EDEXCEL STATISTICS S4 (6686) - JUNE 2002 PROVISIONAL MARK SCHEME

Question Number	Scheme	Marks
1. (*a*)	$b = 2.75$, $a = \dfrac{1}{2.91} = 0.344$ 2.75, reciprocal, 0.344	B1, M1, A1 **(3 marks)**
2.	d: 5, 13, −8, 2, −3, 4, 11, −1 at least 2 correct $(\Sigma d = 23, \Sigma d^2 = 409)$ $\bar{d} = 2.875$, $sd = 6.9987$ (≈ 7.00) $H_0: \mu_d = 0$, $H_1: \mu_d > 0$ both $t = \dfrac{2.875\sqrt{8}}{6.9987} = 1.1618\ldots$ (≈ 1.16) formula and substitution, 1.16 Critical value $t_7(10\%) = 1.415$ (1 tail) Not significant. Insufficient evidence to support the chemist's claim.	M1 A1, A1 B1 M1, A1 B1 A1 ft **(8 marks)**
3. (*a*)	$E(A_1) = E(X_1)\,E(X_2) = \mu^2$ $A_2 = \bar{X}^2$, $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{2}\right)$ $\therefore$ $E(\bar{X}^2) = E(A_2) = ,\ \mu^2 + \dfrac{\sigma^2}{2}$	B1 M1, M1, A1 (4)
(*b*)	A_1 is unbiased, bias for A_2 is $\dfrac{\sigma^2}{2}$	B1, B1 (2)
(*c*)	Used A_1 since it is unbiased	B1 (1)
(*d*)	$E(\bar{X}^2) = \mu^2 + \dfrac{\sigma^2}{2}$; as $n \to \infty$, $E(\bar{X}^2) \to \mu^2$ $\text{Var}(\bar{X}^2) = \dfrac{2\sigma^4}{n^2} + \dfrac{4\sigma^2\mu^2}{n}$; as $n \to \infty$, $\text{Var}(\bar{X}^2) \to 0$ $\bar{X}^2$ is a consistent estimator of μ^2	M1 M1 A1 (3) **(10 marks)**

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4. (*a*)	$H_0: \mu = 150.9$ [accept ≥ 150.9], $H_1: \mu < 150.9$ both	B1
	$s^2 = \frac{1}{29}\left(646\,904.1 - \frac{(4400.7)^2}{30}\right) = \frac{1365.727}{29} = 47.1$	M1
	test statistic $t = \frac{30}{s/\sqrt{30}} = -3.36$	M1 A1
	critical value $t_{29}(5\%) = (-)1.669$	B1
	significant, evidence to confirm doctor's statement	A1 ft (6)
(*b*)	$H_0: \sigma^2 = 36$, $H_1: \sigma^2 \neq 36$ both	B1
	test statistic $\frac{(n-1)s^2}{\sigma^2}$ =, $\frac{1365.727}{36} = 37.9$	M1, A1
	critical values $\chi^2_{29}(5\%)$ upper tail $= 45.722$, $\chi^2_{29}(5\%)$ lower tail $= 16.047$ not significant	B1, B1
	Insufficient evidence that variance of the heights of female Indians is different from that of females in the UK	A1 ft (6)
		(12 marks)
5. (*a*)	$H_0: \sigma_G^2 = \sigma_B^2$, $H_1: \sigma_G^2 \neq \sigma_B^2$,	B1
	$s_B^2 = \frac{1}{6}(56130 - 7 \times 88.9^2) = \frac{807.53}{6} = 134.6$	M1 A1
	$s_G^2 = \frac{1}{7}(55746 - 8 \times 83.1^2) = \frac{501.12}{7} = 71.58$	A1
	$\frac{s_B^2}{s_G^2} = 1.880\ldots$	M1
	critical value $F_{6,7} = 3.87$	B1
	not significant, variances are the same	A1 ft (7)
(*b*)	$H_0: \mu_B = \mu_G$, $H_1: \mu_B > \mu_G$	B1
	pooled estimate of variance $s^2 = \frac{6 \times 134.6 + 7 \times 71.58}{13} = 100.6653\ldots$	M1
	test statistic $t = \frac{88.9 - 83.1}{s\sqrt{\frac{1}{7} + \frac{1}{8}}}$	M1 A1
	critical value $t_{13}(5\%) = 1.771$	B1
	Insufficient evidence to support parent's claim	A1 ft (6)
		(13 marks)

ft = follow through mark

Question Number	Scheme	Marks
6. (*a*)	95% confidence interval for μ is 2.064	B1
	$1.68 \pm t_{24}(2.5\%)\sqrt{\frac{1.79}{25}} = 1.68 \pm 2.064\sqrt{\frac{1.79}{25}} = (1.13, 2.23)$	M1 A1 A1 (4)
(*b*)	95% confidence interval for σ^2 is	
	$12.401, < \frac{24 \times 1.79}{\sigma^2} <, 39.364$	B1, M1, B1
	$\sigma^2 > 1.09,\ \sigma^2 > 3.46$	A1, A1 (5)
(*c*)	Require $\mathrm{P}(X > 2.5) = \mathrm{P}\left(Z > \frac{2.5-\mu}{\sigma}\right)$ to be as small as possible OR	
	$\frac{25-\mu}{\sigma}$ to be as large as possible; both imply lowest σ and μ.	M1 M1
	$\frac{25-1.13}{\sqrt{1.09}} = 1.31$	M1
	$\mathrm{P}(Z > 1.31) = 1 - 0.9049 = 0.0951$	A1 (4)
		(13 marks)
7. (*a*)	X is the number of defectives, $X \sim \mathrm{B}(5, p)$	
	size = $\mathrm{P}(\text{reject } H_0 \mid p = 0.1) = \mathrm{P}(X > 2 \mid p = 0.1)$	M1
	$= 1 - 0.9914 = 0.0086$	A1 (2)
(*b*)	$r = \mathrm{P}(X > 2 \mid p = 0.2)$, $1 - 0.9421$, $= 0.0579$	M1, M1, A1 (3)
(*c*)	Y is the number of defectives, $Y \sim \mathrm{B}(10, p)$	
	P(Type I error) = $\mathrm{P}(Y > 4 \mid p = 0.1) = 1 - 0.9984 = 0.0016$	M1 A1 (2)
(*d*)	$s = \mathrm{P}(Y > 4 \mid p = 0.4) = 1 - 0.6331 = 0.3669$	B1 (1)
(*e*)	Graph	G4 (4)
(*f*)	(i) Intersection 0.32 – 0.33	B1
	(ii) $p > 0.32$; Assistant's test is more powerful (sensible comment)	B1 (2)
(*g*)	Consider costs – smaller sample so test is cheaper	B1
	More powerful for $p < 0.32$ and $p > 0.32$ is unlikely	B1 (2)
		(16 marks)