

1. In a trial for a new cough medicine, a random sample of 8 healthy patients were given steadily increasing doses of a pepper extract until they started coughing. The level of pepper that triggered the coughing was recorded. Each patient completed the trial after taking a standard cough medicine and, at a later time, after taking the new medicine. The results are given in the table below.

Patient	Level of pepper extract that triggers coughing							
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>
Standard medicine	46	12	18	31	23	16	27	9
New medicine	53	16	13	49	11	34	38	22

- (a) Using a suitable test, at the 5% level of significance, state whether or not, on the basis of this trial, you would recommend using the new medicine. State your hypotheses clearly.

(8)

- (b) State an assumption needed to carry out this test.

(1)



3. A farmer is investigating the milk yields of two breeds of cow. He takes a random sample of 9 cows of breed *A* and an independent random sample of 12 cows of breed *B*. For a 5 day period he measures the amount of milk, x gallons, produced by each cow. The results are summarised in the table below.

Breed	Sample size	Mean (\bar{x})	Standard deviation (s_x)
<i>A</i>	9	6.23	2.98
<i>B</i>	12	7.13	2.33

The amount of milk produced by each cow can be assumed to follow a normal distribution.

- (a) Use a two-tail test to show, at the 10% level of significance, that the variances of the yields of the two breeds can be assumed to be equal. State your hypotheses clearly. (4)
- (b) Stating your hypotheses clearly, test, at the 5% level of significance, whether or not there is a difference in the mean yields of the two breeds of cow. (7)
- (c) Explain briefly the importance of the test in part (a) for the test in part (b). (1)



- At the start of each academic year, a large college carries out a diagnostic test on a random sample of new students. Past experience has shown that the standard deviation of the scores on this test is 19.71

The admissions tutor claimed that the new students in 2013 would have more varied scores than usual. The scores for the students taking the test can be assumed to come from a normal distribution. A random sample of 10 new students was taken and the score x , for each student was recorded. The data are summarised as $\sum x = 619$ $\sum x^2 = 42397$

- Stating your hypotheses clearly, and using a 5% level of significance, test the admission tutor's claim. (6)

The admissions tutor decides that in future he will use the same hypotheses but take a larger sample of size 30 and use a significance level of 1%.

- Use the tables to show that, to 3 decimal places, the critical region for S^2 is $S^2 > 664.281$ (3)
- Find the probability of a type II error using this test when the true value of the standard deviation is in fact 22.20 (3)



Question 4 continued

Lined writing area consisting of 25 horizontal lines for providing an answer to Question 4.



6. Emily is monitoring the level of pollution in a river. Over a period of time she has found that the amount of pollution, X , in a 100 ml sample of river water has a continuous distribution with probability density function $f(x)$ given by

$$f(x) = \begin{cases} \frac{2x}{a^2} & 0 \leq x \leq a \\ 0 & \text{otherwise} \end{cases}$$

where a is a constant.

Emily takes a random sample $X_1, X_2, X_3, \dots, X_n$ to try to estimate the value of a .

- (a) Show that $E(\bar{X}) = \frac{2a}{3}$ and $\text{Var}(\bar{X}) = \frac{a^2}{18n}$ (4)

The random variable $S = p\bar{X}$, where p is a constant, is an unbiased estimator of a .

- (b) Write down the value of p and find $\text{Var}(S)$. (2)

Felix suggests using the statistic $M = \max\{X_1, X_2, X_3, \dots, X_n\}$ as an estimator of a .

He calculates $E(M) = \frac{2n}{2n+1}a$ and $\text{Var}(M) = \frac{n}{(n+1)(2n+1)^2}a^2$

- (c) State, giving your reasons, whether or not M is a consistent estimator of a . (3)

The random variable $T = qM$, where q is a constant, is an unbiased estimator of a .

- (d) Write down, in terms of n , the value of q and find $\text{Var}(T)$. (3)

- (e) State, giving your reasons, which of S or T you would recommend Emily use as an estimator of a . (3)

Emily took a sample of 5 values of X and obtained the following:

5.3 4.3 5.7 7.8 6.9

- (f) Calculate the estimate of a using your recommended estimator from part (e). (2)
- (g) Find the standard error of your estimate, giving your answer to 2 decimal places. (2)



