, B1
Approximate confidence interval is given by $10 \pm 1.96\sqrt{2} = 10 \pm 2.77$, i.e. it is (7.23, 12.77).	M1 centred at 10 B1 1.96 M1 Use of √2 A1 c.a.o. Final interval **[6]**

4769 June 2011 Qu 2

(i) $n = 2$ $\quad \mathrm{f}(x) = \frac{1}{2}\mathrm{e}^{-x/2}$	
$\mathrm{M}(\theta) = \mathrm{E}\left(\mathrm{e}^{\theta X}\right) = \int_0^\infty \frac{1}{2}\mathrm{e}^{-x\left(\frac{1}{2}-\theta\right)}\mathrm{d}x$	A1 Any equivalent form
$= \frac{1}{2}\left[\frac{e^{-x\left(\frac{1}{2}-\theta\right)}}{-\left(\frac{1}{2}-\theta\right)}\right]_0^\infty$ **[A1]** $= \frac{\frac{1}{2}}{\frac{1}{2}-\theta}$ **[A1]** $= (1-2\theta)^{-1}$ **[A1]**	A1, A1, A1 for each expression, as shown, **beware printed answer**
$n = 4$ $\quad \mathrm{f}(x) = \frac{1}{4}x\mathrm{e}^{-x/2}$	
$\mathrm{M}(\theta) = \int_0^\infty \frac{1}{4}x\mathrm{e}^{-x\left(\frac{1}{2}-\theta\right)}\mathrm{d}x$	M1 for attempt to integrate this by parts
$= \frac{1}{4}\left\{\left[\frac{x\mathrm{e}^{-x\left(\frac{1}{2}-\theta\right)}}{-\left(\frac{1}{2}-\theta\right)}\right]_0^\infty \textbf{[A1]} - \int_0^\infty \frac{\mathrm{e}^{-x\left(\frac{1}{2}-\theta\right)}}{-\left(\frac{1}{2}-\theta\right)}\mathrm{d}x \textbf{ [A1]}\right\}$	A1, A1 for each component, as shown
$= \frac{1}{4}\left\{[0-0] \textbf{ [A1]} + \frac{1}{\frac{1}{2}-\theta}.2(1-2\theta)^{-1} \textbf{ [A1]}\right\}$	A1, A1 for each component, as shown
$= \frac{1}{2}\frac{1}{\frac{1}{2}(1-2\theta)}(1-2\theta)^{-1} = (1-2\theta)^{-2}$	A1 for final answer, **beware printed answer**
	[10]
(ii) Mean = M'(0) $\quad \mathrm{M}'(\theta) = -2\left(-\frac{n}{2}\right)(1-2\theta)^{-\frac{n}{2}-1} = n(1-2\theta)^{-\frac{n}{2}-1}$	M1 A1
∴ mean = *n*	A1
Variance = M''(0) – {M'(0)}^{2}	
$\mathrm{M}''(\theta) = n\left(-\frac{n}{2}-1\right)(-2)(1-2\theta)^{-\frac{n}{2}-2} = n(n+2)(1-2\theta)^{-\frac{n}{2}-2}$	M1 A1
∴ M''(0) = *n*(*n* + 2)	A1
∴ variance = $n(n + 2) - n^2 = 2n$	A1
[**Note.** This part of the question may also be done by expanding the mgf.]	**[7]**

Solution continued on next page

4769 June 2011 Qu 2 **continued**

(iii)	By convolution theorem, $$M_W(\theta)=\left\{(1-2\theta)^{-\frac{1}{2}}\right\}^k=(1-2\theta)^{-k/2}.$$ This is the mgf of χ^2_k, so (by uniqueness of mgfs) $W \sim \chi^2_k$.	M1 B1 M1 B1 **[4]**
(iv)	$W \sim \chi^2_{100}$ has mean 100, variance 200. Can regard W as the sum of a large "random sample" of χ^2_1 variates. $$\therefore \mathrm{P}\left(\chi^2_{100}<118.5\right)\approx \mathrm{P}\left(\mathrm{N}(0,1)<\frac{118.5-100}{\sqrt{200}}=1.308\right)$$ $= 0.9045$.	M1 for use of N(0,1) A1 c.a.o. for 1.308 A1 c.a.o. **[3]**

4769 June 2011 Qu 3

(i)		8 separate B1 marks for components of answer, as shown
	Type I error: rejecting null hypothesis **[B1]** when it is true **[B1]**	Allow B1 out of 2 for P(...)
	Type II error: accepting null hypothesis **[B1]** when it is false **[B1]**	Allow B1 out of 2 for P(...)
	OC: P(accepting null hypothesis **[B1]** as a function of the parameter under investigation **[B1]**)	P(Type II error | the true value of the parameter) scores B1+B1
	Power: P(rejecting null hypothesis **[B1]** as a function of the parameter under investigation **[B1]**)	P(Type I error | the true value of the parameter) scores B1+B1. "1 – OC" as definition scores zero. **[8]**
(ii)	$X \sim N(\mu, 25)$ $H_0: \mu = 94$ $H_1: \mu > 94$	
	We require $0.02 = P(\text{reject } H_0 \mid \mu = 94) = P(\bar{X} > c \mid \mu = 94)$	M1
	$= P(N(94, 25/n) > c) = P\left(N(0,1) > \dfrac{c-94}{5/\sqrt{n}}\right)$	M1 for first expression M1 for standardising
	$\therefore \dfrac{c-94}{5/\sqrt{n}} = 2.054$	B1 for 2.054
	We also require $0.95 = P(\text{reject } H_0 \mid \mu = 97)$	
	$= P(N(97, 25/n) > c) = P\left(N(0,1) > \dfrac{c-97}{5/\sqrt{n}}\right)$	M1 for first expression M1 for standardising
	$\therefore \dfrac{c-97}{5/\sqrt{n}} = -1.645$	B1 for –1.645
	$\therefore$ we have $c = 94 + \dfrac{10.27}{\sqrt{n}}$ and $c = 97 - \dfrac{8.225}{\sqrt{n}}$	M1 two equations A1 both correct (FT any previous errors)
	Attempt to solve; c = 95.666 [allow 95.7 or awrt] $\sqrt{n}$ = 6.165, n = 38.01 Take n as "next integer up" from candidate's value	M1 A1 c.a.o. A1 c.a.o. A1 **[13]**
(iii)	Power function: step function from 0 with step marked at 94 to height marked as 1	G1 G1 G1 Zero out of 3 if step is wrong way round. **[3]**

4769 June 2011 Qu 4

(a)	Each E2 in this part is available as E2, E1, E0.	
(i)	Description of situation where randomised blocks would be suitable, ie one extraneous factor (eg stream down one side of a field).	E2
	Explanation of why RB is suitable (the design allows the extraneous factor to be "taken out "separately).	E2
	Explanation of why LS is not appropriate (eg: there is only one extraneous factor; LS would be unnecessarily complicated; not enough degrees of freedom would remain for a sensible estimate of experimental error).	E2
(ii)	Description of situation where Latin square would be suitable, ie two extraneous factors (and all with same number of levels) (eg streams down two sides of a field).	E2
	Explanation of why LS is suitable (the design allows the extraneous factors to be "taken out "separately).	E2
	Explanation of why RB is not appropriate (RB cannot cope with two extraneous factors).	E2 **[12]**

(b) Totals are 56.5 57.4 60.6 82.3 from samples of sizes 4 3 5 4

Grand total 256.8 "Correction factor" CF = $256.8^2/16 = 4121.64$

Total SS = 4471.92 – CF = 350.28

Between treatments SS = $\frac{56.5^2}{4}+\frac{57.4^2}{3}+\frac{60.6^2}{5}+\frac{82.3^2}{4} - CF$

= 4324.1103 – CF = 202.47

Residual SS (by subtraction) = 350.28 – 202.47 = 147.81

M1 for attempt to form three sums of squares.
M1 for correct method for any two.
A1 if each calculated SS is correct.

Source of variation	SS	df	MS **[M1]**	MS ratio **[M1]**
Between treatments	202.47	3 **[B1]**	67.49	5.47(92) **[A1 cao]**
Residual	147.81	12 **[B1]**	12.3175	
Total	350.28	15		

5 marks within the table, as shown

Refer MS ratio to $F_{3,12}$. — M1 No FT if wrong
Upper 5% point is 3.49. — A1 No FT if wrong
Significant. — E1
Seems the effects of the treatments are not all the same. — E1

[12]