

June 2006
6686 Statistics S4
Mark Scheme

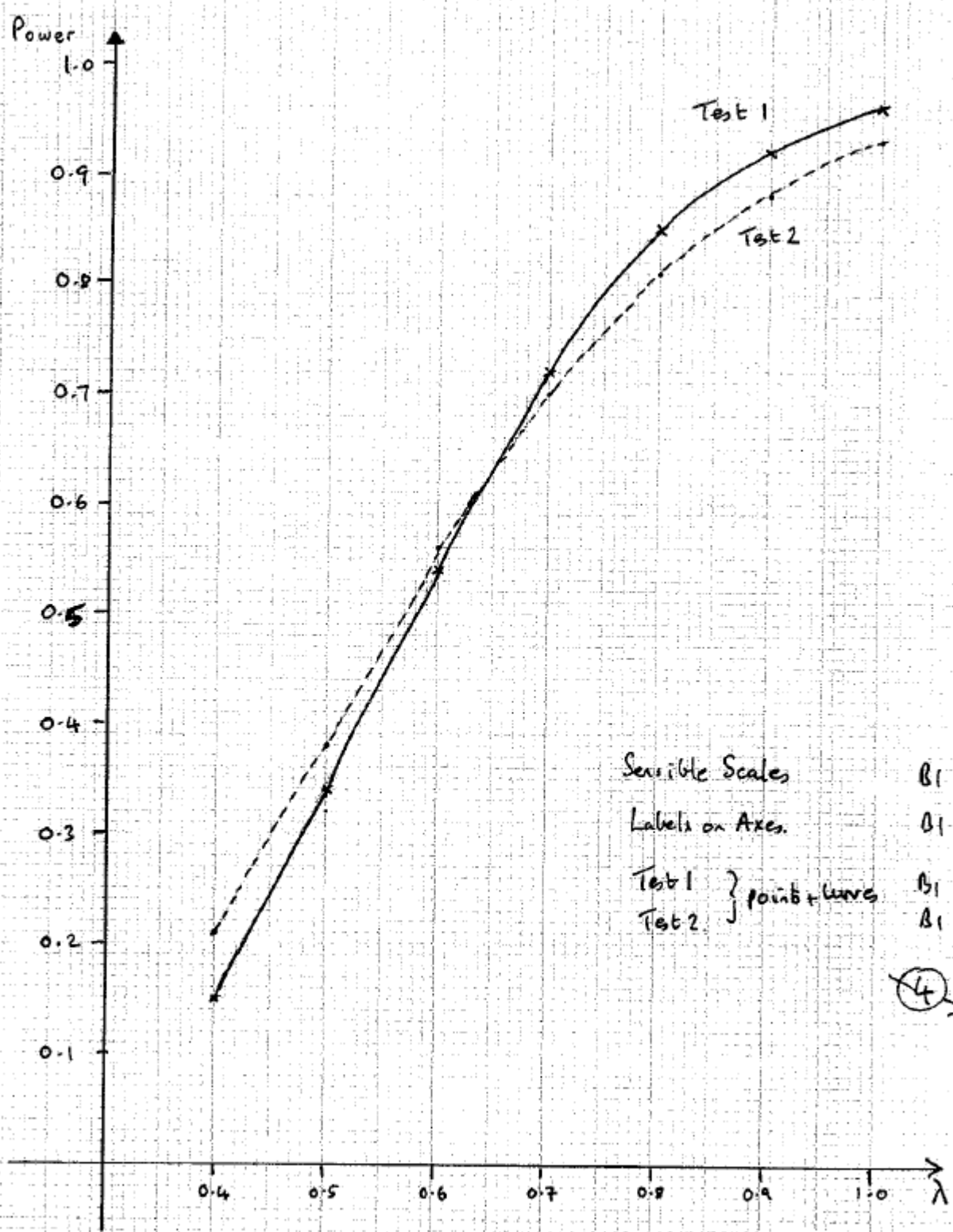
Question Number	Scheme	Marks
1.	$H_0: \mu = 1012 \quad H_1: \mu \neq 1012 \quad \text{both}$ $\bar{x} = \frac{13700}{14} (= 978.57...)$ $S_x^2 = \frac{13\,448\,750 - 14\bar{x}^2}{13} (= 3255.49... \quad (S_x = 57.056...)) \quad S^2 = S$ $t_{13} = \frac{\bar{x} - \mu}{\frac{S_x}{\sqrt{n}}} = \frac{978.6 - 1012}{\frac{57.06}{\sqrt{14}}} = -2.19...$ <p style="text-align: right;">AWRT - 2.19</p> $t_{13}(5\%) \text{ 2tail c.v.} = -2.160$ <p>Significant result - there is evidence of a change in mean weight of squirrel (and one decrease) <u>must mention weight</u></p>	<p>B1 M1 M1 M1 A1 A1 B1 A1</p> <p style="text-align: right;">(7)</p>
2.	<p>(a) $(\bar{x} = \frac{466}{4} = 116.5) \quad S_x^2 = \frac{54\,386 - 4\bar{x}^2}{3} = \frac{32.3}{3} \text{ or } \frac{9.7}{3}$ or AWRT 32.3</p> $0.216 < \frac{3 S_x^2}{\sigma^2} < 9.348$ $10.376... < \sigma^2 < 449.07...$ <p style="text-align: right;">AWRT 10.4, 449</p> <p>(b) $H_0: \sigma_M^2 = \sigma_S^2 \quad H_1: \sigma_M^2 > \sigma_S^2 \quad \text{both}$ <small>$\sigma_M > \sigma_S$ (are OK)</small></p> $\frac{S_M^2}{S_S^2} = \frac{318.8}{32.3} = 9.859...$ <p style="text-align: right;">AWRT 9.86</p> $F_{6,3} (1\% \text{ c.v.}) = 27.91$ <p>$9.85 < 27.91$, insufficient evidence of an increase in variance to say $\sigma_M^2 > \sigma_S^2$ is OK. variance can be assumed to be the same is OK</p> <p>(6) NB. $\frac{32.3}{318.8} = 0.101...$ only gets M1A1 if appropriate F value attempted</p>	<p>M1, A1 B1 M1 B1 A1, A1 (7) B1 A1</p> <p style="text-align: right;">(5) (12)</p>

Question Number	Scheme	Marks
<p>3.</p> <p>(a)</p> <p>$H_0: \mu_D = 0$ $H_1: \mu_D > 0$</p> <p>$d: 6, -3, 7, -2, -8, 6, 5, 11.5$</p> <p>$\bar{d} = 3, \quad s_d = 6 \quad \left(= \sqrt{\frac{369 - 9 \times 3^2}{8}} \right) \quad \left(\frac{\sum d}{n} \quad M1 \right)$</p> <p>$t_8 = \frac{3-0}{\frac{6}{\sqrt{9}}} = 1.5$</p> <p>$t_8$ (5% tail c.v.) = 1.860</p> <p>Not significant - insufficient evidence (that solar heating has) decreased weekly fuel consumption.</p> <p>(b) <u>Difference in weekly fuel consumption is normally distributed.</u></p>	<p>$\mu_1 = \mu_2$ etc is</p> <p>$\begin{bmatrix} \bar{D} = 0 & \bar{D} > 0 \\ D = 0 & D > 0 \end{bmatrix}$</p> <p>(Attempt d.s.)</p> <p>(±)</p> <p>of</p>	<p>B0</p> <p>B0</p> <p>B1</p> <p>M1</p> <p>M1, M1</p> <p>M1 A1 c.a.o</p> <p>B1</p> <p>A1 ✓ (8)</p> <p>B1 (1)</p> <p>9</p>
<p>4. (a)</p> <p>$H_0: \sigma_A^2 = \sigma_B^2$ $H_1: \sigma_A^2 \neq \sigma_B^2$</p> <p>$\frac{S_B^2}{S_A^2} = \frac{0.721^2}{0.572^2} = 1.588 \dots$</p> <p>$F_{8,9}$ (5%) c.v. [= 10% tail] = 3.23</p> <p>Not significant, can assume variances are equal. (accept $\sigma_A^2 = \sigma_B^2$)</p> <p>(b)</p> <p>$S_p^2 = \frac{8 \times 0.721^2 + 9 \times 0.572^2}{8+9} = 0.41784 \dots$</p> <p>$t_{17}$ (2.5%) c.v. = 2.110</p> <p>95% CI = $\bar{x}_B - \bar{x}_A \pm 2.110 \times S_p \times \sqrt{\frac{1}{9} + \frac{1}{10}}$</p> <p>= $0.02 \pm 2.110 \times \sqrt{0.417 \dots} \times \sqrt{\frac{1}{9} + \frac{1}{10}}$</p> <p>= $(-0.6066 \dots, 0.6466 \dots)$ Answer $(-0.607, 0.647)$</p> <p>(c) ± 0.7 is outside interval</p> <p>\therefore manager need <u>not</u> be concerned</p> <p>(allow ✓ if 0.7 inside)</p>	<p>Answer 1.59</p> <p>M1 A1</p> <p>B1</p> <p>B1 c.a.o (4)</p> <p>0.417... or Answer 0.418</p> <p>M1 A1</p> <p>B1</p> <p>M1</p> <p>A1 ✓</p> <p>Answer $(-0.607, 0.647)$ A1, A1 (7)</p> <p>B1 ✓</p> <p>(dep) B1 ✓ (2)</p>	<p>M1 A1</p> <p>B1</p> <p>B1 c.a.o (4)</p> <p>M1 A1</p> <p>B1</p> <p>M1</p> <p>A1 ✓</p> <p>A1, A1 (7)</p> <p>B1 ✓</p> <p>(2)</p> <p>13</p>

5 (a)	$X_1 = \text{no. of defects in 15m}$. $X_1 \sim P_0(4.5)$ Use of $P_0(4.5)$	MI
	Size = $P(X_1 \geq 9) = 1 - P(X_1 \leq 8) = 1 - 0.9597 = 0.0403$ (Answer)	AI (2)
(b)	$r = P(X_2 \geq 9 X_2 \sim P_0(9)) = 1 - P(X_2 \leq 8) = 1 - 0.4557 = 0.5443$ (Answer)	MI, AI (2)
(c)	$Y_1 = \text{no. of defects in 10m}$ $Y_1 \sim P_0(3)$ Use of $P_0(3)$ to find $P(Y_1 \geq c)$	MI
	Require smallest c so that $P(Y_1 \geq c) < 0.10$. Table $Y_1 \geq 6$	AI (2)
(d)	Size = $P(Y_1 \geq 6) = 1 - P(Y_1 \leq 5) = 1 - 0.9161 = 0.0839$	BI (1)
(e)	$s = 1 - P(Y_2 \leq 5 Y_2 \sim P_0(8)) = 1 - 0.1912 = 0.8088$ (Answer)	MI, AI (2)
(f)	See graph	(4) (4)
(g)	(i) $0.62 \sim 0.67$ (ii) Test 1 is more powerful	BI BI (2)
(h)	Test 2 has higher $P(\text{Type I error})$ but cost of this is low Test 2 is more powerful for $\lambda < 0.7$ and $\lambda > 0.7$ is rare	BI Test BI Reason (2)
		(17)

6. (a)	$E(X^n) = \int_0^t x^n \frac{1}{t} dx = \left[\frac{x^{n+1}}{t(n+1)} \right]_0^t = \left(\frac{t^{n+1}}{t(n+1)} - 0 \right) = \frac{t^n}{n+1}$ $\int_0^t x^n \frac{1}{t} dx$	MI AI also (3)
(b)	$(E(X) = \frac{t}{2})$ $E(S) = k E(X) E(Y) = k \cdot \frac{t^2}{4}$ $E(S) = t^2 \Rightarrow k = 4$	MI, AI AI (3)
(c)	$\text{Var}(XY) = E(X^2) E(Y^2) - [E(XY)]^2$ $= \frac{t^2}{3} \times \frac{t^2}{3} - \left(\frac{t^2}{4}\right)^2 = \left\{ \frac{7t^4}{144} \right\}$ $\text{Var}(S) = k^2 \text{Var}(XY) = 16 \times \frac{7t^4}{144} = \frac{7t^4}{9}$	MI MI AI also (3)
(d)	$E(u) = t^2 \Rightarrow 2 E(X^2) q = t^2, \Rightarrow 2 \frac{t^2}{8} q = t^2, \Rightarrow q = \frac{3}{2}$	MI, MI, AI also (3)
(e)	$\text{Var}(u) = q^2 [\text{Var}(X^2) + \text{Var}(Y^2)] = 2q^2 \text{Var}(X^2)$ $\text{Var}(X^2) = E(X^4) - [E(X^2)]^2 = \frac{t^4}{5} - \left(\frac{t^2}{3}\right)^2 = \left(\frac{4}{45} t^4\right)$ $\text{Var}(u) = 2 \times \frac{9}{4} \times \frac{4}{45} t^4 = \frac{2}{5} t^4$	MI MI AI (3)
(f)	$\frac{2}{5} < \frac{7}{9} \therefore u$ is better \therefore smaller variance	BI ✓ (1)
(g)	Using u estimate is: $\frac{3}{2} (2^2 + 3^2) = \frac{3}{2} \times 13 = \frac{39}{2}$ or 19.5	BI ✓ (1) (17)

Power Functions



Sensible Scales B1
Labels on Axes B1
Test 1 } point + curve B1
Test 2 } B1

④ →