

January 2006  
6686 Statistics S4  
Mark Scheme

Question Number	Scheme	Marks
1 (a)	$\Sigma x = 49.6$ ; $\Sigma x^2 = 362.36$ $s^2 = \frac{1}{6} \left( 362.36 - \frac{49.6^2}{7} \right) = 1.8180952\dots$ awrt 1.82	M1A1 (2)
(b)	$CI = \left( \frac{6 \times 1.818\dots}{12.592}, \frac{6 \times 1.818\dots}{1.635} \right)$ $= (0.866\dots, 6.67\dots)$	M1 B1B1 A1A1 (5)
(c)	$0.9^2 < 0.866$ , interval does not support $\sigma = 0.9$ as out of range.	B1 (1)
		TOTAL 8
2 (a)(i)	Type I: $H_0$ rejected when true	B1
(ii)	Type II: $H_0$ accepted when false	B1 (2)
(b)(i)	$p = \frac{7.5}{50} = 0.15$ cr $X \leq 3$	B1 B1 (2)
(ii)	$H_0: p = 0.15$ , $H_1: p < 0.15$ both $x = 2$ in cr $X \leq 3$ so $H_0$ is rejected The new machine has reduced the <sup>mean</sup> number of faulty socks	B1 M1 A1 (3)
(c)	$P(\text{Type I error}) = P(X \leq 3   p = 0.15) = 0.0460$	M1A1 (2)
(d)	$P(\text{Faulty}) = \frac{5}{50} = 0.1$ $P(\text{Type II error}) = P(X \geq 4   p = 0.1) = 1 - 0.2503$ $= 0.7497$ awrt 0.750	B1 M1 A1 (3)
(e)	Critical region changes to $X \leq 2$ . $H_0$ still rejected.	B1 (1)
		TOTAL 13

3(a) (i)	$E\left(\frac{1}{3}\bar{X} + \frac{2}{3}\bar{Y}\right) = \frac{1}{3}E\left(\frac{X_1+X_2+X_3}{3}\right) + \frac{2}{3}E\left(\frac{Y_1+Y_2+Y_3+Y_4}{4}\right)$ $= \frac{1}{3}\mu + \frac{2}{3}\mu$ $= \mu \quad \text{Hence unbiased estimator}$	M1 A1 (2)
(ii)	$E\left(\frac{5\bar{X} + 4\bar{Y}}{9}\right) = \frac{1}{9}(5E(\bar{X}) + 4E(\bar{Y}))$ $= \frac{1}{9}(5\mu + 4\mu)$ $= \mu \quad \text{Hence unbiased estimator}$	M1 A1 (2)
(b)	$\text{Var}(\bar{X}) = \frac{\sigma^2}{3}, \quad \text{Var}(\bar{Y}) = \frac{\sigma^2}{4}$ $\text{Var}(\hat{\mu}_1) = \frac{1}{9} \cdot \frac{\sigma^2}{3} + \frac{4}{9} \cdot \frac{\sigma^2}{4} = \frac{4\sigma^2}{27}$	M1A1 (2)
(c)	$\frac{4}{27}\sigma^2 < \frac{37}{243}\sigma^2 \quad \text{so use } \hat{\mu}_1.$	B1 (1) TOTAL 7
4(a)	$\text{Size of test} = P(X > 4   \lambda = 3)$ $= 1 - P(X \leq 4   \lambda = 3) = 1 - 0.8153$ $= 0.1847 \quad \text{ans } 0.185$	M1 A1 A1 (3)
(b)	$r = 1 - 0.6288 = 0.3712 = 0.37 \text{ (2dp)}$ $s = 1 - 0.2851 = 0.7149 = 0.71 \text{ (2dp)}$	B1 B1 (2)
(c)	<p>When <math>\lambda = 4</math>, power = 0.27 &lt; 0.5          Probability of coming to correct conclusion is less than          probability of coming to wrong conclusion.  <u>Not suitable.</u></p>	B1 (1) TOTAL 6

<p>5(a)</p>	<p><math>d = 0 - c, 2, -4, 18, 16, 0, 15, 9</math></p> <p><math>\bar{d} = \frac{56}{7} = 8</math></p> <p><math>s_d^2 = \frac{906 - 7 \times 8^2}{6} = 76\frac{1}{2}</math></p> <p><math>H_0: \mu_d = 0, H_1: \mu_d \neq 0</math></p> <p><math>t = \frac{8}{\sqrt{\frac{76.5}{7}}} = 2.42260\dots</math> <small>correct 2.42</small></p> <p><math>t_{2}(2.5\%) = 2.447</math></p> <p>Insufficient evidence to reject <math>H_0</math>. No evidence of a difference between the mean amount of corrosion on coated and uncoated pipes.</p>	<p>M1 B1 M1A1 B1 M1A1 B1 A1 (9)</p>
<p>(b) (i) (ii) (c)</p>	<p>Differences are normally distributed</p> <p>Values do not appear <sup>to be</sup> normally distributed</p> <p><math>t_{2}(5\%) = 1.943</math>. There is evidence to reject <math>H_0</math>. There is evidence to suggest that there is a greater corrosion on coated pipes.</p>	<p>B1 (1) B1 (1) B1 B1 (2) TOTAL 13</p>
<p>6(a)</p>	<p><math>s_1^2 = \frac{1}{19} (274050 - \frac{(2340)^2}{20}) = 14.2</math></p> <p><math>s_0^2 = \frac{1}{36} (645282 - \frac{(4884)^2}{37}) = 16.5</math> <small>*AG</small> <small>York</small></p>	<p>M1 A1 (2)</p>
<p>(b)</p>	<p><math>s_p = \sqrt{\frac{19 \times 14.2 + 36 \times 16.5}{55}} = \sqrt{15.705} = 3.963\dots</math></p> <p>Mean outside = <math>\frac{2340}{20} = 117</math>, Mean Inside = 132</p> <p>Confidence limits = <math>(132 - 117) \pm 2.009 \times 3.93\dots \sqrt{\frac{1}{20} + \frac{1}{37}}</math></p> <p>= (12.8, 17.2)</p>	<p>M1A1 B1 B1 M1A1 ✓ A1A1 (8)</p>
<p>(c)</p>	<p>0 lies outside confidence interval. The means are different.</p>	<p>B1 B1 (2) TOTAL 12</p>

<p>7(a)</p>	<p><math>S_A^2 = 5.11, S_B^2 = 5.14</math>  <math>H_0: \sigma_A^2 = \sigma_B^2, H_1: \sigma_A^2 \neq \sigma_B^2</math>                  Critical value <math>F_{6,8} = 3.58</math></p> <p><math>\frac{S_B^2}{S_A^2} = 1.0062112...</math>      a/crt 1.01</p> <p>No evidence to reject <math>H_0</math>. The variances are equal.</p>	<p>B   B                    B                    B                    M   A                    A   (7)</p>
<p>(b)</p>	<p><math>S_p^2 = \frac{8 \times 5.14 + 6 \times 5.11}{9 + 7 - 2} = 5.1247</math>      a/crt 5.12</p> <p><math>M_A = 14.11..., M_B = 11.857...</math>  <math>H_0: M_A = M_B, H_1: M_A &gt; M_B</math>                  Critical value <math>t_{14}(5\%) = 1.761</math></p> <p><math>t = \frac{14.11... - 11.857...}{\sqrt{5.1247... \left(\frac{1}{9} + \frac{1}{7}\right)}} = 1.9757...</math>      a/crt 1.98</p> <p>There is evidence to reject <math>H_0</math>.                  Mean time taken from school A is greater than school B.</p>	<p>M   A                    B                    B                    M   A                    A   (7)</p>
<p>(c)</p>	<p>Equal variances are a condition for the test in part (b)</p>	<p>B   (1)</p>
<p>(d)</p>	<p>Groups not equal ability</p>	<p>B   (1)</p>
		<p>TOTAL 16</p>