Edexcel Maths S4

Mark Scheme Pack

2002-2015

Question Number	Scheme	Marks
1. (*a*)	$b = 2.75$, $a = \dfrac{1}{2.91} = 0.344$ 2.75, reciprocal, 0.344	B1, M1, A1
		(3 marks)
2.	d: 5, 13, −8, 2, −3, 4, 11, −1 at least 2 correct	M1
	$(\Sigma d = 23, \Sigma d^2 = 409)$ $\bar{d} = 2.875$, $sd = 6.9987$ (≈ 7.00)	A1, A1
	H_0: $\mu_d = 0$, H_1: $\mu_d > 0$ both	B1
	$t = \dfrac{2.875\sqrt{8}}{6.9987} = 1.1618\ldots$ (≈ 1.16) formula and substitution, 1.16	M1, A1
	Critical value $t_7(10\%) = 1.415$ (1 tail)	B1
	Not significant. Insufficient evidence to support the chemist's claim.	A1 ft
		(8 marks)
3. (*a*)	$E(A_1) = E(X_1)\,E(X_2) = \mu^2$	B1
	$A_2 = \bar{X}^2$, $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{2}\right)$ $\therefore$ $E(\bar{X}^2) = E(A_2) =$, $\mu^2 + \dfrac{\sigma^2}{2}$	M1, M1, A1 (4)
(*b*)	A_1 is unbiased, bias for A_2 is $\dfrac{\sigma^2}{2}$	B1, B1 (2)
(*c*)	Used A_1 since it is unbiased	B1 (1)
(*d*)	$E(\bar{X}^2) = \mu^2 + \dfrac{\sigma^2}{2}$; as $n \to \infty$, $E(\bar{X}^2) \to \mu^2$	M1
	$\text{Var}(\bar{X}^2) = \dfrac{2\sigma^4}{n^2} + \dfrac{4\sigma^2\mu^2}{n}$; as $n \to \infty$, $\text{Var}(\bar{X}^2) \to 0$	M1
	$\bar{X}^2$ is a consistent estimator of μ^2	A1 (3)
		(10 marks)

Question Number	Scheme	Marks
4. (*a*)	H_0: $\mu = 150.9$ [accept ≥ 150.9], H_1: $\mu < 150.9$ both	B1
	$s^2 = \frac{1}{29}\left(646904.1 - \frac{(4400.7)^2}{30}\right) = \frac{1365.727}{29} = 47.1$	M1
	test statistic $t = \frac{30}{s/\sqrt{30}} = -3.36$	M1 A1
	critical value $t_{29}(5\%) = (-)1.669$	B1
	significant, evidence to confirm doctor's statement	A1 ft (6)
(*b*)	H_0: $\sigma^2 = 36$, H_1: $\sigma^2 \neq 36$ both	B1
	test statistic $\frac{(n-1)s^2}{\sigma^2}$ =, $\frac{1365.727}{36} = 37.9$	M1, A1
	critical values $\chi^2_{29}(5\%)$ upper tail $= 45.722$, $\chi^2_{29}(5\%)$ lower tail $= 16.047$ not significant	B1, B1
	Insufficient evidence that variance of the heights of female Indians is different from that of females in the UK	A1 ft (6)
		(12 marks)
5. (*a*)	H_0: $\sigma^2_G = \sigma^2_B$, H_1: $\sigma^2_G \neq \sigma^2_B$,	B1
	$s^2_B = \frac{1}{6}(56130 - 7 \times 88.9^2) = \frac{807.53}{6} = 134.6$	M1 A1
	$s^2_G = \frac{1}{7}(55746 - 8 \times 83.1^2) = \frac{501.12}{7} = 71.58$	A1
	$\frac{s^2_B}{s^2_G} = 1.880\ldots$	M1
	critical value $F_{6,7} = 3.87$	B1
	not significant, variances are the same	A1 ft (7)
(*b*)	H_0: $\mu_B = \mu_G$, H_1: $\mu_B > \mu_G$	B1
	pooled estimate of variance $s^2 = \frac{6 \times 134.6 + 7 \times 71.58}{13} = 100.6653\ldots$	M1
	test statistic $t = \frac{88.9 - 83.1}{s\sqrt{\frac{1}{7} + \frac{1}{8}}}$	M1 A1
	critical value $t_{13}(5\%) = 1.771$	B1
	Insufficient evidence to support parent's claim	A1 ft (6)
		(13 marks)

Question Number	Scheme	Marks
6. (*a*)	95% confidence interval for μ is 2.064	B1
	$1.68 \pm t_{24}(2.5\%)\sqrt{\frac{1.79}{25}} = 1.68 \pm 2.064\sqrt{\frac{1.79}{25}} = (1.13, 2.23)$	M1 A1 A1 (4)
(*b*)	95% confidence interval for σ^2 is	
	$12.401, < \frac{24 \times 1.79}{\sigma^2} <, 39.364$	B1, M1, B1
	$\sigma^2 > 1.09,\ \sigma^2 > 3.46$	A1, A1 (5)
(*c*)	Require $P(X > 2.5) = P\left(Z > \frac{2.5-\mu}{\sigma}\right)$ to be as small as possible OR	
	$\frac{25-\mu}{\sigma}$ to be as large as possible; both imply lowest σ and μ.	M1 M1
	$\frac{25-1.13}{\sqrt{1.09}} = 1.31$	M1
	$P(Z > 1.31) = 1 - 0.9049 = 0.0951$	A1 (4)
		(13 marks)
7. (*a*)	X is the number of defectives, $X \sim B(5, p)$	
	size = P(reject H_0 | $p = 0.1$) = $P(X > 2 \mid p = 0.1)$	M1
	$= 1 - 0.9914 = 0.0086$	A1 (2)
(*b*)	$r = P(X > 2 \mid p = 0.2),\ 1 - 0.9421,\ = 0.0579$	M1, M1, A1 (3)
(*c*)	Y is the number of defectives, $Y \sim B(10, p)$	
	P(Type I error) = $P(Y > 4 \mid p = 0.1) = 1 - 0.9984 = 0.0016$	M1 A1 (2)
(*d*)	$s = P(Y > 4 \mid p = 0.4) = 1 - 0.6331 = 0.3669$	B1 (1)
(*e*)	Graph	G4 (4)
(*f*)	(i) Intersection 0.32 – 0.33	B1
	(ii) $p > 0.32$; Assistant's test is more powerful (sensible comment)	B1 (2)
(*g*)	Consider costs – smaller sample so test is cheaper	B1
	More powerful for $p < 0.32$ and $p > 0.32$ is unlikely	B1 (2)
		(16 marks)

Question number	Scheme		Marks
1. (a)	$H_0 : \sigma_A^2 = \sigma_B^2,\ H_1 : \sigma_A^2 \neq \sigma_B^2$	both	B1
	critical values $F_{24,25} = 1.96$ and $\frac{1}{F_{24,25}} = 0.510$	both	B1
	$\frac{s_B^2}{s_A^2} = 2.10$ or $\frac{s_A^2}{s_B^2} = 0.476$	both	M1A1
	Since 2.10 or 0.476 are in the critical region we reject H_0 and conclude there is evidence that the two variances are different.		A1∫
			(5)
(b)	The populations of pebble lengths are normal.		B1
			(1)
			6
2.	$H_0 : \mu = 5.1,\ H_1 : \mu < 5.1$	both	B1
	$\nu = 9$	9	B1
	Critical Region t<-2.262		B1
	$\bar{x} = 4.91$	4.91	B1
	$s^2 = \frac{241.89 - 10 \times (4.91)^2}{9} = 0.0899$		M1
	$s = 0.300$	0.0899 or 0.300	A1
	$t = \frac{4.91 - 5.1}{\frac{0.3}{\sqrt{10}}} = -2.00$		M1A1
	There is no evidence to suggest that the mean height is less than those grown previously	context	A1∫
			(9)
			9

Question number	Scheme		Marks	
3 (a)	1-0.8891=0.1109		B1	
				(1)
(b)	1-(P(0)+P(1)+P(2))		M1	
	$=1-((1-p)^{12}+12p(1-p)^{11}+66p^2(1-p)^{10})$		M1A1	
	$=1-(1-p)^{10}((1-p)^2+12p(1-p)+66p^2)$			
	$=1-(1-p)^{10}(1+10p+55p^2)$ ****given****	cso	A1	
				(4)
(c) (i)	1-0.5583=0.442		M1A1	
	1-0.00281=0.997		A1	
				(3)
(ii)	The test is more discriminating for the larger value of p		B1	
				(1)
			9	
4 (a)	$s^2=\frac{2962-15\times\left(\frac{208}{15}\right)^2}{14}=5.55$ or $(n-1)s^2=2962-\frac{208^2}{15}=77.3$	either	M1A1	
	$\frac{14\times5.55}{23.685}<\sigma^2<\frac{14\times5.55}{6.571}$	23.685,6.571	M1B1,B1	
	$3.28<\sigma^2<11.83$		A1A1	
				(7)
(b)	Since 9 lies in the interval, yes		B1,B1(dep)	
				(2)
			9	

Question number	Scheme	Marks	
5 (a)(i)	Type I - H_0 rejected when it is true	B1	
(ii)	Type II - H_0 is accepted when it is false	B1	
			(2)
(b)	$\mathrm{H}_0 : \lambda = 5,\ \mathrm{H}_1 : \lambda > 5$ both	B1	
	$\mathrm{P}(X \geq 7 \mid \lambda = 5) = 1 - 0.7622 = 0.2378$ >0.05	M1A1	
	(**OR** $\mathrm{P}(X \geq 9) = 0.0681,\ \mathrm{P}(X \geq 10) = 0.0318$, CV=10, 7 not in CR. probabs, 10	M1A1)	
	No evidence of an increase in the number of chicks reared per year. context **A1**		
			(4)
(c)	$\mathrm{P}(X \geq c \mid \lambda = 5) < 0.05$	M1	
	$\mathrm{P}(X \geq 9) = 0.0681,\ \mathrm{P}(X \geq 10) = 0.0318,\ c=10$ may be seen in (b)	M1	
	P(Type I Error)=0.0318	A1	
			(3)
(d)	$\lambda=8$		
	$\mathrm{P}(X \leq 9 \mid \lambda = 8) = 0.7166$	M1A1	
	(OR if *c*=9 in (d), $\mathrm{P}(X \leq 8 \mid \lambda = 8) = 0.5925$	M1A1)	
		11	(2)
6 (a)	$\mathrm{E}\left(\frac{2}{3}X_1 - \frac{1}{2}X_2 + \frac{5}{6}X_3\right) = \frac{2}{3}\mu - \frac{1}{2}\mu + \frac{5}{6}\mu = \mu$	M1A1	
	$\mathrm{E}(Y) = \mu \Rightarrow$ unbiased	B1	
			(3)
(b)	$\mathrm{E}(aX_1 + bX_2) = a\mu + b\mu = \mu$	M1	
	$a + b = 1$	A1	
	$\mathrm{Var}(aX_1 + bX_2) = a^2\sigma^2 + b^2\sigma^2$	M1A1	
	$= a^2\sigma^2 + (1-a)^2\sigma^2$	M1	
	$= (2a^2 - 2a + 1)\sigma^2$	A1	
			(6)
(c)	Min value when $(4a-2)\sigma^2 = 0$ $\frac{\mathrm{d}}{\mathrm{d}a}(\mathrm{Var}) = 0$,all correct **M1A1**		
	$\Rightarrow 4a - 2 = 0$	A1	
	$a = \frac{1}{2}, b = \frac{1}{2}.$	A1A1∫	
			(5)
		14	

Question number	Scheme		Marks
7 (a)	$s_p^2 = \dfrac{7\times 7.84 + 7\times 4}{7+7} = 5.92$		M1
	$s_p = 2.433105$	awrt 2.43	A1
	$H_0 : \mu_A = \mu_B,\ H_1 : \mu_A \neq \mu_B$	both	B1
	$t = \dfrac{26.125 - 25}{2.43\sqrt{\frac{1}{8}+\frac{1}{8}}} = 0.92474$	awrt 0.925	M1A1
	$t_{14}(2.5\%) = 2.145$	2.145	B1
	Insufficient evidence to reject H_0 that there is no difference in the means.		A1∫
			(7)
(b)	d=M1-M2		
	2,5,-2,1,3,-4,1,3		M1
	$\bar{d} = \dfrac{9}{8} = 1.125$	1.125	B1
	$s_d^2 = \dfrac{69 - 8\times 1.125^2}{7} = 8.410714$	awrt 8.41	M1A1
	$H_0 : \delta = 0,\ H_1 : \delta \neq 0$	both	B1
	$t = \dfrac{1.125}{\sqrt{\frac{8.41}{8}}} = 1.0972$	awrt 1.10	M1A1
	$t_7(2.5\%) = 2.365$	2.365	B1
	There is no significant evidence of a difference between method A and method B.		A1∫
			(9)
(c)	Paired sample as they are two measurements on the same orange		B1
			(1)
			17

Question number	Scheme	Marks
1.	$P(X > 2.85) = 0.05$	B1
	$P(X < \frac{1}{5.67}) = 0.01$	B1
	$\therefore P(\frac{1}{5.67} < X < 2.85) = 1 - 0.05 - 0.01$	M1
	$= 0.94$	A1
		(4 marks)
2.	$H_0: \sigma^2 = 4$; $H_1: \sigma^2 > 4$ both	B1
	$\upsilon = 19$, $X^2_{19}(0.05) = 30.144$ 30.144	B1
	$\frac{(n-1)S^2}{\sigma^2} = \frac{19 \times 6.25}{4} = 29.6875$ use of $\frac{(n-1)S^2}{\sigma^2}$	M1
	AWRT 29.7	A1
	Since 29.6875 < 30.144 there is insufficient evidence to reject H_0.	A1 ft
	There is insufficient evidence to suggest that the standard deviation is greater than 2.	B1 ft
		(6 marks)
3. (a)	$P(X \le c_1) \le 0.05$; $P(X \le 3 \mid \lambda = 8) = 0.0424 \Rightarrow X \le 3$	M1; A1
	$P(X \ge c_2) \le 0.05$; $P(X \ge 4 \mid \lambda = 8) = 0.0342 \Rightarrow X \ge 13$	M1; A1
	$P(X \ge 13 \mid \lambda = 8) = 0.0638$	
	$\therefore$ critical region is $\{X \le 3 \cup X \ge 13\}$	A1 ft (5)
(b) (i)	$P(4 \le X \le 12 \mid \lambda = 10) = P(X \le 12) - P(X \le 3)$	M1 M1
	$= 0.7916 - 0.0103$	
	$= 0.7813$	A1
(ii)	Power $= 1 - 0.7813 = 0.2187$	B1 ft (4)
		(9 marks)

Question number	Scheme	Marks
4.	d: 7 2 –3 1 –1 –2 10 5	M1
	$\Sigma d = 19$; $\Sigma d^2 = 193$	
	$\therefore\ \bar{d} = \frac{19}{8} = 2.375$; $S_d^2 = \frac{1}{7}\left\{193 - \frac{19^2}{8}\right\} = 21.125$	B1; M1 A1
	H_0: $\mu_D = 0$; H_1: $\mu_D > 0$ both	B1
	$t = \frac{2.375 - 0}{\sqrt{\frac{21.125}{8}}} = 1.4615\ldots$ AWRT 1.46	M1 A1
	$\upsilon = 7 \Rightarrow$ critical region: $t > 1.895$ 1.895	B1
	Since 1.4915… is not in the critical region there is insufficient evidence to reject H_0 and we conclude that there is in sufficient evidence to support the doctors' belief.	A1 ft **(9 marks)**
	Alternative:	
	Use of 2 sample t-test $\Rightarrow$ B0 B0 B0 M1 A1 M1 A1 B1 A1 i.e : 6/9 max	
	$S_p^2 = \frac{7\times 440.125 + 7\times 501.357}{8+8-2} = 470.74$	M1 A1
	$t = \frac{216.125 - 213.75}{\sqrt{470.74\left(\frac{1}{8}+\frac{1}{8}\right)}} = 0.0547$	M1 A1
	critical region: $t > 1.761$	B1
	Conclusion as above	A1 ft

Question number	Scheme	Marks
5. (a)(i)	E $(\hat{\theta}) = \theta$	B1
(ii)	E $(\hat{\theta}) = \theta$ or E $(\hat{\theta}) \to \theta$	B1
	and Var $(\hat{\theta}) \to 0$ as n $\to \infty$ where n is the sample size	B1 (3)
(b)	$\mathrm{E}(\hat{p}_1) = p, \therefore$ Bias = 0	B1
	$\mathrm{E}(\hat{p}_2) = \frac{5p}{6}, \therefore$ Bias = $\frac{1}{6}p$	B1 B1
	$\mathrm{E}(\hat{p}_3) = p, \therefore$ Bias = 0	B1 (4)
(c)	$\mathrm{Var}\ (\hat{p}_1) = \frac{1}{9n^2}\ \{npq + npq + npq\}$	M1
	$= \frac{pq}{3n}$	A1
	$\mathrm{Var}(\hat{p}_2) = \frac{1}{36n^2}\ \{npq + 9npq + npq\} = \frac{11pq}{36n}$	A1
	$\mathrm{Var}(\hat{p}_3) = \frac{1}{36n^2}\ \{4npq + 9npq + npq\} = \frac{7pq}{18n}$	A1 (4)
(d) (i)	$\hat{p}_1$; unbiased and smallest variance	B1 dep; B1
(ii)	$\hat{p}_2$; biased	B1 dep; B1 (4)
		(15 marks)

Question number	Scheme	Marks
6. (a)	$\bar{x} = 123.1$	B1
	$s = 5.87745….$	B1
	(NB: $\Sigma x = 1231$; $\Sigma x^2 = 151847$)	
(i)	95% confidence interval is given by	
	$123.1 \pm 2.262 \times \dfrac{5.87745....}{\sqrt{10}}$	M1
	2.262	B1
	i.e: (118.8958…, 127.30418…)	A1 ft
	AWRT (119, 127)	A1 A1
(ii)	95% confidence interval is given by	
	$\dfrac{9 \times 5.87745...^2}{19.023} < \sigma^2 < \dfrac{9 \times 5.87745...^2}{2.700}$ use of $\dfrac{(n-1)s^2}{\sigma^2}$	M1
	19.023	B1
	2.700	B1
	i.e; (16.34336…, 115.14806….)	A1ft
	AWRT (16.3, 115)	A1 A1 (13)
(b)	130 is just outside confidence interval	B1
	16 is just outside confidence interval	B1
	Thus supervisor should be concerned about the speed of the new typist	B1 (3)
		(16 marks)

Question number	Scheme	Marks
7. (a)	$S_A^2 = \frac{1}{10}\{3960540 - \frac{6600^2}{11}\} = 54.0$	B1
	$S_B^2 = \frac{1}{12}\{7410579 - \frac{9815^2}{13}\} = 21.1\dot{6}$	B1
	H_0: $\sigma_A^2 = \sigma_B^2$; H1: $\sigma_A^2 \neq \sigma_B^2$	B1
	CR: $F_{10,\,12} > 2.75$	
	$S_A^2 / S_B^2 = \frac{54.0}{21.1\dot{6}} = 2.55118\ldots$	M1 A1
	Since 2.55118… is not in the critical region we can assume that the variances are equal.	B1 (6)
(b)	H_0: $\mu_B = \mu_A + 150$; H_1: $\mu_B > \mu_A + 150$ both	B1
	CR: $t_{22}(0.05) > 1.717$ 1.717	B1
	$S_p^2 = \frac{10 \times 54.0 + 12 \times 21.1\dot{6}}{22} = 36.0\dot{9}0\dot{9}$	M1 A1
	$t = \frac{1755 - 6001 - 150}{\sqrt{36.0909\ldots\left(\frac{1}{11} + \frac{1}{13}\right)}} = 2.03157$	M1 A1
	AWRT 2.03	A1
	Since 2.03… is in the critical region we reject H_0 and conclude that the mean weight of cauliflowers from *B* exceeds that from *A* by at least 50g.	A1 ft (8)
(c)	Samples from normal populations	
	Equal variances Any two sensible verifications	B1 B1 (2)
	Independent samples	
		(16 marks)

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6686 Statistics S4
Mark Scheme

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Question Number	Scheme	Marks
1.	(a) $P(X > 19.023) = 0.025$ or $P(X < 19.023) = 0.975$ both	B1
	$P(X > 2.088) = 0.990$ or $P(X < 2.088) = 0.010$	
	$\therefore P(2.088 < X < 19.023) = 0.990 - 0.025$ or $0.975 - 0.010$	M1
	$= \underline{0.965}$	A1 (3)
	(b) Upper Critical value of $F_{12,5} = 4.68$	B1
	Lower Critical value of $F_{12,5} = \frac{1}{F_{5,12}}$	M1
	$= \frac{1}{3.11} = 0.3215\ldots$ AWRT 0.322	A1 (3)
2.	(a) $H_0: \sigma_1^2 = \sigma_2^2$; $H_1: \sigma_1^2 \neq \sigma_2^2$ both	B1
	$\frac{s_1^2}{s_2^2} = \frac{14^2}{8^2} = \underline{3.0625}$ or $\frac{8^2}{14^2} = 0.32653\ldots$ AWRT 3.06 or 0.327	M1 A1
	C.V $F_{12,7} = 3.57$ CV: $F_{7,12} = \frac{1}{3.57} = 0.28011$	B1
	Since 3.0625 is not in the Critical Region their is insufficient evidence to reject H_0. There is insufficient evidence of a difference in the variances of the lengths of the fence posts.	A1√ (5)
	(b) The distribution of the population of lengths of fence posts is normally distributed.	B1 (1)

Question Number	Scheme	Marks
3.	Let x represent weight of flour.	
	$\sum x = 8055 \quad \therefore \quad \bar{x} = 1006.875$ AWRT 1006.9	B1
	$\sum x^2 = 8110611 \quad \therefore \quad s^2 = \frac{1}{7}\left\{8110611 - \frac{8055^2}{8}\right\} = 33.26785\ldots$ AWRT 33.7	M1
	(Allow from Calculator) $\therefore S = 5.767828\ldots$ or AWRT 5.77	A1
	$H_0: \mu = 1010$; $H_1: \mu < 1010$ both	B1
	CV: $\lvert t \rvert = 1.895$	B1
	$t = \left\lvert \frac{1006.875 - 1010}{5.767828\ldots/\sqrt{8}} \right\rvert = \pm 1.5324$ Using $\frac{\bar{x}-\mu}{s/\sqrt{n}}$	M1
	AWRT -1.53	A1
	Since -1.53 is not in the critical region ($t < -1.895$) there is insufficient evidence to reject H_0 and (consistent) thus the mean weight of flour delivered by the machine is 1010g.	A1√ (8)

4.	(a) The data were not collected in pairs.	B1 (1)
	(b) Use data from twin lambs.	B1 (1)
	(c) Age, weight, gender — Any Two sensible factors.	B1; B1 (2)
	(d) $d = B - A$	
	$d: 2, 1.2, 1, 1.8, -1, 2.2, 2, -1.2, 1.1, 2.8$	M1
	$\sum d = 11.9$; $\sum d^2 = 30.01$	
	$\therefore \bar{d} = 1.19$; $s^2 = 1.761$ $(s = 1.327)$	A1; A1
	$H_0: \delta = 0$; $H_1: \delta \neq 0$ Allow μ_D for δ — both	B1
	$t = \dfrac{1.19 - 0}{\sqrt{\dfrac{1.761}{10}}} = 2.83574\ldots$	M1
	AWRT 2.84	A1
	$\nu = 9$; CV: $t = 2.262$	B1
	Since 2.8357… is in the critical region $(t > 2.262)$ there is evidence to reject H_0. The (mean) weight gained by the lambs is different for each diet.	A1$\sqrt{}$ (8)
	(e) Diet B – it has the higher mean	B1 (1)
	(d) Using non-paired t-test.	
	$H_0: \mu_A = \mu_B$; $H_1: \mu_A \neq \mu_B$	B1
	$t = \dfrac{\mu_A - \mu_B}{\sqrt{s_p^2\left(\frac{1}{10} + \frac{1}{10}\right)}} = -2.30$ — AWRT −2.30	B1
	CV: $\lvert t \rvert = 2.101$	B1
	Conclusion: Mean weight gained is different	B1 (4)
	NB $\mu_A = 5.6$ $\mu_B = 6.79$ $S_p^2 = 1.342722\ldots$	

5.	(a) A Type I error occurs when H_0 is rejected when in fact it is true.	B1 (1)
	(b) The size of a test is the probability of a Type I error	B1 (1)
	(c) $X \sim B(8, 0.25)$ Can be implied	B1
	Size $= P(X > 6) = 1 - P(X \leq 6 \mid n=8, p=0.25)$	M1
	$= 1 - 0.9996$	
	$= \underline{0.0004}$	A1 (3)
	(d) Power $= P(X > 6 \mid p = p, n = 8)$	
	$= P(X=7) + P(X=8)$	M1
	$= \frac{8!}{7!1!} p^7(1-p) + p^8$	A1
	$= 8p^7 - 8p^8 + p^8$	
	$* \; = \underline{8p^7 - 7p^8} \; *$	A1 (3)
	(e) Power when $p = 0.3 = 8 \times 0.3^7 - 7 \times 0.3^8$	B1 (1)
	$= \underline{0.00129}$ AWRT 0.0013	
	(f) P(Type II error) = 1 − Power(0.3)	M1
	$= \underline{0.99870...}$	A1 (2)
	(g) Increase the number of trials	B1
	Increase the critical region	B1 (2)

6.	(a) Confidence interval is given by			
	$\bar{x} \pm t_{19} \times s/\sqrt{n}$	2.539	B1	
	ie:- $207.1 \pm 2.539 \times \sqrt{\frac{3.2}{20}}$	Using $\bar{x} \pm t \times s/\sqrt{n}$	M1	
		All correct	A1	
	ie:- 207.1 ± 1.0156			
	ie:- $(206.08..., 208.1156)$	AWRT (206, 208)	A1	(4)
	(b) $s_p^2 = \frac{19 \times 3.2 + 9 \times 10.217\dot{3}}{28}$	$10.217\dot{3}$ AWRT 10.2	B1	
			M1	
	$= 5.45557...$	AWRT 5.46	A1	
	Confidence interval is given by			
	$\bar{x}_B - \bar{x}_G \pm t_{28} \times \sqrt{5.45557\left(\frac{1}{20} + \frac{1}{10}\right)}$			
		1.701	B1	
	ie:- $(207.1 - 204.62) \pm 1.701\sqrt{5.45557\left(\frac{1}{20} + \frac{1}{10}\right)}$			
	ie:- 2.48 ± 1.53875	Using $\bar{x} - \bar{y} \pm t\sqrt{s^2\left(\frac{1}{n_x} + \frac{1}{n_y}\right)}$	M1	
	ie:- $(0.94125, 4.0187)$	All correct	A1	
		AWRT 0.941; 4.02	A1;A1	(8)

7.	(a) $E(X) = np$, $E(Y) = mp$ both; can be implied	B1
	$E(p_1) = \frac{a\,E(X)}{n} + \frac{b\,E(Y)}{m} = p$; $\Rightarrow \frac{anp}{n} + \frac{bmp}{m} = p$	M1 A1
	$\Rightarrow (a+b) = 1$	A1 (4)
	(b) $E(p_2) = \frac{1}{n+m}\{E(X) + E(Y)\}$	M1
	$= \frac{1}{n+m}\{np + mp\}$	A1
	$= \frac{1}{(n+m)} \cdot p(n+m) = p \Rightarrow p_2$ is unbiased	A1 (3)
	(c) $Var(X) = np(1-p)$; $Var(Y) = mp(1-p)$ both; can be implied	B1
	$Var(p_1) = \frac{a^2 Var(X)}{n^2} + \frac{b^2 Var(Y)}{m^2}$ Use of $Var(aX) = a^2 Var(X)$	M1
	$= \frac{a^2 np(1-p)}{n^2} + \frac{b^2 mp(1-p)}{m^2}$	
	$= p(1-p)\left\{\frac{a^2}{n} + \frac{b^2}{m}\right\}$	A1 (3)
	(d) $Var(p_2) = \frac{1}{(n+m)^2}\{np(1-p) + mp(1-p)\}$	M1 A1
	$= \frac{p(1-p)}{n+m}$	A1 (3)
	(e) $Var(p_1) = 0.044p(1-p)$; $Var(p_2) = 0.033\dot{3}\,p(1-p)$	B1; B1
	Use p_2 ; $Var(p_2) < Var(p_1)$	B1; B1 (4) dep

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January 2006
6686 Statistics S4
Mark Scheme

Question Number	Scheme	Marks
1 (a)	$\Sigma x = 49.6$; $\Sigma x^2 = 362.36$	
	$s^2 = \frac{1}{6}\left(362.36 - \frac{49.6^2}{7}\right) = 1.8180952\ldots$ awrt 1.82	M1 A1 (2)
(b)	$CI = \left(\frac{6 \times 1.818..}{12.592}, \frac{6 \times 1.818..}{1.635}\right)$	M1
	12.592, 1.635	B1 B1
	$= (0.866\ldots, 6.67\ldots)$ awrt (0.866, 6.67)	A1 A1 (5)
(c)	$0.9^2 < 0.866$, interval does not support $\sigma = 0.9$ as out of range.	B1 (1)
		TOTAL 8
2 (a)(i)	Type I: H_0 rejected when true	B1
(ii)	Type II: H_0 accepted when false	B1 (2)
(b)(i)	$p = \frac{7.5}{50} = 0.15$	B1
	cr $X \leq 3$	B1 (2)
(ii)	$H_0: p = 0.15$, $H_1: p < 0.15$ both	B1
	$x = 2$ in cr $X \leq 3$ so H_0 is rejected	M1
	The new machine has reduced the mean number of faulty sacks	A1 (3)
(c)	$P(\text{Type I error}) = P(X \leq 3 \mid p = 0.15) = 0.0460$	M1 A1 (2)
(d)	$P(\text{Faulty}) = \frac{5}{50} = 0.1$	B1
	$P(\text{Type II error}) = P(X \geq 4 \mid p = 0.1) = 1 - 0.2503$	M1
	$= 0.7497$ awrt 0.750	A1 (3)
(e)	Critical region changes to $X \leq 2$. H_0 still rejected.	B1 (1)
		TOTAL 13

3(a) (i)	$E(\frac{1}{3}\bar{X}+\frac{2}{3}\bar{Y}) = \frac{1}{3}E\left(\frac{X_1+X_2+X_3}{3}\right)+\frac{2}{3}E\left(\frac{Y_1+Y_2+Y_3+Y_4}{4}\right)$	M1	
	$= \frac{1}{3}\mu + \frac{2}{3}\mu$		
	$= \mu$ therefore unbiased estimator	A1	(2)
(ii)	$E\left(\frac{5\bar{X}+4\bar{Y}}{9}\right) = \frac{1}{9}(5E(\bar{X})+4E(\bar{Y}))$	M1	
	$= \frac{1}{9}(5\mu+4\mu)$		
	$= \mu$ therefore unbiased estimator	A1	(2)
(b)	$Var(\bar{X}) = \frac{\sigma^2}{3}$, $Var(\bar{Y}) = \frac{\sigma^2}{4}$		
	$Var(\hat{\mu}_1) = \frac{1}{9}\cdot\frac{\sigma^2}{3}+\frac{4}{9}\cdot\frac{\sigma^2}{4} = \frac{4\sigma^2}{27}$	M1A1	(2)
(c)	$\frac{4}{27}\sigma^2 < \frac{37}{243}\sigma^2$ so use $\hat{\mu}_1$.	B1	(1)
		TOTAL 7	
4(a)	Size of test $= P(X>4 \mid \lambda=3)$	M1	
	$= 1 - P(X \le 4 \mid \lambda=3) = 1 - 0.8153$	A1	
	$= 0.1847$ awrt 0.185	A1	(3)
(b)	$r = 1 - 0.6288 = 0.3712 = 0.37$ (2dp)	B1	
	$s = 1 - 0.2851 = 0.7149 = 0.71$ (2dp)	B1	(2)
(c)	When $\lambda=4$, power $= 0.27 < 0.5$ Probability of coming to correct conclusion is less than probability of coming to wrong conclusion. Not suitable.	B1	(1)
		TOTAL 6	

5(a)	$d = U - C$, 2, −4, 18, 16, 0, 15, 9	M1
	$\bar{d} = \frac{56}{7} = 8$	B1
	$s_d^2 = \frac{906 - 7 \times 8^2}{6} = 76\frac{1}{3}$	M1A1
	$H_0: \mu_d = 0$, $H_1: \mu_d \neq 0$	B1
	$t = \frac{8}{\sqrt{\frac{76.\dot{3}}{7}}} = 2.42260\ldots$ awrt 2.42	M1A1
	$t_6(2.5\%) = 2.447$	B1
	Insufficient evidence to reject H_0. No evidence of a difference between the mean amount of corrosion on coated and uncoated pipes.	A1 (9)
(b)(i)	Differences are normally distributed	B1 (1)
(ii)	Values do not appear to be normally distributed	B1 (1)
(c)	$t_6(5\%) = 1.943$. There is evidence to reject H_0. There is evidence to suggest that there is a greater corrosion on coated pipes.	B1 B1 (2)
		TOTAL 13
6 (a)	$s_I^2 = \frac{1}{19}\left(274050 - \frac{(2340)^2}{20}\right) = 14.2$	M1
	$s_o^2 = \frac{1}{36}\left(645282 - \frac{(4884)^2}{37}\right) = 16.5$ *AG both	A1 (2)
(b)	$s_p = \sqrt{\frac{19 \times 14.2 + 36 \times 16.5}{55}} = \sqrt{15.705} = 3.963\ldots$	M1A1
	Mean outside $= \frac{2340}{20} = 117$, Mean inside $= 132$	B1 B1
	confidence limits $= (132 - 117) \pm 2.004 \times 3.963\ldots\sqrt{\frac{1}{20} + \frac{1}{37}}$	M1A1ft
	$= (12.8, 17.2)$	A1A1 (8)
(c)	0 lies outside confidence interval. The means are different.	B1 B1 (2)
		TOTAL 12

7(a)	$s_A^2 = 5.11$, $s_B^2 = 5.14$	B1 B1
	$H_0: \sigma_A^2 = \sigma_B^2$, $H_1: \sigma_A^2 \neq \sigma_B^2$	B1
	Critical Value $F_{6,8} = 3.58$	B1
	$\frac{s_B^2}{s_A^2} = 1.0062112...$ awrt 1.01	M1 A1
	No evidence to reject H_0. The variances are equal.	A1 (7)
(b)	$s_p^2 = \frac{8 \times 5.14 + 6 \times 5.11}{9+7-2} = 5.1247$ awrt 5.12	M1 A1
	$\mu_A = 14.11..$, $\mu_B = 11.857...$	
	$H_0: \mu_A = \mu_B$, $H_1: \mu_A > \mu_B$	B1
	Critical value $t_{14}(5\%) = 1.761$	B1
	$t = \frac{14.11.. - 11.857...}{\sqrt{5.1247..\left(\frac{1}{9}+\frac{1}{7}\right)}} = 1.9757...$ awrt 1.98	M1 A1
	There is evidence to reject H_0. Mean time taken from school A is greater than school B.	A1 (7)
(c)	Equal variances are a condition for the test in part (b)	B1 (1)
(d)	Groups not equal ability	B1 (1)
		TOTAL 16

June 2006
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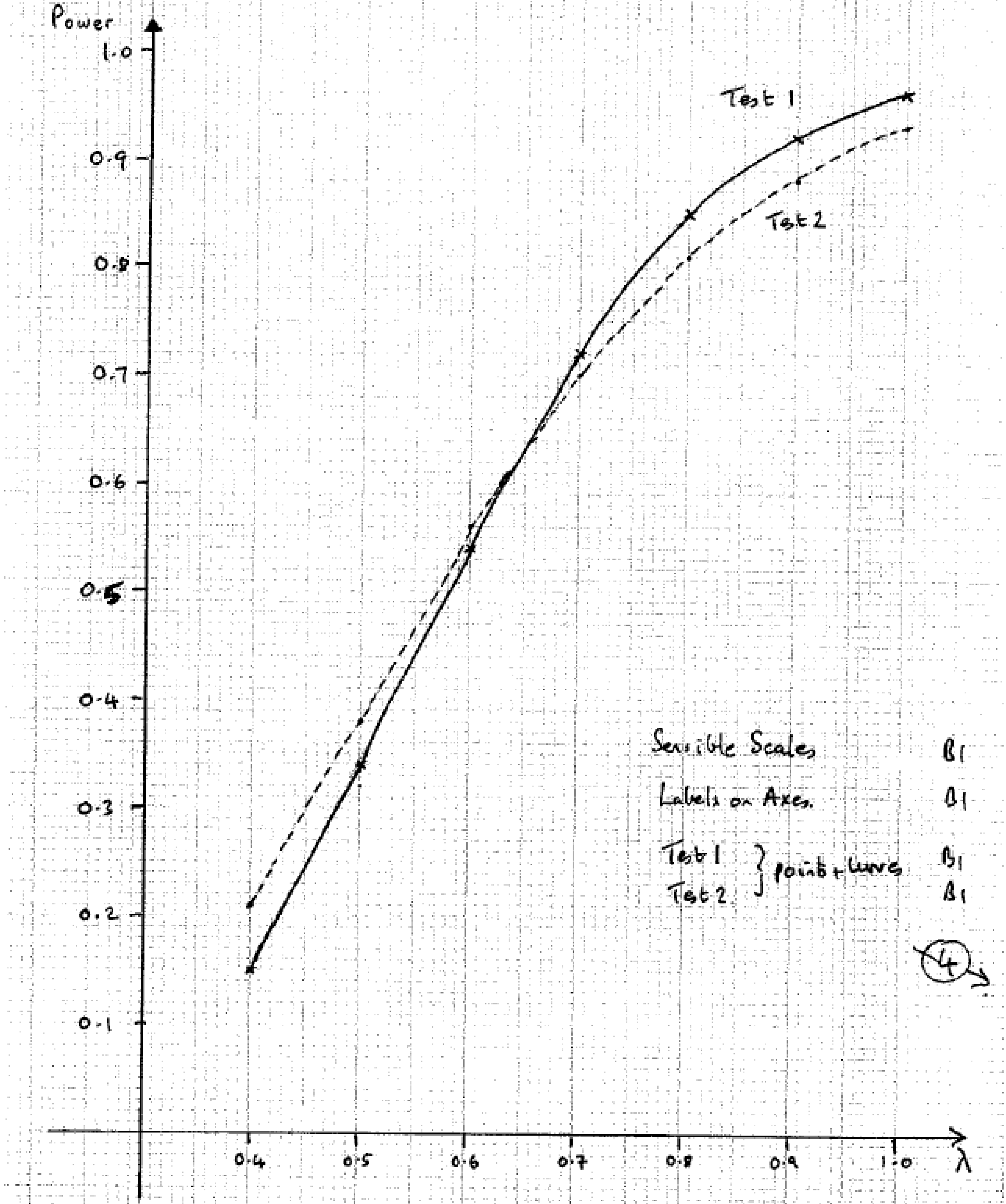
Mark Scheme

Question Number	Scheme	Marks
1.	$H_0: \mu = 1012 \quad H_1: \mu \neq 1012$ both	B1
	$\bar{x} = \frac{13700}{14}$ $(= 978.57\ldots)$	M1
	$S_x^2 = \frac{13\,448\,750 - 14\bar{x}^2}{13}$ $(= 3255.49\ldots)$ $(S_x = 57.056\ldots)$ S^2 or S	M1
	$t_{13} = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{978.6 - 1012}{57.06/\sqrt{14}} = -2.19\ldots$ AWRT -2.19	M1 A1
	$t_{13}(5\%)$ 2tail c.v. $= -2.160$ $(\pm)$	B1
	significant result – there is evidence of a change in mean weight of squirrels (condone decrease) must mention weight	A1√ (7)
2.	(a) $\left(\bar{x} = \frac{466}{4} = 116.5\right)$ $S_x^2 = \frac{54306 - 4\bar{x}^2}{3}$, $= 32.\dot{3}$ or $\frac{97}{3}$ or AWRT 32.3	M1, A1
	$0.216 < \frac{3S_x^2}{\sigma^2} < 9.348$	B1 M1 B1
	$10.376\ldots < \sigma^2 < 449.07\ldots$ AWRT 10.4, 449	A1, A1 (7)
	(b) $H_0: \sigma_M^2 = \sigma_S^2$ ($\sigma_m = \sigma_s$) $H_1: \sigma_M^2 > \sigma_S^2$ ($\sigma_m > \sigma_s$) (are OK) both	B1
	$\frac{S_M^2}{S_S^2} = \frac{318.8}{32.\dot{3}} = 9.859\ldots$ AWRT 9.86	M1 A1
	$F_{6,3}$ (1% c.v.) $= 27.91$	B1
	$9.86 < 27.91$, insufficient evidence of an increase in variance; to say $\sigma_M^2 > \sigma_S^2$ is OK; variance can be assumed to be the same is OK	A1√ (5) (12)
	(b) [NB. $\frac{32.\dot{3}}{318.8} = 0.101\ldots$ only gets M1A1 if appropriate F value attempted]	

Question Number	Scheme	Marks
3.	(D = Without Solar heating − with Solar heating) $\quad \mu_1 = \mu_2$ etc is B0 $\left[\begin{matrix}\bar{D}=0 & \bar{D}>0 \\ D=0 & D>0\end{matrix}\right]$	B0 B0 B0
(a)	$H_0: \mu_D = 0 \qquad H_1: \mu_D > 0$	B1
	d: 6, −3, 7, −2, −8, 6, 5, 11, 5 (Attempt ds)	M1
	$\bar{d} = 3$, $s_d = 6$ $\left(= \sqrt{\frac{369 - 9\times3^2}{8}}\right)$ $\left(\frac{\Sigma d}{9}\ \text{M1}\right)$ or $s_d^2 = 36$	M1, M1
	$t_8 = \frac{3-0}{6/\sqrt{9}} = 1.5$ (±)	M1 A1 cao
	t_8 (5% 1tail c.v.) = 1.860	B1
	Not significant – insufficient evidence (that solar heating has) decreased of weekly fuel consumption.	A1√ (8)
(b)	Difference in weekly fuel consumption is normally distributed.	B1 (1) (9)
4. (a)	($H_0: \sigma_A^2 = \sigma_B^2 \qquad H_1: \sigma_A^2 \neq \sigma_B^2$)	
	$\frac{S_A^2}{S_B^2} = \frac{0.721^2}{0.572^2} = 1.588...$ Awrt 1.59	M1 A1
	$F_{8,9}$ (5%) c.v. [= 10% 2tail] = 3.23	B1
	Not significant, can assume variances are equal. (accept $\sigma_A^2 = \sigma_B^2$)	B1 cao (4)
(b)	$S_p^2 = \frac{8\times0.721^2 + 9\times0.572^2}{8+9} = 0.41784...$ 0.417... or Awrt 0.418	M1 A1
	t_{17} (2.5%) c.v. = 2.110	B1
	95% CI = $\bar{x}_B - \bar{x}_A \pm 2.110 \times s_p \times \sqrt{\frac{1}{9} + \frac{1}{10}}$	M1
	$= 0.02 \pm 2.110 \times \sqrt{0.417...} \times \sqrt{\frac{1}{9} + \frac{1}{10}}$	A1√
	= (−0.6066..., 0.6466...) Awrt (−0.607, 0.647)	A1, A1 (7)
(c)	± 0.7 is outside interval	B1√
	∴ manager need not be concerned (dep) (allow √ if 0.7 inside)	B1√ (2) (13)

5 (a)	X_1 = no. of defects in 15 m. $\quad X_1 \sim Po(4.5)$ $\quad$ Use of $Po(4.5)$	M1
	Size = $P(X_1 \geq 9) = 1 - P(X \leq 8) = 1 - 0.9597 = \underline{0.0403}$ (AWRT)	A1 (2)
(b)	$r = P(X_2 \geq 9 \mid X_2 \sim Po(9))$, $= 1 - P(X_2 \leq 8) = 1 - 0.4557 = \underline{0.54}(43)$ (AWRT)	M1, A1 (2)
(c)	Y_1 = no. of defects in 10 m $\quad Y_1 \sim Po(3)$ $\quad$ Use of $Po(3)$ to find $P(Y \geq c)$	M1
	Require smallest c so that $P(Y_1 \geq c) < 0.10$. Tables $\underline{Y_1 \geq 6}$	A1 (2)
(d)	Size = $P(Y_1 \geq 6) = 1 - P(Y_1 \leq 5) = 1 - 0.9161 = \underline{0.0839}$	B1 (1)
(e)	$s = 1 - P(Y_2 \leq 5 \mid Y_2 \sim Po(8))$, $= 1 - 0.1912 = 0.8088$ (AWRT 0.81)	M1, A1 (2)
(f)	See graph	(4) (4)
(g)	(i) $0.62 \sim 0.67$ $\quad$ (ii) Test 1 is more powerful	B1 B1 (2)
(h)	Test 2 has higher P(Type I error) but cost of this is low	B1 Test 2
	Test 2 is more powerful for $\lambda < 0.7$ and $\lambda > 0.7$ is rare	B1 Reason (2)
		(17)
6. (a)	$E(X^n) = \int_0^t x^n \frac{1}{t}\,dx = \left[\frac{x^{n+1}}{t(n+1)}\right]_0^t = \left(\frac{t^{n+1}}{t(n+1)} - 0\right) = \underline{\underline{\frac{t^n}{n+1}}}$ $\quad \int x^n \frac{1}{t}$, $\int_0^t dx$	M1, M1, A1 cso (3)
(b)	$\left(E(X) = \frac{t}{2}\right)$ $\quad \underline{E(S) = kE(X)E(Y)}$, $= k \cdot \frac{t^2}{4}$	M1, A1
	$E(S) = t^2 \quad \Rightarrow \quad \underline{k = 4}$	A1 (3)
(c)	$Var(XY) = E(X^2)E(Y^2) - [E(XY)]^2$	M1
	$= \frac{t^2}{3} \times \frac{t^2}{3} - \left(\frac{t^2}{4}\right)^2 = \left\{\frac{7t^4}{144}\right\}$	M1
	$Var(S) = k^2 Var(XY) = 16 \times \frac{7t^4}{144} = \underline{\frac{7t^4}{9}}$	A1 cso (3)
(d)	$E(U) = t^2 \Rightarrow 2E(X^2)q = t^2, \Rightarrow 2\frac{t^2}{3}q = t^2, \Rightarrow \underline{q = \frac{3}{2}}$	M1, M1, A1 cso (3)
(e)	$Var(U) = q^2[Var(X^2) + Var(Y^2)] = 2q^2 Var(X^2)$	M1
	$Var(X^2) = E(X^4) - [E(X^2)]^2 = \frac{t^4}{5} - \left(\frac{t^2}{3}\right)^2 = \left(\frac{4}{45}t^4\right)$	M1
	$Var(U) = 2 \times \frac{9}{4} \times \frac{4}{45} t^4 = \underline{\underline{\frac{2}{5}t^4}}$	A1 (3)
(f)	$\frac{2}{5} < \frac{7}{9}$ $\quad \therefore$ U is better $\quad \because$ smaller variance	B1 ft (1)
(g)	Using U estimate is: $\frac{3}{2}(2^2 + 3^2) = \frac{3}{2} \times 13 = \underline{\underline{\frac{39}{2}}}$ or $\underline{\underline{19.5}}$	B1 ft (1)
		(17)

Power Functions



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Mark Scheme (Final)

Summer 2007

GCE

GCE Mathematics (6686/01)

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June 2007
6686 Statistics S4
Mark Scheme

Question Number	Scheme	Marks
1. a	d: 14 2 18 25 0 -8 4 4 12 20	M1
	$\bar{d} = \pm 9.1$ sd = $\sqrt{106.7} = 10.332..$ $(\sum d = 91, \quad \sum x^2 = 1789)$	A1 A1
	$H_0 : \mu_d = 0 \quad H_1 : \mu_d \neq 0$	B1
	$t = \pm\frac{9.1\sqrt{10}}{10.332} = \pm\ 2.785$ awrt ± 2.78 or 2.79	M1 A1
	Critical value $t_9 = \pm 1.833$	B1
	Significant. There is a difference between blood pressure measured by arm cuff and finger monitor.	A1
		(8)
b.	The difference in measurements of blood pressure is normally distributed	B1
		(1)
	Notes. (a) One tail test Loses the first B1 . CV is 1.383 in this case. Can get 7/8 (b) looking for the difference in measurements. Not just it is normally distributed.	

Question Number	Scheme	Marks
2. a)	$\text{E}(\overline{X}) = \mu$	B1
	$\text{Var}(\overline{X}) = \text{Var}\left(\frac{X_1 + X_2 + X_3 + ... + X_n}{n}\right)$	
	$= \frac{\sigma^2}{n}$	B1
		(2)
b)	$\text{E}(U) = \frac{1}{n+m}\left(n\text{E}(\overline{X}) + m\text{E}(\overline{Y})\right)$	M1
	$= \frac{1}{n+m}(n\mu + m\mu)$	A1
	$= \mu \quad \underline{\Rightarrow U \text{ is unbiased}}$ state unbiased	A1
		(3)
c)	$\text{Var}(\overline{Y}) = \frac{\sigma^2}{m}$	B1
	$\text{Var}(U) = \frac{n^2\text{Var}(\overline{X}) + m^2\text{Var}(\overline{Y})}{(n+m)^2}$	M1
	$= \frac{n^2\frac{\sigma^2}{n} + m^2\frac{\sigma^2}{m}}{(n+m)^2}$	A1
	$= \frac{n\sigma^2 + m\sigma^2}{(n+m)^2}$	
	$= \frac{\sigma^2}{n+m}$ * cso	A1
		(4)
d)	$\frac{n\overline{X} + m\overline{Y}}{n+m}$ is a better estimate since variance is smaller.	B1 B1
		(2)

Question Number	Scheme	Marks
3. a	$H_0 : \sigma^2_F = \sigma^2_M \qquad H_1 : \sigma^2_F \neq \sigma^2_M$	B1
	$s^2_F = \frac{1}{6}(17956.5 - 7 \times 50.6^2) = \frac{33.98}{6} = 5.66333....$	B1
	$s^2_M = \frac{1}{9}(28335.1 - 10 \times 53.2^2) = \frac{32.7}{9} = 3.63333....$	B1
	$\frac{s^2_F}{s^2_M} = 1.5587...$ (Reciprocal 0.6415)	M1 A1
	$F_{6,9} = 3.37$ (or 0.24)	B1
	Not in critical region. Variances of the two distributions are the same	A1
		(7)
b.	$H_0 : \mu_F = \mu_M \qquad H_1 : \mu_F < \mu_M$	B1
	Pooled estimate $s^2 = \frac{6 \times 5.66333.... + 9 \times 3.63333}{15}$	M1
	$= 4.44533$	
	$s = 2.11$	
	$t = \frac{50.6 - 53.2}{2.11\sqrt{\frac{1}{7} + \frac{1}{10}}} = \pm 2.50$	M1 A1
	C.V. $t_{15}(5\%) = \pm 1.753$	B1
	Significant. The mean length of the females forewing is less than the length of the males forewing	A1
		(6)
	Notes (a) need to have variance and the same o.e (b) need female and forewing(wing)	

Question Number	Scheme	Marks
4.a)	$H_0: \sigma^2 = 0.9$ $H_1: \sigma^2 \neq 0.9$	B1
	$\nu = 19$	
	CR (Lower tail 10.117)	B1
	Upper tail 30.144	B1
	Test statistic = $\frac{19 \times 1.5}{0.9} = 31.6666$, significant	M1 A1
	There is sufficient evidence that the variance of the length of spring is different to 0.9	A1
		(6)
b)	$H_0: \mu = 100$ $H_1: \mu > 100$	B1
	$t_{19} = 1.328$	B1
	$t = \frac{100.6 - 100}{\sqrt{\frac{1.5}{20}}} = 2.19$	M1 A1 A1
	Significant. The mean length of spring is greater than 100	B1
		(6)
	Notes (a) only need to see 30.144 need variance in conclusion (b) conclusion must be in context. Length of spring needed	

Question Number	Scheme	Marks
5.a)	Power = P $(X \leq 3 / \lambda)$	M1
	$= e^{-\lambda} + e^{-\lambda}\lambda + \frac{e^{-\lambda}\lambda^2}{2} + \frac{e^{-\lambda}\lambda^3}{6}$	A1
	$= \frac{e^{-\lambda}}{6}(6 + 6\lambda + 3\lambda^2 + \lambda^3)$	A1 **(3)**
b)	CR is $X \leq 3$	M1
	Size = $P[X \leq 3 / \lambda = 7]$ = 0.0818	A1 **(2)**
c)	P(Type II error) = 1 – power	
	$= 1 - \frac{e^{-4}}{6}(6 + 6\times 4 + 3\times 4^2 + 4^3)$	M1
	= 0.5665..	A1 **(2)**
6.a)	$\frac{\overline{X} - 250}{\frac{4}{\sqrt{15}}} > 2.3263$ or $\frac{\overline{X} - 250}{\frac{4}{\sqrt{15}}} < -2.3263$ ± 2.3262	B1 M1
	$\overline{X} > 252.40...$ or $\overline{X} < 247.6...$ awrt 252 and 248	A1 **(3)**
b)	$P(\overline{X} < 252.4 / \mu = 254)$ - $P(\overline{X} < 247.6 / \mu = 254)$ using their '252.4' and '247.6	M1
	$= P\left(Z < \frac{252.4 - 254}{\frac{4}{\sqrt{15}}}\right) - P\left(Z < \frac{247.6 - 254}{\frac{4}{\sqrt{15}}}\right)$ stand using 4/√15, 254 their '252.4' or '247.6	M1
	= P(Z < -1.5492) – P(Z < -6.20) -1.5492 and -6.20 o.e.	A1
	= (1 – 0.9394) – (1 – 1)	M1
	= 0.0606	A1 **(5)**
	Notes (a) only needs to try and find one side for M1 (b) only need to see one of the standardisation for second M1 if consider only 252.4 and get 0.0606 they get M0 M1 A0 M1 A1 ie they can get 3/5	

Question Number	Scheme		Marks
7.	$\bar{x} = 4.01$ $s = 0.7992\ldots$		B1 M1 A1
(a)	$4.01 \pm t_9\,(2.5\%)\dfrac{0.7992..}{\sqrt{10}} = 4.01 \pm 2.262\dfrac{0.7992..}{\sqrt{10}}$	2.262	B1
		their $\bar{x}$ and s and $\sqrt{10}$	M1 A1√
	$= 4.5816\ldots.$ and $3.4383\ldots$	awrt 4.58 and 3.44	A1
			(7)
(b)	$2.700 < \dfrac{9\times 0.7992..^2}{s^2} < 19.023$	2.7,19.023	B1 B1
		$9\times s^2/\sigma^2$	M1
	$\sigma^2 < 2.13,\quad \sigma^2 > 0.302$	both awrt 2.13, 0.302	A1
			(4)
(c)	$\text{P}(X > 7) = \text{P}\left(Z > \dfrac{7-\mu}{\sigma}\right)$ needs to be as high as possible		M1
	Therefore μ and σ must be as big as possible		M1
	$= \text{P}\left(Z > \dfrac{7-4.581}{\sqrt{2.13}}\right)$		A1√
	$= 1 - 0.9515$		
	$= 0.0485$		
	$= 4.85\%$	4.8 to 4.9	A1
			(4)
	Notes (a) $s^2 = 0.63877\ldots$ (c) M1 may be implied by them using their highest μ and σ.		

GCE

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Mathematics

Statistics 4 (6686)

June 2008

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Mark Scheme (Final)

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Mark Scheme

Question Number	Scheme	Marks
1 a	$E(\theta_1) = \frac{E(X_3)+E(X_4)+E(X_5)}{3}$	
	$= \frac{3\mu}{3}$	
	$= \mu$ Bias $= 0$ allow unbiased	B1
	$E(\theta_2) = \frac{E(X_{10}) - E(X_1)}{3}$	
	$= 1/3(\mu - \mu)$	
	$= 0$ Bias $= -\mu$ allow $\pm\mu$	B1,B1
	$E(\theta_3) = \frac{3E(X_1)+2E(X_2)+E(X_{10})}{6}$	
	$= \frac{3\mu+2\mu+\mu}{6}$	
	$= \mu$ Bias $= 0$ allow unbiased	B1
		(4)
b	$Var(\theta_1) = \frac{1}{9}\{(Var\ X_2) + Var(X_3) + Var(X_4)\}$	M1
	$= \frac{1}{9}\{\sigma^2 + \sigma^2 + \sigma^2\}$	
	$= \frac{1}{3}\sigma^2$	A1
	$Var(\theta_2) = \frac{2}{9}\sigma^2$	B1
	$Var(\theta_3) = \frac{1}{36}\{9\sigma^2 + 4\sigma^2 + \sigma^2\}$	M1
	$= \frac{7}{18}\sigma^2$	A1
		(5)
ci)	θ_1 is the better estimator. It has a lower var. and no bias	B1 depB1
ii)	θ_2 is the worst estimator. It is biased	B1 depB1
		(4)

Question Number	Scheme	Marks
2 a	$H_1 : \sigma_A^2 = \sigma_B^2 \quad H_0 : \sigma_A^2 \neq \sigma_B^2$	B1
	$S_A^2 = 22.5 \qquad s_B^2 = 21.6$ awrt	M1 A1A1
	$\frac{s_1^2}{s_2^2} = 1.04$	M1 A1
	$F_{(8,\,6)} = 4.15$	B1
	$1.04 < 4.15$ do not reject H_0. The variances are the same.	B1 (8)
b	Assume the samples are selected at random, (independent)	B1 (1)
c	$s^2_p = \frac{8(22.5)+6(21.62)}{14} = 22.12$ awrt 22.1	M1 A1
	$H_0 : \mu_A = \mu_B \qquad H_1 : \mu_A \neq \mu_B$	B1
	$t = \frac{40.667-39.57}{\sqrt{22.12}\sqrt{\frac{1}{9}+\frac{1}{7}}}$	M1
	$= 0.462$ 0.42 – 0.47	A1
	Critical value = $t_{14}(2.5\%) = 2.145$	B1
	$0.462 < 2.145$ No evidence to reject H_0. The means are the same	B1 (7)
d	Music has no effect on performance	B1 (1)

Question Number	Scheme	Marks
3	Differences 2.1 -0.7 2.6 -1.7 3.3 1.6 1.7 1.2 1.6 2.4	M1
	$\bar{d}$ = 1.41	M1
	$H_0 : \mu_d = 0 \quad H_1 : \mu_d > 0$	B1
	$s = \sqrt{\dfrac{40.65 - 10 \times 1.41^2}{9}}$ = 1.5191…	M1
	$t = \dfrac{1.41}{\left(\dfrac{1.519...}{\sqrt{10}}\right)}$ = 2.935… awrt 2.94 /2.93	M1 A1
	t_9 (1%) = 2.821	B1
	2.935.. > 2.821 Evidence to reject H_0. There has been an increase in the mean weight of the mice.	B1ft
		(8)

2 sample test can score
M0 M0
B1 for $H_0 : \mu_A = \mu_B \quad H_1 : \mu_A < \mu_B$

M1 $\dfrac{9 \times 24.5 + 9 \times 17.16}{18}$
M0 A0
B1 2.552
B1 ft

ie 4/8

Question Number	Scheme	Marks
4a	$\bar{x} = 668.125$ $s = 84.428$	M1 M1
	$T_7(5\%) = 1.895$	B1
	Confidence limits = 668.125 $\pm \dfrac{1.895 \times 84.428}{\sqrt{8}}$	M1
	= 611.6 and 724.7 Confidence interval = (612, 725)	A1A1
		(6)
b	Normal distribution	B1 (1)
c	£650 is within the confidence interval. No need to worry.	B1 √ B1 √ (2)

Question Number	Scheme	Marks
5 a		
	Confidence interval = $\left(\frac{15\times 0.003}{27.488}, \frac{15\times 0.003}{6.262}\right)$	M1 B1B1
	= (0.00164, 0.00719)	A1 A1 (5)
b	0.07^2 =0.0049	M1
	0.0049 is within the 95% confidence interval.	A1
	There is no evidence to reject the idea that the standard deviation of the volumes is not 0.07 or The machine is working well.	A1 (3)

Question Number	Scheme Marks	
6 a	H_0 $p = 0.35$ $H_1 : p \neq 0.35$	B1 B1 (2)
b	Let X = Number cured then $X \sim$ B(20, 0.35)	B1
	α = P(Type I error) = P($x \leq 3$) +P($x \geq 11$) given p= 0.35	M1
	= 0.0444 + 0.0532	
	= 0.0976	A1 (3)
c	β = P(Type II error) = P($4 \leq x \leq 10$)	M1
	p 0.2 0.3 0.4 0.5	
	β 0.5880 **0.8758** 0.8565 **0.5868**	A1A1 (3)
d	Power = 1 – B	M1
	0.4120 0.1435	A1 (2)
e	Not a good procedure.	B1
	Better further away from 0.35 or	B1dep
	This is not a very powerful test (power = 1 - β)	(2)

Question Number	Scheme	Marks
7 a	$H_0 : \mu = 230$ $H_1 : \mu < 230$	B1
	$\nu = 9$	
	From table critical value $= \pm 1.833$	B1√
	$\bar{x} = 228.3$ $S = 17.858$	B1 B1
	$t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$	M1
	$= \pm \dfrac{228.3 - 230}{\frac{17.858}{\sqrt{10}}} = \pm 0.301$	A1
	$\pm 0.301 > \pm 1.833$. No evidence to reject H_0. Mean is 230 N/mm^2	B1 (7)
b	Since the tensile strength is the same and the price is cheaper recommend use new supplier.	B1 (1)

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Mark Scheme (Results)

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Mark Scheme

Question Number	Scheme		Marks
Q1	H_0: $\mu = 5$; H_1: $\mu < 5$	both	B1
	CR: $t_9(0.01) > 2.821$		B1
	$\bar{x} = 4.91$		B1
	$s^2 = \frac{1}{9}\left(241.2 - \frac{49.1^2}{10}\right) = 0.0132222$	s= awrt 0.115	M1 A1
	$t = \frac{\lvert 4.91 - 5 \rvert}{\frac{\sqrt{0.013222}}{\sqrt{10}}} = \pm 2.475$	2.47 – 2.48	M1 A1
	Since 2.475 is not in the critical region there is insufficient evidence to reject H_0 and conclude that the mean diameter of the bolts is not less than (not equal to) 5mm.		A1ft **[8]**

Question Number	Scheme	Marks
Q2 (a)	The differences are normally distributed	B1 (1)
(b)	The data is collected in pairs or small sample size and variance unknown or samples not independent	B1 (1)
(c)	d: 2.5, 1.6, 1.6, –1.9, –0.6, 4.5 at least 2 correct	M1
	$(\Sigma d = 7.7, \Sigma d^2 = 35.59)$ $\bar{d} = \pm 1.2833$, sd = 2.2675. (Var = 5.141)	A1, A1
	H_0: $\mu_d = 0$, H_1: $\mu_d > 0$ (H_1: $\mu_d < 0$ if d - 2.5, -1.6, -1.6 etc) both depend on their d's	B1
	$t = \dfrac{\pm 1.2833\sqrt{6}}{2.2675} = \pm 1.386……$ formula and substitution, 1.38 – 1.39	M1, A1
	Critical value $t_5(5\%) = 2.015$ (1 tail)	B1
	Not significant. Insufficient evidence to support that the device reduces CO_2 emissions.	A1 ft (8)
(d)	The idea that the device reduces $C0_2$ emissions has been rejected when in fact it does reduce emissions. OR Concluding that the device does not reduce emissions when in fact it does (if not in context can get B1 only)	B1 B1 (2) **[12]**
	(b) Allow because the same car has been used (c) awrt ± 1.28 , 2.27	

Question Number	Scheme	Marks
3 (a)	Size is the probability of H_0 being rejected when it is in fact true. or P(reject H_0/ H_0 is true) oe	B1 (1)
(b)	The power of the test is the probability of rejecting H_0 when H_1 is true. or P(rejecting H_0/H_1 is true) / P(rejecting H_0/H_0 is false) oe	B1 (1)
(c)	$X \sim B(12,0.5)$ $P(X \le 2) = 0.0193$ $P(X \ge 10) = 0.0193$ $\therefore$ critical region is $\{X \le 2 \cup X \ge 10\}$	B1 M1 A1A1 (4)
(d)(i)	P(Type II error) $= P(3 \le X \le 9 \mid p = 0.4)$ $= P(X \le 9) - P(X \le 2)$ $= 0.9972 - 0.0834$ $= 0.9138$	M1 M1dep A1
(ii)	Power $= 1 - 0.9138$ $= 0.0862$	B1 ft (4)
(e)	Increase the sample size Increase the significance level/larger critical region	B1 B1 (2)
		[12]
Notes	(d) (i) first M1 for either correct area or follow through from their critical region 2nd M1 dependent on them having the first M1. for finding their area correctly A1 cao (ii) B1 follow through from their (i)	

Question Number	Scheme	Marks
Q4 (a)	$H_0 : \sigma_A^2 = \sigma_B^2,\ H_1 : \sigma_A^2 \neq \sigma_B^2$	B1
	critical values $F_{12,8}$ =3.28 and $\frac{1}{F_{8,12}} = 0.35$	B1
	$\frac{s_B^2}{s_A^2} = 2.40 \quad \left(\frac{s_A^2}{s_B^2} = 0.416\right)$	M1A1
	Since 2.40 (0.416) is not in the critical region we accept H_0 and conclude there is no evidence that the two variances are different.	A1ft (5)
(b)	$S_p^2 = \frac{8\times 1.02 + 12\times 2.45}{20}$	M1
	= 1.878	A1
	$(27.94 - 25.54) \pm 2.086\times\sqrt{1.878}\times\sqrt{\frac{1}{9}+\frac{1}{13}}$	B1M1 A1ft
	(1.16, 3.64)	A1 A1 (7)
(c)	To calculate the confidence interval the variances need to be equal. In part (a) the test showed they are equal.	B1 B1 (2)
		[14]

Question Number	Scheme	Marks
Q5 (a)	95% confidence interval for μ is 2.145	B1
	$560 \pm t_{14}(2.5\%)\sqrt{\frac{25.2}{15}} = 560 \pm 2.145\sqrt{\frac{25.2}{15}} = (557.2, 562.8)$	M1 A1 A1 (4)
(b)	95% confidence interval for σ^2 is	
	$5.629 < \frac{14 \times 25.2}{\sigma^2} < 26.119$	B1, M1, B1
	$\sigma^2 < 62.675$ $\sigma^2 > 13.507$	
	$13.507 < \sigma^2 < 62.675$ awrt 13.5, 62.7	A1, A1 (5)
(c)	Require $P(X > 565) = P\left(Z > \frac{565-\mu}{\sigma}\right)$ to be as large as possible OR	
	$\frac{565-\mu}{\sigma}$ to be as small as possible; both imply highest σ and μ.	M1
	$\frac{565-562.8}{\sqrt{62.675}} = 0.28$	M1A1
	$P(Z > 0.28) = 1 - 0.6103 = 0.3897$ awrt 0.39 – 0.40	M1 A1 (5)
		[14]
	(c) M1 for using their largest σ and μ M1 for using $\frac{x-\mu}{\sigma}$ M1 1 – their prob	

Question Number	Scheme	Marks
Q6 (a)	$E(\frac{2}{3}X_1 + \frac{1}{2}X_2 + \frac{5}{6}X_3) = \frac{2}{3} \times \frac{k}{2} + \frac{1}{2} \times \frac{k}{2} + \frac{5}{6} \times \frac{k}{2} = k$	M1 A1
	$E(X_1 + X_2 + X_3) = k \Rightarrow$ unbiased	B1
		(3)
(b)	$E(aX_1 + bX_2) = a\frac{k}{2} + b\frac{k}{2} = k$	M1
	$a + b = 2$	A1
	$\text{Var}(aX_1 + bX_2) = a^2\frac{k^2}{12} + b^2\frac{k^2}{12}$	M1A1
	$= a^2\frac{k^2}{12} + (2-a)^2\frac{k^2}{12}$	M1
	$= (2a^2 - 4a + 4)\frac{k^2}{12}$	
	$= (a^2 - 2a + 2)\frac{k^2}{6}$ (*) since answer given	A1 cso
(c)		(6)
	Min value when $(2a-2)\frac{k^2}{6} = 0$ $\frac{d}{da}(\text{Var}) = 0$, all correct, condone missing $\frac{k^2}{6}$	M1A1
	$\Rightarrow 2a - 2 = 0$ $a = 1, b = 1.$	A1A1
	$\frac{d^2(\text{Var})}{da^2} = \frac{2k^2}{6} > 0$ since $k^2 > 0$ therefore it is a minimum	M1
	min variance $= (1 - 2 + 2)\frac{k^2}{6}$	
	$= \frac{k^2}{6}$	B1
		(6)
	Alternative	
	$\frac{k^2}{6}(a-1)^2 - \frac{k^2}{6} + \frac{2k^2}{6}$	M1 A1
	$\frac{k^2}{6}(a-1)^2 + \frac{k^2}{6}$	M1
	Min when $\frac{k^2}{6}(a-1)^2 = 0$	A1A1
	$a = 1$ $b = 1$	B1
	min var = $k^2/6$	

Mark Scheme (Results)

Summer 2010

GCE

GCE Statistics S4 (6686/01)

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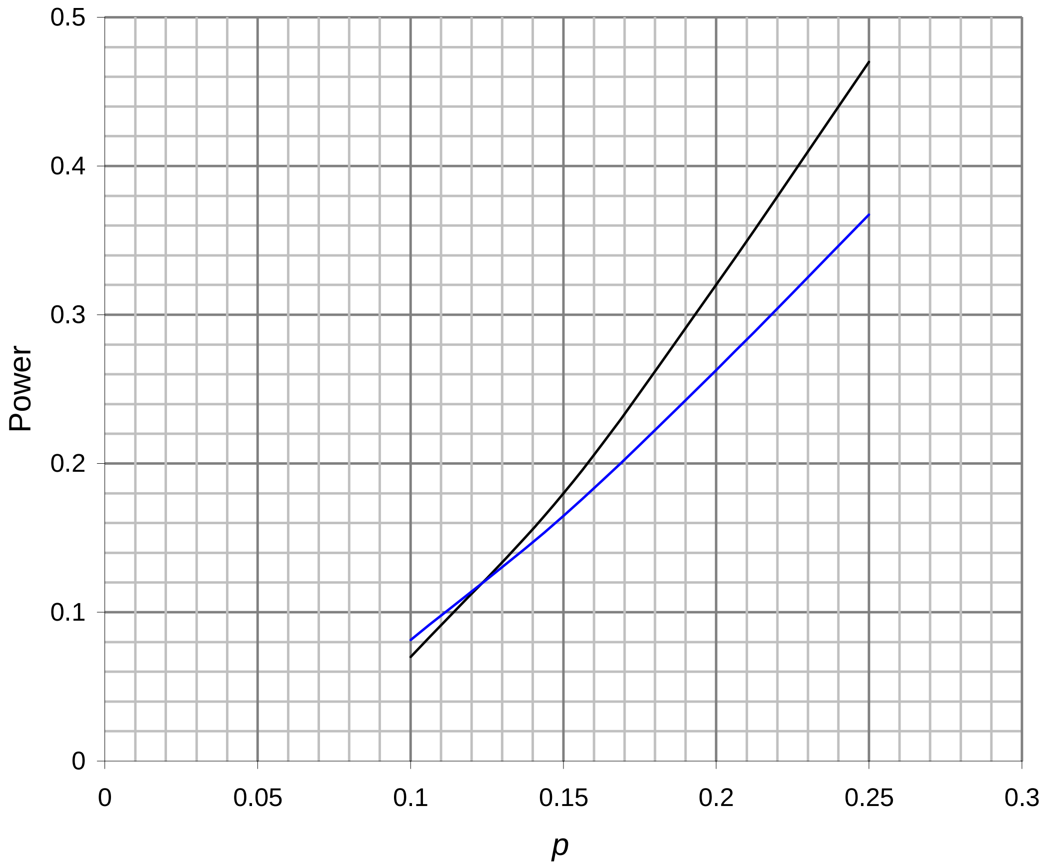
Question Number	Scheme	Marks
Q1 (a)	$H_0: \sigma_1^2 = \sigma_2^2$, $H_1: \sigma_1^2 \neq \sigma_2^2$	B1
	critical values $F_{6,7} = 3.87$ $\left(\frac{1}{F_{6,7}} = 0.258\right)$	B1
	$\frac{s_2^2}{s_1^2} = \frac{5.2^2}{4.1^2}; = 1.61$ $\left(\frac{s_1^2}{s_2^2} = \frac{4.1^2}{5.2^2} = 0.622\right)$	M1; A1
	Since 1.61 (0.622) is not in the critical region we accept H_0 and conclude there is no evidence that the two variances are different	A1ft (5)
(b)	$Sp^2 = \frac{7 \times 4.1^2 + 6 \times 5.2^2}{7+6} = 21.53\ldots$	M1A1
	t_{13}=3.012	B1
	99% CI = (17.9 – 15.9) $\pm 3.012 \times \sqrt{21.53} \times \sqrt{\frac{1}{8}+\frac{1}{7}}$	M1A1ft
	= $\pm$(9.23, -5.233), [or accept: [0,9.23] or [-9.23,0]] awrt 9.23, -5.23	A1A1 (7)
(c)	a person will be quicker at the task second time through/ times not independent/ familiar with the task/groups are not independent	B1 (1)
		[13]
	Notes B1 Allow $\sigma_1 = \sigma_2$ and $\sigma_1 \neq \sigma_2$ B1 must match their F M1 for $\frac{s_2^2}{s_1^2}$ or other way up A1 awrt 1.61(0.622) M1 A1 Sp^2 may be seen in part a B1 3.012 only M1 for (17.9 – 15.9) $\pm$ t value $\times \sqrt{S_p^2} \times \sqrt{\frac{1}{8}+\frac{1}{7}}$ A1ft their Sp^2 A1 awrt 9.23/-9.23 A1 awrt -5.23/5.23 (c) B1 any correct sensible comment	

Question Number		Scheme	Marks
Q2	(a)	The differences in the mean heart rates are normally distributed.	B1 (1)
	(b)	D = standing up – lying down	
		H_0: $\mu_D = 5$ H_1: $\mu_D > 5$	B1
		d: 9, 6, 4, 2, 8, 9, 3, 5, 7, 7	M1
		$\bar{d} = 6$; $s_d = \sqrt{\frac{414 - 10\times 36}{9}} = 2.45$	M1;M1
		$t_9 = \frac{6-5}{2.45/\sqrt{10}} = 1.29$	M1A1
		$t_9(5\%) = 1.833$	B1
		insignificant. There is no evidence to suggest that heart rate rises by more than 5 beats when standing up.	A1 ft (8)
			[9]
		Notes must have "The differences in (mean heart rate) are normally distributed) B1 both correct allow μ_D -5 >0 (μ_D = -5 H_1: μ_D < -5) M1 finding differences M1 finding $\bar{d}$ M1 $\sqrt{\frac{\sum d^2 - 10\times(\bar{d})^2}{9}}$ o.e M1 $\pm\left(\frac{6-5}{s_d/\sqrt{10}}\right)$ need to see full expression with numbers in A1 awrt ± 1.29 . B1 ± 1.833 only A1 ft their CV and t. Need context. Heart rate and 5 beats	

Question Number	Scheme	Marks
Q3 (a)	$X \sim B(5, p)$	
	Size = P(reject H_0/ p = 0.05)	
	= P($X > 1$/ p = 0.05)	
	= 1 – 0.9774	M1
	= 0.0226	A1 (2)
(b)	Power = 1 – P(0) – P(1)	M1
	$= 1 - (1-p)^5 - 5(1-p)^4 p$	M1
	$= 1 - (1-p)^4 (1 - p + 5p)$	
	$= 1 - (1-p)^4 (1 + 4p)$	A1cso
		(3)
(c)	$Y \sim B(10, p)$	
	P (Type I error) = P($Y > 2$/ p = 0.05)	M1
	= 1 – 0.9885	
	= 0.0115	A1 (2)
(d)	$s = 0.18$	B1 (1)
(e)		B1ft (1)

Question Number	Scheme	Marks
(f)	**i** intersection 0.12 – 0.13 "their graphs intersection"	B1ft
	ii if $p > 0.12$ the deputy's test is more powerful.	B1 (2)
(g)	More powerful for $p < 0.12$ and p unlikely to be above 0.12	
	Allow it would cost more/take longer/more to sample	B1 (1)
		[12]
	Notes (a) M1 for finding P (X>1) A1 awrt 0.0226 (b) M1 for 1-P(0) – P(1) M1 for $1 - (1 - p)^5 - 5(1 - p)^4 p$ A1 cso (a) M1 for finding P(Y > 2) A1 awrt0.0115 (b) B1 0.18 cao (c) B1 graph. ft their value of s (d) B1 ft their intersection. B1 deputy test more powerful o.e. (e) If give first statement they must suggest p unlikely to be above 0.12	

Question Number	Scheme	Marks
Q4 (a)	$\bar{x} = \frac{291}{15} = 19.4 \quad s = \sqrt{\frac{5968 - 15\bar{x}^2}{14}} = 4.800$	M1M1
	i $t_{14} = 2.145$	B1
	95% CI = $19.4 \pm 2.145 \times \frac{4.800}{\sqrt{15}}$	M1 A1ft
	= (16.7, 22.1)	A1A1
	ii 95% CI is given by	
	$\frac{14 \times 4.800^2}{26.119} < \sigma^2 < \frac{14 \times 4.800^2}{5.629}$	M1 B1B1
	(12.4, 57.3) accept 12.3	A1A1 (12)
(b)	Require P(X > 23) = P$\left(Z > \frac{23-\mu}{\sigma}\right)$ to be as large as possible OR $\frac{23-\mu}{\sigma}$ to be as small as possible; both imply highest σ and μ. $\frac{23-22.1}{\sqrt{57.3..}} = 0.124$	M1M1
	P(Z > 0.124) = 1 – 0.5478	M1
	= 0.4522	A1 (4)
		[16]
	Notes (a)(i) M1 $\frac{291}{15}$ M1 $\sqrt{\frac{5968-15\bar{x}^2}{14}}$ B1 2.145 M1 $(19.4) \pm t \times \frac{\text{"}\textit{their}\text{ s"}}{\sqrt{15}}$ A1ft $19.4 \pm 2.145 \times \frac{\text{"}\textit{their}\text{ s"}}{\sqrt{15}}$ A1 awrt 16.7 A1 awrt 22.1 (ii) M1 $\frac{14 \times s^2}{\chi^2}$ B1 26.119 B1 5.629 A1 awrt12.4/12.3 A1 awrt 57.3 (b) M1 use of highest mean and sigma M1 standardising using values of mean and sigma from intervals M1 finding 1 – P(z > their value) A1 awrt 0.45	

Question Number	Scheme	Marks
Q5 (a)	H_0: $\mu = 70$ [accept ≤ 70], H_1: $\mu > 70$	B1
	$t = \dfrac{71.2 - 70}{3.4/\sqrt{20}} = 1.58$	M1A1
	critical value $t_{19}(5\%) = 1.729$	B1
	not significant, insufficient evidence to confirm manufacturer's claim	A1 ft (5)
(b)	H_0: $\sigma^2 = 16$, H_1: $\sigma^2 \neq 16$	B1
	test statistic $\dfrac{(n-1)s^2}{\sigma^2}$ =, $\dfrac{219.64}{16}$ = 13.7..	M1 A1
	critical values $\chi^2_{19}(5\%)\text{ upper tail} = 32.852$ $\chi^2_{19}(5\%)\text{ lower tail} = 8.907$ not significant	B1 B1
	Insufficient evidence to suggest that the variance of the miles per gallon of the panther is different from that of the Tiger.	A1ft (6)
		[11]
	Notes (a) B1 both hypotheses using μ M1 $\dfrac{71.2 - 70}{3.4/\sqrt{20}}$ A1 awrt 1.73 A1 correct conclusion ft their *t* value and CV (b) B1 both hypotheses and 16. accept $\sigma = 4$ and $\sigma \neq 4$ M1 $\dfrac{(19)\times 3.4^2}{16}$ allow $\dfrac{(19)\times 3.4^2}{4}$ A1 awrt 13.7 B1 32.852 B1 8.907 A1 correct contextual comment NB those who use $\sigma^2 = 4$ throughout can get B0 M1 A0B1 B1 A1	

Question Number	Scheme	Marks
Q6 (a)	$X_1 \sim \text{Po}(3\lambda)$ $X_2 \sim \text{Po}(7\lambda)$ $X_3 \sim \text{Po}(10\lambda)$	M1
	$\text{E}(\hat{\lambda}) = k\,[\text{E}(X_1) + \text{E}(X_2) + \text{E}(X_3)]$ $= 20\lambda k$	M1
	$\hat{\lambda}$ unbiased therefore $20\lambda k = \lambda$	M1
	$k = \frac{1}{20}$	A1 (4)
(b)	$\text{Var}(\hat{\lambda}) = \frac{1}{20^2}\text{Var}(X_1 + X_2 + X_3)$	M1
	$= \frac{1}{20^2}(3\lambda + 7\lambda + 10\lambda)$	M1
	$= \frac{\lambda}{20}$	A1ft (3)
(c)	$Y \sim \text{Po}(4\lambda)$ $\text{E}\left(\frac{1}{4}\bar{Y}\right) = \frac{1}{4} \times 4\lambda = \lambda$ therefore unbiased	M1 A1 (2)
(d)	$\text{Var}\left(\frac{1}{4}\bar{Y}\right) = \frac{1}{16} \times \frac{4\lambda}{n}$	M1 B1
	$= \frac{\lambda}{4n}$	A1 (3)
(e)	$\frac{\lambda}{4n} < \frac{\lambda}{20}$	M1
	$n > 5$ therefore $n = 6$	A1 (2)
		[14]

Question Number	Scheme	Marks
Q6	Notes (a) M1 all 3 needed. Poisson and mean M1 adding their means M1 putting their E($\hat{\lambda}$) = λ A1 cao (b) M1 use of k^2Var (X_1+ X_2+ X_3) M1 using their means from part(a) as Variances and adding together A1 cao (c) M1 use of 4λ A1 cso plus conclusion. Accept working out bias to = 0 (d) M1 $\frac{1}{16}\times \text{Var}\bar{Y}$ B1 for $\text{Var}\bar{Y} = \frac{4\lambda}{n}$ A1 cao (e) M1 for $\text{Var}\left(\frac{1}{4}\bar{Y}\right) <$ Var ($\hat{\lambda}$) A1 n = 6	

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Mark Scheme (Results)

June 2011

GCE Statistics S4 (6686) Paper 1

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EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - B marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\surd$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper
   - □ The second mark is dependent on gaining the first mark

June 2011
Statistics S4 6686
Mark Scheme

Question Number	Scheme	Marks
1.	$\mathrm{P}\left(F_{8,10} > 3.07\right) = 0.05$	
	So need $\mathrm{P}\left(F_{10,8} > x\right) = 0.01$ so $x = 5.81$	B1
	So $a = \frac{1}{5.81} =$ **<u>0.172</u>** awrt 0.172	B1
		2
2.	$s_p^{\ 2} = \frac{6s_x^{\ 2} + 3s_y^{\ 2}}{9}$ $(=192.03\ldots)$	M1
	$1.735 < \frac{9s_p^{\ 2}}{\sigma^2} < 23.589$	B1M1B1
	So 99% confidence interval is $(73.26\ldots,\ 996.14\ldots)$ **awrt (<u>73.3, 996)</u>**	A1
		5
Notes:	1^{st} M1 for attempting $s_p^{\ 2}$ 1^{st} B1 for 1.735 (or better) 2^{nd} M1 for use of $\frac{9s_p^{\ 2}}{\sigma^2}$, follow through their $s_p^{\ 2}$ 2^{nd} B1 for 23.589 (or better) A1 for both values correct to awrt 3 sf	

Question Number	Scheme	Marks
3.	$d = B - A : 1, 2, 3, -1, 3, -1, -2, 2$	M1
	$\bar{d} = 0.875$	M1
	$s_d^{\ 2} = \frac{33 - 8 \times 0.875^2}{7} = (3.8392\ldots)$	M1
	$H_0 : \mu_d = 0 \qquad H_1 : \mu_d > 0$	B1
	$t_7 = \frac{0.875}{\frac{s_p}{\sqrt{8}}} = 1.263\ldots$ **awrt <u>1.26</u>**	M1A1
	$t_7(10\%)$ one tail critical value is **<u>1.415</u>**	B1
	Not significant. There is insufficient evidence to support the claim of manufacturer *B* or machine *B* does not produce more juice (than machine *A*)	A1 **8**
	1st M1 for attempting the *d*s 2nd M1 for attempting $\bar{d}$ 3rd M1 for attempting s_d or $s_d^{\ 2}$ 4th M1 for attempting the correct test statistic 3rd A1 contextual statement only required. Allow The juice provided by machine *A* is the same as by machine *B*	
	NB 2 sample test can score 3/8 M0 M0 M1 $\frac{7 \times 9.27 + 7 \times 16.79}{14}$ B1 for $H_0 : \mu_A = \mu_B \quad H_1 : \mu_A < \mu_B$ M0 A0 B1 1.345 A0	

Question Number	Scheme	Marks
4. **(a)**	[X = no. of incorrectly addressed letters. $X \sim B(40,0.05)$] $P(X > 3) = 1 - P(X \le 3), = 1 - 0.8619 = 0.1381$ **awrt <u>0.138</u>**	M1, A1 (2)
(b)	$P(\text{Type II Error}) = P(X \le 3 \mid p = 0.10)$ $= 0.4231$ **awrt <u>0.423</u>**	M1 A1 (2)
(c)	Power = 1 - P(Type II error) so s = **<u>0.58</u>** (0.5769)	B1 (1)
(d)	Y = no. of incorrectly addressed letters in a sample of 15. $Y \sim B(15, 0.05)$ Size = $P(Y \ge 2) + P(Y = 1) \times P(Y \ge 2)$ $= [1 - 0.8290] \times [1 + 0.8290 - 0.4633]$ $= 0.23353\ldots$ **awrt <u>0.23</u>**	M1 A1 A1 (3)
(e)	(use overlay)	B1B1 (2)
(f)	2nd/ consultants test is quicker (since it uses fewer letters) 2nd / consult test is more powerful for $p < 0.125$ (and values greater than this should be unlikely)	B1 B1 (2) **12**
Notes: **(a)** **(b)** **(c)** **(d)** **(e)** **(f)**	M1 for 1- $P(X \le 3)$ and $X \sim B(40, 0.05)$ M1 for a correct interpretation of P(Type II error) B1 must be 2dp M1 for a correct strategy 1st A1 for a correct numerical expression 1st B1 for correct points (accept $\pm$ one 2mm square) 2nd B1 for curve 1st B1 for selecting 2nd test 2nd B1 for a suitable supporting reason eg more powerful for small values of p/p around 0.05	

Question Number	Scheme	Marks
5.		
(a)	$s_x^2 = \dfrac{1559691 - 6\times\left(\dfrac{3059}{6}\right)^2}{5} = 22.1666...$	M1
	$H_0 : \sigma_x^2 = \sigma_y^2 \quad H : \sigma_x^2 \neq \sigma_y^2$	B1
	$\dfrac{s_x^2}{s_y^2} = 1.895....$	M1
	$F_{5,4} = 6.26$	B1
	$\dfrac{s_x^2}{s_y^2} = 1.895....$ **awrt <u>1.90</u>** and comment : not significant - variances of **<u>weights</u>** of the two **<u>boxes</u>** can be assumed equal.	A1 (5)
(b)	$\bar{x} = 509.833... \Rightarrow \bar{x} - \bar{y} = 5.03333$	M1
	$s_p^2 = \dfrac{5s_x^2 + 4s_y^2}{9} = 17.513....$ **awrt <u>17.5</u>**	M1A1
	5% two tail t value is $t_9 = 1.833$	B1
	90% confidence interval is $5.03... \pm 1.833 \times \sqrt{17.513...} \times \sqrt{\tfrac{1}{6}+\tfrac{1}{5}}$	M1
	(0.388…., 9.6782…) **awrt <u>(0.388, 9.68)</u>**	A1, A1 (7)
(c)	Zero is not in CI, there <u>is</u> evidence to <u>reject</u> the manufacturer's claim Or the weight of the contents of the boxes has changed.	B1ft, B1ft (2) **14**
Notes: (a)	1st M1 for use of the correct formula for s_x^2 with reasonable attempt at $\sum x^2$ and $\sum x$ 2nd M1 for use of the correct test statistic. Allow use of 3.42 instead of 3.42^2. Top must be their variance.	
(b)	1st M1 for attempting $\bar{x} - \bar{y}$ can follow through their $\bar{x}$ 2nd M1 for attempt to find pooled estimate of variance 3rd M1 for use of correct formula for CI allow any t value and ft their $\bar{x}$ and s_p	

Question Number	Scheme	Marks
6. **(a)**	$\mathrm{E}\left(Y^m\right)=\frac{n}{\beta^n}\int y^m \times y^{n-1}\,\mathrm{d}y\ =,\ \left[\frac{n}{\beta^n}\times\frac{1}{m+n}\times y^{m+n}\right]_0^{\beta}$ $=\frac{n}{\beta^n}\times\frac{1}{m+n}\times\beta^{m+n}=\frac{n}{m+n}\beta^m$ **(*)**	M1, A1 A1cso (3)
(b)	$\mathrm{E}\left(Y\right)=\frac{n}{n+1}\beta$	B1 (1)
(c)	$\mathrm{E}\left(Y^2\right)=\frac{n}{n+2}\beta^2,\quad \mathrm{Var}\left(Y\right)=\mathrm{E}\left(Y^2\right)-\left[\mathrm{E}(Y)\right]^2$ $\mathrm{Var}\left(Y\right)=\frac{n}{n+2}\beta^2-\frac{n^2}{(n+1)^2}\beta^2=\frac{n}{(n+1)^2(n+2)}\beta^2$ **(*)**	B1,M1 A1cso (3)
(d)	As $n\to\infty$ E(Y) $\to\beta$, Var(Y) $\to$ 0 So Y <u>is</u> a consistent estimator for β.	M1,A1 A1 (3)
(e)	$k=\frac{n+1}{n}$	B1 (1)
(f)	$\mathrm{Var}\left(M\right)=4\mathrm{Var}(\bar{X})\ =4\frac{\sigma^2}{n}=\frac{4}{n}\times\frac{\beta^2}{12}=\frac{\beta^2}{3n}$ $\frac{(n+1)^2}{n^2}\times\frac{n}{(n+1)^2(n+2)}\beta^2=\frac{\beta^2}{n(n+2)}<\frac{\beta^2}{3n}$ so S is better (n > 1)	B1 M1A1 (3)
(g)	Max = 9.1, s = $\frac{6}{5}\times 9.1$ = <u>**10.9(2)**</u>	M1A1 (2) **16**

Question Number	Scheme	Marks
Notes:		
(a)	M1 for attempt to integrate $y^m \mathrm{f}(m)$ 1st A1 for correct integration (limits not needed yet) 2nd A1 for use of correct limits and proceeding to printed answer. No incorrect working seen.	
(c)	M1 for use of their E(Y) and E(Y^2) in a correct formula for Var(Y)	
(d)	M1 for examining both E(Y) and Var(Y) for $n \to \infty$ 1st A1 for correct limits for both the above 2nd A1 for a correct statement following correct working	
(f)	M1 for attempting Var(S)	
(g)	M1 for correct use of S to find estimate	
7. **(a)**	$s_x{}^2 = \dfrac{214856 - 20 \times \left(\dfrac{2072}{20}\right)^2}{19} = 10.357\ldots$ **awrt** **<u>10.4</u>**	B1
	$\mathrm{H}_0 : \sigma = 2.8$ (or $\sigma^2 = \ldots$) $\mathrm{H}_1 : \sigma \neq 2.8$ (or $\sigma^2 \neq \ldots$)	B1
	$\dfrac{(n-1)s^2}{\sigma^2} \sim \chi^2_{19}$ test statistic = 25.102… **awrt** **<u>25.1</u>**	M1A1
	$\chi^2_{19}(0.025) = 32.852$, $\chi^2_{19}(0.975) = 8.907$	B1B1
	Not significant so no evidence of a change in standard deviation	A1
		(7)

Question Number	Scheme	Marks
(b) (i)	$H_0 : \mu = 102.3 \quad H_1 : \mu \neq 102.3$	B1
	$z = \dfrac{\frac{2072}{20} - 102.3}{\frac{2.8}{\sqrt{20}}} = 2.0763...$ **awrt <u>2.08</u>**	M1A1
	Critical value is z =1.96 or awrt 0.019 < 0.025	B1
	So a significant result, there is evidence of a change in mean length	A1ft
(ii)	$t = \dfrac{\frac{2072}{20} - 102.3}{\sqrt{\frac{10.357...}{20}}} = 1.8064...$ **awrt <u>1.81</u>**	M1A1
	Critical value of t_{19} = 2.093	B1
	Not significant, there is insufficient evidence of a change in mean length	A1
		(9)
(c)	(a) suggests that σ is unchanged so can use σ = 2.8 so normal test can be used	B1ft
	So using (i) conclude that there is evidence of an increase in mean length	B1ft
		(2) **18**
Notes: **(a)**	M1 for use of the correct test statistic	
(b)	1st and 2nd M1 for use of correct test statistics	
(c)	1st B1 for reason for selecting (i) or (ii) based on their conclusion from test in (a).	
	2nd B1 For a final conclusion about mean lengths based on their (a) and (b)	
	NB if both conclusions are the same it needs to be clear they have chosen (i)	

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Mark Scheme (Results)

Summer 2012

GCE Statistics S4
(6686) Paper 1

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Summer 2012
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

June 2012
6686 Statistics S4
Mark Scheme

Question Number	Scheme	Marks
1 (a)	H_0: $\mu_d = 0$, H_1: $\mu_d > 0$ (or H_1: $\mu_d < 0$)	B1
	where μ_d is the (population) mean difference :- BP sitting down – BP standing. (BP standing – BP sitting down)	
	Assume the differences are normally distributed	B1
		(2)
(b)	d: 4, -1, 6, 6, 3, -2, 9, -1, 4, 7, -11, 7	M1
	($\Sigma d = 31$, $\Sigma d^2 = 419$) $\bar{d} = \pm 2.5833$; sd = 5.55073. (or Var = 30.8106)	A1; A1
	$t = \frac{\pm 2.5833\sqrt{12}}{5.55073} = \pm 1.612\ldots$ Formula and substitution, 1.61	M1, A1
	Critical value $t_{11}(1\%) = 2.718$(1 tail)	B1
	Not significant. Insufficient evidence to support that the blood pressure of a person sitting down is more than the blood pressure of a person after standing up.	A1 ft
		(7) **Total 9 marks**

	Notes
1a	B1 both hypotheses. B1 must be differences
1b	M1 at least 2 correct or may be implied by correct Σd or Σd^2 or $\bar{d}$ or sd or var or implied by correct t value A1 correct $\bar{d}$ awrt ± 2.58- may be implied by correct t value A1 correct sd awrt 5.55 or var awrt 30.8 - may be implied by correct t value M1 $\frac{\pm \text{their } \bar{d}\sqrt{12}}{\text{their sd}}$ A1 awrt 1.61 B1 CV A1ft follow through their t value – need context of **blood pressure** and **sitting** and **standing**

Question Number	Scheme	Marks
2 (a)	$S_F^2 = \frac{1}{5}\{2308.01 - 6\times19.6^2\} = 0.61$	B1
	$S_M^2 = \frac{1}{11}\{2262.57 - 12\times13.7^2\} = 0.93545..$	B1
	$H_0: \mu_F = \mu_M + 5$; $H_1: \mu_F \neq \mu_M + 5$ both	B1
	CR: $t_{16}(0.025) > 2.120$ 2.12	B1
	$S_p^2 = \frac{5\times0.61+11\times0.93545...}{16} = 0.83375$	M1 A1
	$t = \frac{19.6-13.7-5}{\sqrt{0.83375\left(\frac{1}{6}+\frac{1}{12}\right)}} = 1.971$	M1 A1ftA1
	Since 1.971 is not in the critical region we accept H_0 and conclude that the mean shell length of female turtles does exceed the shell length of male turtles by 5cm.(or Biologists claim is correct)	A1 ft
		(10)
(b)(i)	$-1.96 < \frac{\overline{X}_F - \overline{X}_M - 5}{\sqrt{\left(\frac{0.9}{6}+\frac{0.9}{12}\right)}} < 1.96$	B1 M1
	$4.07 < \overline{X}_F - \overline{X}_M < 5.93$	A1cso
(ii)	P(Type II error) = P($4.07 < \overline{X}_F - \overline{X}_M < 5.93$ | N(6, 0.225))	M1
	$= P\left(\frac{4.07-6}{\sqrt{0.225}} < z < \frac{5.93-6}{\sqrt{0.225}}\right)$	M1
	= 0.44 awrt 0.44	A1
		(6)
		Total 16 marks
2(a)	B1 – awrt 0.61 B1 – awrt 0.935 Both may be implied by correct t value or S_p B1 allow rearrangements eg $\mu_F - \mu_M = 5$. If M and F not used then they must make clear what each letter is. B1 CV (if using one tail test allow 1.746) M1 $\frac{5\times\text{their } 0.61+11\times\text{their } 0.93545...}{16}$ A1 awrt 0.834 M1 $\pm\left(\frac{19.6-13.7-5}{\sqrt{p\left(\frac{1}{6}+\frac{1}{12}\right)}}\right)$ where p is either their 0.61 or 0.94 or their S_p^2 (awrt 0.834) (Allow 13.7 - 19.6 - 5) A1 ft their S_p^2 A1 awrt 1.97	
(b)	B1 1.96 M1 must use z value	
(c)	M1 writing or using N(6, 0.225) M1 finding correct area and standardising (must use 6 but allow use of 0.9 and (0.9/18) for var)	

Question Number	Scheme	Marks
3.	H$_0$: $\sigma_A^2 = \sigma_B^2$; H$_1$: $\sigma_A^2 \neq \sigma_B^2$	B1
	$S_A^2 / S_B^2 = \frac{225}{36} = 6.25 \quad \left(\frac{36}{225} = 0.16\right)$	M1A1
	CR: $F_{10,8} > 3.35 \quad \left(\frac{1}{F_{10.8}} = 0.299\right)$	B1
	Since 6.25 is in the critical region we can assume that the lengths of paving slabs sold by the builders merchant differ in variability.	A1ft
		(5)
		Total 5 marks
	B1 both correct. Must use σ .May use different notation to *A* and *B* M1 $\frac{225}{36}$ or $\frac{36}{225}$ allow $\frac{15}{6}$ or $\frac{6}{15}$ A1 either 6.25 or 0.16 B1 CR must match their method A1 context must include “lengths of slabs”	

Question Number	Scheme	Marks
4 (a)	$\bar{x} = 4.9$	B1
	$s = \sqrt{0.191..}$ (0.437...)	B1
	(NB: $\Sigma x = 49$; $\Sigma x^2 = 241.82$)	
(i)	95% confidence interval is given by	
	$4.9 \pm 2.262 \times \sqrt{\frac{0.191..}{10}}$	M1A1ft B1
	i.e: (4.587…, 5.212 …)	A1 A1
(ii)	95% confidence interval is given by	
	$\frac{9\times 0.437...^2}{19.023} < \sigma^2 < \frac{9\times 0.437...^2}{2.7}$ use of $\frac{(n-1)s^2}{\chi^2_{n-1}}$	M1B1B1A1
	i.e; (0.0904, 0.63704)	A1 A1
(b)		**(13)**
	5 lies inside the confidence interval	B1ft
	$0.49(0.7^2)$ lies inside the confidence interval	B1ft
	Yes it does meet the time requirement	B1 ft
		(3)
		Total 16 marks

Question Number	Scheme	Marks
(a)	B1 B1 may be implied by correct a correct answer to (i) or (ii)	
(i)	M1 - "their 4.9" ± t value × $\sqrt{\frac{\text{their } 0.191..}{10}}$	
	A1ft - "their 4.9" ± 2.262 × $\sqrt{\frac{\text{their } 0.191..}{10}}$	
	B1 2.262	
	A1 either correct to 3sf or better or both correct to 2sf or better A1 both correct to 3sf or better	
(ii)	M1 – writing and attempting to use $\frac{(n-1)s^2}{\chi^2_{n-1}}$ or may be implied by correct formula used with their 0.437	
	B1 19.023	
	B1 2.7 A1ft follow through their 0.437 and two chi squared values A1 either correct to 2sf or better A1 awrt (0.09, 0.637)	
(b)	For the second B1. If both 0.7 and 0.49 lie in interval they must state variance = 0.49 or the interval for standard deviation.	
	For the third B1 their must not be two conflicting conclusions unless they give just one overall as well.	

Question Number	Scheme	Marks
5.(a)	$H_0: \sigma^2 = 36$; $H_1: \sigma^2 > 36$	B1
	$\upsilon = 24$, X^2_{24} (0.05) = 36.415	B1
	$\frac{(n-1)S^2}{\sigma^2} = \frac{24 \times 55}{36} = 36.67$	M1 A1
	Since 36.67 > 36.415 there is sufficient evidence to reject H_0.	A1 ft
	There is evidence to suggest that the variance is greater than 36.	A1 ft
		(6)
(b)		
	$H_0 : \mu = 450$ $H_1 : \mu > 450$	B1
	$t_{24} = 1.711$	B1
	$t = \pm\frac{455-450}{\sqrt{\frac{55}{25}}} = \pm 3.37\ldots$	M1 A1
	Significant; The **<u>mean weight</u>** of chocolates is **<u>greater than 450,</u>** Or **<u>μ is more than 450</u>**	A1ft; A1ft **(6)**
(c)	The **<u>weights</u>** are normally distributed	B1 **(1)**
		Total 13 marks
	Notes	
(a)	B1 both correct. Also allow $H_0: \sigma = 6$; $H_1: \sigma > 6$ B1 36.415 M1 use of $\frac{(n-1)S^2}{\sigma^2}$ A1 awrt 36.7	
(b)	M1 $\pm\frac{455-450}{\sqrt{\frac{55}{25}}}$ A1 awrt 3.4	
	A1ft any statement – no conflicting A1ft contextual statement must include " weight of chocolate" and is "greater than 50"	

Question Number	Scheme	Marks	
6(a)(i)	$\mathrm{E}(\hat{p}_1) = \mathrm{E}\left(\frac{X}{n}\right)$		
	$= \frac{1}{n}\mathrm{E}(X)$		
	$= \frac{1}{n} \times np$	M1	
	$= p$ unbiased	A1cso	
(ii)	$\mathrm{Var}(\hat{p}_1) = \mathrm{Var}\left(\frac{X}{n}\right)$		
	$= \frac{1}{n^2}\mathrm{Var}(X)$	M1	
	$= \frac{1}{n^2} \times np(1-p)$		
	$= \frac{p(1-p)}{n}$	A1	
			(4)
b(i)	$\mathrm{E}(\hat{p}_3) = 3a\,\mathrm{E}(\hat{p}_1) + 2a\,\mathrm{E}(\hat{p}_2)$	M1	
	$= 3ap + 2ap$		
	$= 5ap$		
	$5ap = p$	M1	
	$a = \frac{1}{5}$	A1	
(ii)	$\mathrm{Var}(\hat{p}_3) = \frac{9}{25}\mathrm{Var}(\hat{p}_1) + \frac{4}{25}\mathrm{Var}(\hat{p}_2)$	M1	
	$= \frac{9p(1-p)}{25n} + \frac{4p(1-p)}{25m}$	M1d	
	$= \frac{p(1-p)}{25}\left(\frac{9}{n} + \frac{4}{m}\right)$	A1	
			(6)
(c)	$\frac{p(1-p)}{25}\left(\frac{9}{n} + \frac{4}{m}\right) < \frac{p(1-p)}{n}$	M1	
	$9m + 4n < 25m$		
	$4n < 16m$		
	$\frac{n}{m} < 4$		
	$\frac{p(1-p)}{25}\left(\frac{9}{n} + \frac{4}{m}\right) < \frac{p(1-p)}{m}$	M1	
	$9m + 4n < 25n.$		

Question Number	Scheme	Marks
	$9m<21n$ $\frac{9}{21}<\frac{n}{m}$ or $\frac{3}{7}<\frac{n}{m}$	
	$\frac{3}{7}<\frac{n}{m}<4$	A1 **(3)**
(d)	Var($\hat{p}_1$) = 0.05 $p(1-p)$ Var($\hat{p}_2$) = 0.0167 $p(1-p)$ Var($\hat{p}_3$) = 0.0207 $p(1-p)$	M1
	Or since $\frac{1}{3}$ is not in the range $\frac{9}{21}<\frac{n}{m}<4$ Var($\hat{p}_3$) is not the smallest variance. Var($\hat{p}_1$) = 0.05 $p(1-p)$ Var($\hat{p}_2$) = 0.0167 $p(1-p)$	
	Therefore $\hat{p}_2$; is the best estimator as it has the smallest variance	A1ft; A1ft **(3)** **Total 16 marks**
	Notes	
(a) (i)	M1 either $\frac{1}{n}$ E(X) or $\frac{1}{n}\times np$	
	A1 cso	
(ii)	M1 either $\frac{1}{n^2}$ Var(X) or $\frac{1}{n^2}\times np(1\text{-}p)$ A1 cso	
(b) (i)	M1 For either 3a E($\hat{p}_1$) + 2a E($\hat{p}_2$) or 3ap + 2ap M1 Putting their E($\hat{p}_3$) = p	
(ii)	M1 for $\frac{9}{25}$ Var($\hat{p}_1$) + $\frac{4}{25}$ Var($\hat{p}_2$) M1d for substituting (aii) for Var($\hat{p}_1$) and (aii) with m instead of n for Var($\hat{p}_2$) A1 cso	
(c)	M1 Putting Var($\hat{p}_3$) < their Var($\hat{p}_1$) leading to an inequality of the form $\frac{n}{m}<a$ or $\frac{n}{m}>a$ where a is a constant.	
	M1 Putting Var($\hat{p}_3$) < their Var($\hat{p}_2$) leading to an inequality of the form $\frac{n}{m}>a$ or	

	$\frac{n}{m} < a$ where a is a constant.		
(d)	1/3 is not in their range in part(c) M1 attempt to find all 3 variances or eliminating Var($\hat{p}_3$) with reason and finding the other 2 variances. A1ft correct estimator chosen. A1ft correct supporting reason from correct working for their var formulae SC if 1/3 is in their range in part(c) they may get B1 for stating $\hat{p}_3$ B1dependent on the previous B being awarded- stating smallest variance award first two marks on epen.		

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Mark Scheme (Results)

Summer 2013

GCE Statistics 4 (6686/01R)

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Summer 2013
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General Marking Guidance

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- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
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EDEXCEL GCE MATHEMATICS

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- **B** marks are unconditional accuracy marks (independent of M marks)
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These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
- ft – follow through
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- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

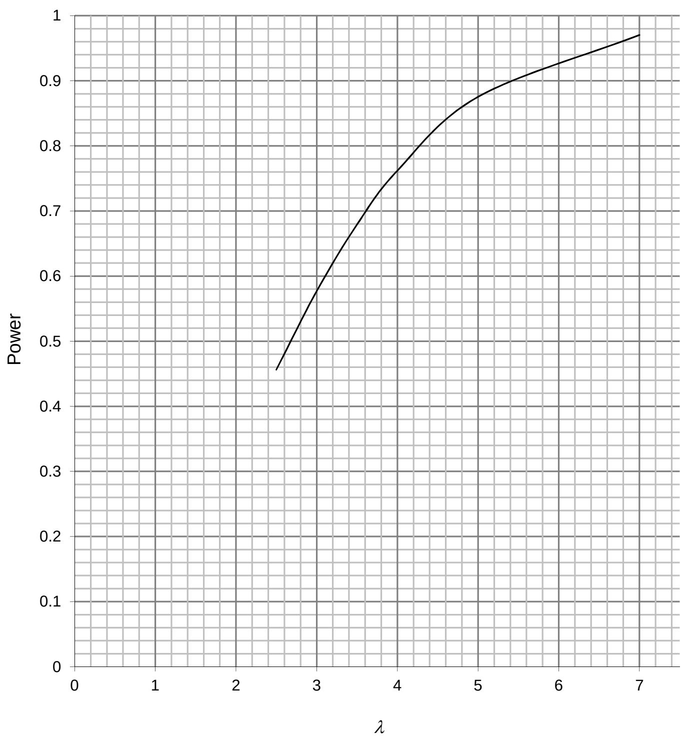
6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

Question Number	Scheme	Marks
1.(a)	P(X > 1.690) = 0.975 P($X > a$) = 0.025 a = 16.013	 M1 A1 **(2)**
(b)	Upper critical value of $F_{6,4}$ = 15.21 Lower critical value of $F_{6,4} = \frac{1}{9.15} = 0.109$	B1 B1 **(2)** **[4]**
	Notes	
(a)	M1 for using 0.025	
(b)	2nd B1 either $\frac{1}{9.15}$ or awrt 0.109	

Question Number	Scheme	Marks
2.		
(a)	$\frac{29\times 0.36}{45.722}<\sigma^2<\frac{29\times 0.36}{16.047}$	M1B1,B1
	$0.228<\sigma^2<0.651$	M1 A1
		(5)
(b)	Since 0.495 lies in the interval or 0.228 < 0.495 < 0.651 yes	B1ft B1ftd
		(2)
		[7]
	Notes	
(a)	1st M1 use of $\frac{29\times s^2}{\chi^2}$ $\left(\text{May use } \frac{s^2}{F_{29,\infty}} \text{ or } s^2\times F_{29,\infty}\right)$	
	$\left(\text{Based on } \frac{s^2}{\sigma^2}=F_{29,\infty}\right)$	
	1st B1 45.722 $\left(\text{using } \frac{s^2}{F_{29,\infty}} \text{ and } s^2\times F_{29,\infty}\right)$	
	2nd B1 16.047 (may use $F_{29,\infty}$ = 1.4686)	
	2nd M1 correct answer using their χ^2 value (correct using their $F_{29,\infty}$)	
	A1 awrt 0.228 and awrt 0.651 (awrt 0.245 and awrt 0.529)	
(b)	ft their interval	

Question Number	Scheme	Marks
3. (a)	X~Po(2)	
	Size = P($X \geq 3 / \lambda = 2$)	
	$= 1 - 0.6767$	M1
	$= 0.3233$ awrt 0.323	A1 **(2)**
(b)	Power = 1 – P(0) – P(1) – P(2)	M1
	$= 1 - \mathrm{e}^{-\lambda} - \lambda \mathrm{e}^{-\lambda} - \frac{\lambda^2 \mathrm{e}^{-\lambda}}{2!}$	A1
	$= 1 - \frac{1}{2}\mathrm{e}^{-\lambda}\left(2 + 2\lambda + \lambda^2\right)$	A1 cso **(3)**
(c)	$r = 0.58$ $s = 0.76$	B1, B1 **(2)**
(d)		B1ft points B1ft curve **(2)**
(e)	$\lambda > 3.1$ allow numbers in range 3.1-3.2	B1 **(1)**
		[10]
	Notes	
(a)	M1 for correct expression for size using Po(2)	
(b)	1st M1 for a correct expression in terms of probabilities. Allow 1- P($X \leq 2$) or 1- P($X < 3$)	
	1st A1 for correct equation in λ	
	2nd A1 cso	
(c)	SC if both correct but not to 2dp award B1B0	
(d)	1st B1ft points	
	2nd B1ft curve (or straight lines) through points	

Question Number	Scheme	Marks
4. (a)(i)	Ardo $s^2 = \frac{1}{6}(1257.78 - 7(13.4)^2)$	M1
	$= 0.143....$ awrt 0.143	A1
(ii)	Bards $0.261 = \frac{6\times 0.143... + 8\times s^2}{7+9-2}$	M1
	$s^2 = 0.349...$	A1 (4)
(b)	$H_0 : \sigma_1^2 = \sigma_2^2,\ H_1 : \sigma_1^2 \neq \sigma_2^2$	B1
	critical values $F_{8,6} = 4.15$ $\left(\frac{1}{F_{8,6}} = 0.241\right)$	B1
	$\frac{s_2^2}{s_1^2} = \frac{0.349}{0.143} =$ awrt 2.44 $\left(\frac{s_1^2}{s_2^2} = \frac{0.143}{.349} = 0.41\right)$	M1; A1
	Since 2.44… (0.424) is not in the critical region we accept H_0 and conclude there is no evidence that the two variances are different	A1cso **(5)**
(c)	H_0: $\mu_B - \mu_A = 0.9$; H_1: $\mu_B - \mu_A > 0.9$ both	B1
	CR: $t_{14}(0.05) > 1.761$ 1.761	B1
	$t = \pm\frac{14.8 - 13.4 - 0.9}{\sqrt{0.261\left(\frac{1}{7}+\frac{1}{9}\right)}} = \pm 1.94...$	M1 A1
	awrt $\pm$ 1.94	A1
	Since 1.94… is in the critical region we reject H_0 and conclude that the **mean strength** of rods from ***Bards*** **is more than 0.9 kN** than that from ***Ardo***.	A1 ft **(6)**
	Notes	**[15]**
(a)(i)	M1 for attempt to calculate s^2	
(ii)	M1 use of correct formula for $s_p{}^2$ A1 awrt 0.349 / 0.3495	
(b)	1st B1 allow $H_0 : \sigma_1 = \sigma_2,\ H_1 : \sigma_1 \neq \sigma_2$	
	M1 For use of a correct formula	
(c)	B1 must use μ. If not use A and B it must be clear which is which	
	M1 for attempt at correct test statistic – matching their hypotheses	
	1st A1 correct test statistic for their hypotheses	

Question Number	Scheme	Marks
5.		
	D = Paper I score – paper II score	
	H_0: $\mu_D = 1$ H_1: $\mu_D > 1$	B1
	d: 4, 1, 7, 3, -1, 1, 9, 2	M1
	$\bar{d} = 3.25$; $s^2 = \frac{162 - 8 \times 3.25^2}{7} = 11.07..$ (s = 3.32)	M1;M1
	$t_7 = \frac{3.25 - 1}{3.32/\sqrt{8}} = 1.9126...$ awrt 1.91	M1A1
	$t_7(5\%) = 1.895$	B1
	There is evidence to support the **teacher's** belief or the score on **paper I** is more than one mark **higher** than on **paper II**	A1 ft **(8)** **[8]**
	Notes	
(a)	1^{st} M1 for attempting differences 2^{nd} M1 for attempting $\bar{d}$ 3^{rd} M1 for attempting s_d or $s_d^{\ 2}$, correct expression with their $\sum d^2$ and $\bar{d}$ or correct calculation (to 2 sf or better) 4^{th} M1 for use of $\frac{\bar{d} - 1}{s/\sqrt{8}}$, ft their values. 1^{st} A1 awrt 1.91 2^{nd} B1 for 1.895 2^{nd} A1 contextual conclusion ft their values. SC if they use a 2 sample test they may get the first B1 for H_0: $\mu_{I} - \mu_{II} = 1$ and H_1: $\mu_{I} - \mu_{II} > 1$	

Question Number	Scheme	Marks
6.		
(a)	$H_0: \mu = 500$ [accept ≤ 500], $H_1: \mu > 500$	B1
	$t = \dfrac{502-500}{\sqrt{5.6}/\sqrt{12}} = 2.93$	M1A1
	critical value $t_{11}(1\%) = 2.718$	B1
	sufficient evidence that the mean **amount** of **water** is more than **500** ml	A1 ft **(5)**
(b)	$H_0: \sigma^2 = 9$ or ($\sigma = 3$), $H_1: \sigma^2 < 9$ or . ($\sigma < 3$)	B1
	test statistic $\dfrac{11s^2}{\sigma^2}$ =, $\dfrac{61.6}{9}$ = 6.84	M1 A1
	critical values $\chi^2_{11}(1\%)\ \text{lower tail} = 3.053$	B1
	Insufficient evidence to suggest that the **standard deviation** of the amount of **water** is less than 3	A1cso **(5)** **[10]**
	Notes	
(a)	M1 attempt at correct statistic	
	1st A1 awrt 2.93	
	2nd A1ft correct contextual comment including amount , water and 500	
(b)	1st B1 Both hypotheses, must use σ	
	2nd B1 for critical value, this should be compatible with their alternative hypothesis	
	3rd A1cso cso. contextual comment, include standard deviation/ variance and water	

Question Number	Scheme	Marks
7.		
(a)	$\dfrac{CV - 202}{2/\sqrt{n}} = -2.3263$	M1 B1
	CR $\leq 202 - \dfrac{4.6526}{\sqrt{n}}$ or $202 - 2.3263\sqrt{\dfrac{4}{n}}$	A1
		(3)
(b)	$\dfrac{CV - 200}{2/\sqrt{n}} = 1.6449$ or $\dfrac{2 - \dfrac{4.6526}{\sqrt{n}}}{\dfrac{2}{\sqrt{n}}} > 1.6449$	M1 B1
	CV = $200 + \dfrac{3.2898}{\sqrt{n}}$	
	Solving simultaneously	
	$2 = \dfrac{7.9424}{\sqrt{n}}$ or $\sqrt{n} - \dfrac{4.6526}{2} > 1.6449$	M1
	$\sqrt{n} = 3.9712$	A1
	n = 15.77	A1
	n = 16	A1
		(6)
		[9]
	Notes	
	Note only lose one B1 for not reading from points table. This should be deducted the first time it is done	
(a)	1st M1 use correct formula equal a z value	
	A1 allow use of <	
(b)	1st M1 use correct formula equal a z value	
	B1 – if B mark lost in part (a) allow 1.64 or 1.65	
	1st A1 awrt 3.97 may be implied by an answer of 15.77 **or** an answer of 16 and using 1.6449	
	2nd A1 awrt 15.8 may be implied by an answer of 16	

Question Number	Scheme	Marks
8.		
(a)	$\mathrm{E}\left(\sum_{i=1}^{n} W_i\right) = n\mu$	B1
	$\mathrm{E}\left(W_i^2\right) = \mathrm{Var}(W_i) + \left(\mathrm{E}(W_i)\right)^2$	M1
	$= \sigma^2 + \mu^2$	A1
	$\mathrm{E}\left(\sum_{i=1}^{n} W_i^2\right) = \mathrm{E}(W_1^2 + W_2^2 + \ldots.\ W_n^2)$	
	$= n\left(\sigma^2 + \mu^2\right)$	A1 cso
		(4)
(b)	$\mathrm{E}\left(\frac{1}{n}\sum_{i=1}^{n} W_i\right) = \frac{1}{n}\mathrm{E}\left(\sum_{i=1}^{n} W_i\right)$	
	$= \mu$	B1
	$\mathrm{Var}\left(\frac{1}{n}\sum_{i=1}^{n} W_i\right) = \frac{1}{n^2}\mathrm{Var}\left(W_1 + W_2 + \ldots + W_n\right)$	
	$= \frac{1}{n^2} n\sigma^2$	
	$= \frac{\sigma^2}{n}, \ \to 0 \text{ as } n \to \infty$	B1,B1d
		(3)
(c)	$\mathrm{E}\left[\frac{1}{n}\left(\sum w_i^2\right) - \left(\overline{w}\right)^2\right] = \frac{1}{n} \times n\left(\sigma^2 + \mu^2\right) - \mathrm{E}(\overline{w}^2)$	M1
	$\mathrm{Var}\ (\overline{w}) = \mathrm{E}\ (\overline{w}^2) - [\mathrm{E}(\overline{w})]^2 \ \Rightarrow\ \mathrm{E}\ (\overline{w}^2) - \mu^2 = \frac{\sigma^2}{n}$	M1
	Hence expected value is $\left(\sigma^2 + \mu^2\right) - \frac{\sigma^2}{n} - \mu^2 = \frac{(n-1)\sigma^2}{n}$	A1
	Bias = $(-)\frac{\sigma^2}{n}$	A1
		(4)
(d)	$\frac{n}{(n-1)} U$	B1
		(1)
		[12]

	Notes	
(a)	1st M1 using $\mathrm{E}\left(W_i^{\,2}\right) = \mathrm{Var}\left(W_i\right) + \left(\mathrm{E}\left(W_i\right)\right)^2$	
(b)	2nd B1 stating $\mathrm{Var}\left(\frac{1}{n}\sum_{i=1}^{n} W_i\right) = \frac{\sigma^2}{n}$	
	3rd B1 dependent on 2nd B1, stating $\frac{\sigma^2}{n} \to 0$ as $n \to \infty$	
(c)	1st M1 attempting correct method with their answer to part (a) – award for $\left(\sigma^2 + \mu^2\right) - E\left(\frac{1}{n}\sum_{i=1}^{n} w_i\right)^2$	
	2nd M1 using Var ($\overline{w}$) = E ($\overline{w}^2$) – [E($\overline{w}$)]^2	
(d)	Allow $\frac{n}{(n-1)}\sigma^2$	

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Summer 2013
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   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

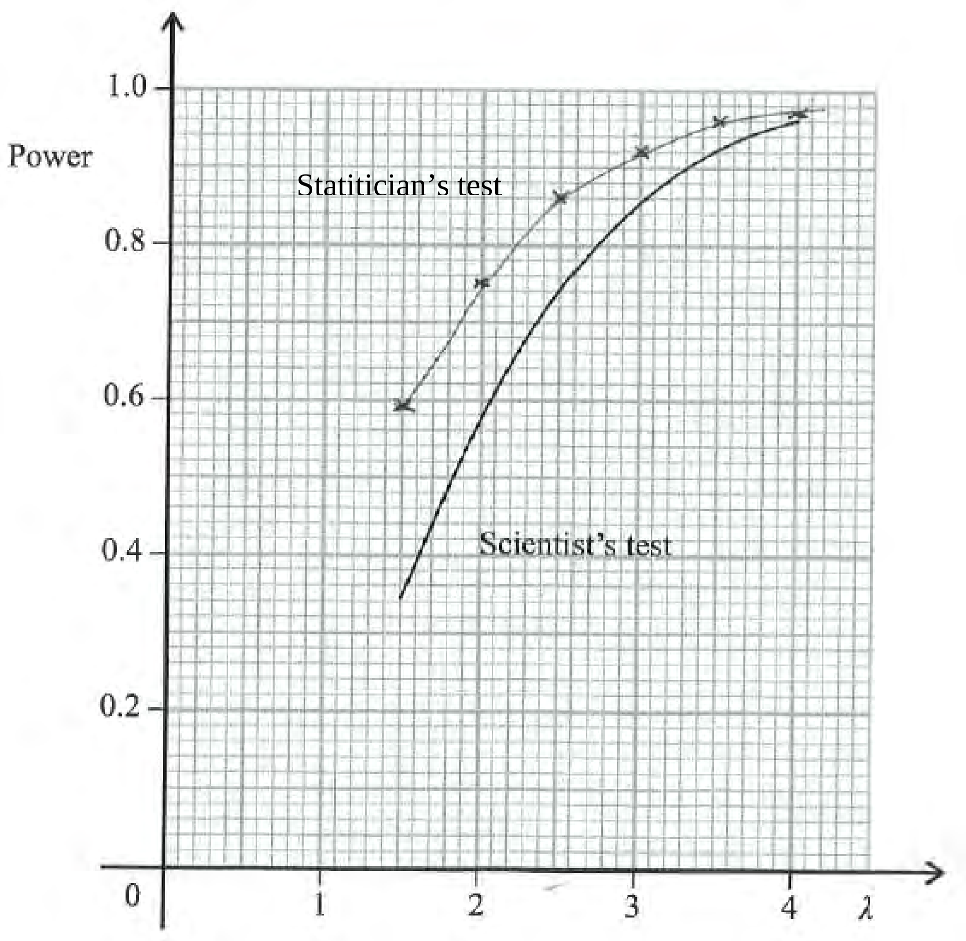
8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme

Question Number	Scheme	Marks
1.	$H_0: \sigma^2 = 2.4^2 \quad H_1: \sigma^2 \neq 2.4^2$	B1
	$s^2 = \dfrac{1414.08 - 10\times\left(\dfrac{113.4}{10}\right)^2}{9} = 14.236$	M1 A1
	$\chi^2 = \dfrac{9s^2}{\sigma^2} = \dfrac{9\times 14.236}{2.4^2} = 22.24375$	M1 A1
	Critical Value $\chi_9^2(0.025) = 19.023$	B1
	Significant result, there is evidence of a change in standard deviation _or_ the data do not support George's belief	A1cso **(7)** **[7]**
	Notes	
	1st B1 Both hypotheses, must use σ. Allow $H_0: \sigma = 2.4 \quad H_1: \sigma \neq 2.4$ 1st M1 correct method used 1st A1 awrt 14.2 2nd M1 $\chi^2 = \dfrac{9\times"their\ s^2"}{2.4^2}$ 2nd A1 awrt 22.2 2nd B1 for critical value, this should be compatible with their alternative hypothesis (16.919 for one tail test) 3rd A1ft fully correct solution only	

Question Number	Scheme	Marks
2. (a)	d = Jan - June: -2, 1, -3, 2, -2, 3, 2, 2	M1
	$\bar{d} = 0.375, \quad \sum d^2 = 39 \Rightarrow s^2 = 5.4107...$ or $s = 2.326\ldots$	M1, M1
	$t_7(0.025) = 2.365$	B1
	Confidence Interval: $0.375 \pm 2.365 \times \frac{2.326...}{\sqrt{8}}$	M1
	= **(-1.57, 2.32)** (o.e.)	A1,A1 **(7)**
(b)	$H_0 : \mu_D = 0 \quad H_1 : \mu_D \neq 0$	B1
	Comment that 0 is in the interval	M1
	Not sig, no evidence of a change in **mean time** to **assemble component**	A1ft **(3)** **[10]**
	Notes	
(a)	1st M1 for attempting differences 2nd M1 for attempting $\bar{d}$ 3rd M1 for attempting $s_d{}^2$, correct expression with their $\sum d^2$ and $\bar{d}$ or correct calculation (to 2 sf or better) 4th M1 for use of a correct CI formula, using a value for t and ft their values. 1st A1 for lower limit of -1.57 or -2.32 2nd A1 for corresponding upper limit	
S.C.	Allow A1A1 for (0, 2.32)	
(b)	B1 for both hypotheses using μ_D M1 for a comment about 0 being in (or out) of their interval A1 contextual conclusion – must include assemble components	
S.C.	If they have used difference in means test in part (a) to get the confidence interval then award the B1 for $H_0 : \mu_x - \mu_y = 0 \quad H_1 : \mu_x - \mu_y \neq 0$ or the correct hypotheses.	

Question Number	Scheme	Marks
3. (a)	$H_0: \sigma^2_A = \sigma^2_B \quad H_1: \sigma^2_A \neq \sigma^2_B$	B1
	$F = \frac{s_B^{\ 2}}{s_A^{\ 2}} = \frac{4.37^2}{4.24^2} = 1.0622...$	M1A1
	$F_{12,6}$ (0.01) = 7.72	B1
	Not sig, so no evidence of a difference in variances	A1ft **(5)**
(b)	$H_0: \mu_A = \mu_B \quad H_1: \mu_A < \mu_B$	B1
	$s_p^{\ 2} = \frac{6\times 4.24^2 + 12\times 4.37^2}{18} = 18.7238$ or $s_p = 4.327...$	M1
	$t = \pm\frac{14.31-8.43}{s_p\sqrt{\frac{1}{7}+\frac{1}{13}}} = \pm\ 2.8985\ldots$ awrt 2.9	M1A1
	$t_{18}(0.01) = 2.552$	B1
	sig, there is evidence to support **archaeologist's** claim <u>or</u> there is evidence that bricks for site *B* have higher **mean compression** strength than those from site *A*.	A1ft **(6)**
(c)	The test in (b) requires $\sigma^2_A = \sigma^2_B$ and the test in part (a) shows that this is a reasonable assumption. (o.e.)	B1 **(1)**
		[12]
	Notes	
(a)	M1 for use of a correct formula Allow $F = \frac{s_A^{\ 2}}{s_B^{\ 2}} = \frac{4.24^2}{4.37^2} = 0.941...$ with 0.1295..	
(b)	B1 if *A* and *B* not used it must be clear which is *A* and which is *B* 1st M1 for attempt to calculate s_p or s^2_p 2nd M1 for attempt correct test statistic 2nd A1 ft need **archaeologist's or compression**	
(c)	Need to refer to 'allows us to assume variances the same' and this is needed in for test. oe	

Question Number	Scheme	Marks
4. (a)	$\text{E}(X)=\mu=\frac{2a-a}{2}=\frac{a}{2}$; $\text{E}(\overline{X})=\mu=\frac{a}{2}$ so biased estimator for a	M1;A1
	Bias = $\frac{a}{2}-a=-\frac{a}{2}$	B1(accept $\pm$)
		(3)
(b)	$k=2$	B1 **(1)**
(c)	$\text{Var}(X)=\sigma^2=\frac{(2a--a)^2}{12}=\frac{9a^2}{12}=\frac{3a^2}{4}$; $\text{Var}(\overline{X})=\frac{\sigma^2}{2}$	B1;B1
	Var(Y) = $k^2\text{Var}(\overline{X})=4,\times\frac{\sigma^2}{2}=4\times\frac{3a^2}{4\times 2}=\frac{3}{2}a^2$	M1,A1 **(4)**
(d)	E(M) = $\int\frac{2x(x+a)}{9a^2}\,\text{d}x=\left[\frac{2x^3}{27a^2}+\frac{ax^2}{9a^2}\right]_{-a}^{2a},=\left(\frac{16a}{27}+\frac{4a}{9}\right)-\left(-\frac{2a}{27}+\frac{a}{9}\right)$ [$=a$]	M1A1,M1d
	So E(M) = a and therefore M is an unbiased estimator for a	A1cso **(4)**
(e)	Var(M) = $\frac{3}{2}a^2-a^2=\frac{1}{2}a^2$	B1 **(1)**
(f)	Var(M) < Var(Y) , so M is the better estimator of a	M1, A1 **(2)**
(g)	Maximum value = **5**	B1ft **(1)** **[16]**
	Notes	
(a)	M1 for use of formula or integration or symmetry to find E(X)	
(c)	1st B1 for use of formula for variance 2nd B1 for use of $\frac{\sigma^2}{n}$ formula M1 for $k^2\text{Var}(\overline{X})$ and ft their k	
(d)	1st M1 for attempt at correct integration of correct expression 1st A1 for correct integration 2nd M1d dependent on previous M, for attempting to use correct limits 2nd A1 need statement that M is therefore unbiased	
(f)	M1 for comparison of their Var(Y) and their Var(M)	
(g)	B1ft for calculation of their estimate based on their choice in (f). If they choose Y answer is 4 (or twice their k)	

Question Number	Scheme	Marks
5. (a)	Y = no. of organisms in 20 ml. Y~Po(2λ) Size = P($Y \geq 4$| Y~Po(2)) , = 1 - P($Y \leq 3$) = 1- 0.8571 = **<u>0.1429</u>**	M1, A1 **(2)**
(b)	P(Type II error) = 1 - P($Y \geq 4$| Y~Po(5)) , = P($Y \leq 3$) = **<u>0.2650</u>**	M1, A1 **(2)**
(c)	X = no. of organisms in 10 ml. X ~ Po(λ) Power = P($X \geq 2$) + P(X =1) × P($X \geq 2$) = P($X \geq 2$) [1+ P(X = 1)] = $\left[1-e^{-\lambda}(1+\lambda)\right]\times\left[1+\lambda e^{-\lambda}\right]$ = $1-e^{-\lambda}-\lambda e^{-\lambda}+\lambda e^{-\lambda}-\lambda(1+\lambda)e^{-2\lambda} = 1-e^{-\lambda}-\lambda(1+\lambda)e^{-2\lambda}$	 M1 M1A1 A1cso **(4)**
(d)	r = 0.92	B1 **(1)**
(e)	See Graph paper  	B1B1 **(2)**

Question Number	Scheme	Marks
(f)	Expected time for statistician's test: $30\times P(X=1)+15\times\left[1-P(X=1)\right]$	M1

	$= 30\lambda e^{-\lambda} + 15\left(1 - \lambda e^{-\lambda}\right) = 15\left(1 + \lambda e^{-\lambda}\right)$	A1
	slower if: $15\left(1+\lambda e^{-\lambda}\right) > 20, \quad \Rightarrow \lambda e^{-\lambda} > \frac{1}{3}$	M1,A1cso **(4)**
(g)	$\lambda e^{-\lambda}$ with $\lambda = 1$ is 0.36…, with $\lambda = 2$ is 0.27…so second(statisticians) test is slower if $\lambda = 1$ but faster for $\lambda = 2$. Second test is more powerful for all λ	B1
	Choose second test - more powerful and faster for $\lambda \geq 2$	B1 **(2)** **[17]**
	Notes	
(a)	M1 for correct expression for size using Po(2)	
(b)	M1 for correct expression using Po(5)	
(c)	1st M1 for a correct expression in terms of probabilities Alternate answer 1 – [P(X = 0) + P(X = 1) x P($X \leq 1$)] 2nd M1 for an attempt at a correct equation in λ 1st A1 for a correct expression in λ	
(e)	1st B1 points 2nd B1 curve (or straight lines)	
(f)	1st M1 for an attempt to calculate expected time Alternate method 15 + 15x P(X = 1) 1st A1 for a correct expression in terms of λ 2nd M1 for attempt at correct inequality	
(g)	1st B1 for a comment about power & timings 2nd B1 for selecting second test	

Question Number	Scheme	Marks
6. (a)	$H_0 : \sigma^2{}_A = \sigma^2{}_B \qquad H_1 : \sigma^2{}_A \neq \sigma^2{}_B$	B1
	$s_A{}^2 = (0.25)^2 = 0.0625 \qquad s_B{}^2 = (0.178885...)^2 = 0.032$	B1B1

	$F = \frac{0.0625}{0.032} = 1.953...$	M1A1
	Critical Value: $F_{3,5} = 5.41$	B1
	not sig, samples come from populations with common variance	A1cso **(7)**
(b)	$s_p^{\ 2} = \frac{3\times 0.25^2 + 5\times 0.032}{8} = 0.04343... = (0.0284...)^2$	M1A1
	Use $\frac{8s_p^{\ 2}}{\sigma^2} \sim \chi_8^{\ 2}$	M1
	$1.344 < \frac{8\times 0.0434...}{\sigma^2} < 21.955$	B1,B1
	99% confidence interval is **<u>(0.0158, 0.259)</u>**	A1 **(6)** **[13]**

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Mark Scheme (Results)

Summer 2014

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎣ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

Ignore wrong working or incorrect statements following a correct answer.

Question	Scheme	Marks
1. (a)	[New – standard =] d : 7, 4, –5 , 18, –12, 18, 11, 13.	M1
	$\bar{d} = 6.75$	M1
	$s_d{}^2 = \frac{1172 - 8\times 6.75^2}{7} = 115.3571...$ or $s_d = 10.7404...$	M1
	$H_0 : \mu_d = 0 \quad H_1 : \mu_d > 0$	B1
	$t_7 = \frac{6.75}{\frac{s_d}{\sqrt{8}}} = 1.7775...$ or $\frac{c}{s_d/\sqrt{8}} = 1.895 \therefore$ CR $c >$ awrt 7.2	M1 A1
	awrt **<u>1.78</u>**	
	$t_7(5\%)$ one tail critical value is **<u>1.895</u>** (or prob. = 0.05935…)	B1
	Not significant.	
	There is insufficient evidence that the new medicine is better or the new medicine is not recommended.	A1ft (8)
(b)	Need the <u>differences</u> between levels triggering coughing to be <u>normally</u> distributed	B1 (1)
		(9 marks)
	Notes	
(a)	1st M1 for attempting the ds 2nd M1 for attempting $\bar{d}$ 3rd M1 for attempting s_d or $s_d{}^2$ 1st B1 for both hypotheses correct in terms of μ or μ_d 4th M1 for attempting the correct test statistic $\frac{6.75}{s_d/\sqrt{8}}$ or p = awrt 0.06 or $\frac{c}{10.7/\sqrt{8}} = t$ value 1st A1 1.78 or awrt 0.06 or awrt 7.2 2nd B1 1.895 or awrt 0.06 2nd A1ft for a correct comment in context based on their test statistic and their cv.	
(b)	B1 for a comment that mentions “differences” and “normal” distribution	

Question	Scheme	Marks
2. (a)	[X = no. of defects in 4 square metres.] X ~Po(6)	
	[Size =] P(X > 10) + P(X = 9 or 10)P(X > 10)	M1
	= (1 – 0.9574) + (0.9574 – 0.8472)(1 – 0.9574)	M1A1
	= 0.04729… = awrt **0.0473**	A1 (4)
(b)	Y ~ Po(8)	B1
	Power = 1 – (P$(X \le 8)$ + [P(X = 9) + P(X = 10] × P$(X \le 10)$)	
	Or (1 - P$(X \le 10)$) + [P(X = 9) + P(X = 10] ×(1 - P$(X \le 10)$)	M1
	= (1 – 0.8159) + (0.8159 – 0.5925)(1 – 0.8159)	
	= 0.22522… = awrt **0.225**	A1 (3)
		(7 marks)
	Notes	
(a)	1st M1 for a correct expression/selection of probabilities	
	2nd M1 for use of Po(6) and at least one correct prob. seen	
	May see $\text{P}(X=9)=\frac{e^{-6}6^9}{9!}$=0.06883... or $\text{P}(X=10)=\frac{e^{-6}6^{10}}{10!}$ = 0.04130...	
	1st A1 for a fully correct expression	
	2nd A1 for awrt 0.0473	
(b)	B1 for evidence of use of Po(8)	
	M1 for an expression of the correct form with at least one correct prob.	
	A1 for awrt 0.225	

Question	Scheme	Marks
3. (a)	$H_0 : \sigma_A{}^2 = \sigma_B{}^2 \quad H_1 : \sigma_A{}^2 \neq \sigma_B{}^2$	B1
	$\left(F_{8,11} =\right)\dfrac{2.98^2}{2.33^2} = \left(1.6357...\right)$	M1
	$F_{8,11}$ 10% (two-tail) cv = **2.95** (or prob. = awrt 0.22)	B1
	Not significant so can accept the assumption that variances are equal.	A1
		(4)
(b)	$H_0 : \mu_A = \mu_B \quad H_1 : \mu_A \neq \mu_B$	B1
	$s_p{}^2 = \dfrac{8\times 2.98^2 + 11\times 2.33^2}{19}, = 6.88216...$ or $s_p = 2.62338...$	M1, A1
	$\left(t_{19} =\right)(\pm)\dfrac{7.13-6.23}{s_p\sqrt{\dfrac{1}{9}+\dfrac{1}{12}}} = (\pm)0.7780047...$ = awrt **<u>0.778</u>**	M1 A1
	t_{19} (0.05) two-tail cv = **2.093**	B1
	[Not significant] Insufficient evidence of a **<u>difference in mean</u>** milk **<u>yields</u>** between the two **<u>breeds</u>**	A1
		(7)
(c)	Test in part(b) requires the variances to be equal. The test in part (a) showed that the variances could be assumed to be equal.	B1
		(1)
		(12 marks)
	Notes	
(a)	1st B1allow σ or σ^2 M1 for use of the correct test statistic	
(b)	1st M1 for attempting s_p or $s_p{}^2$ 1st A1 for awrt 6.90 or 2.63 2nd M1 for use of a correct test statistic 2nd A1 for awrt 0.77 (accept <u>+</u>) 2nd B1 for 2.093 (allow <u>+</u> 1.729 for one-tailed H_1)	

Question	Scheme	Marks
4. (a)	$s^2 = \dfrac{42397 - 10\times\left(\dfrac{619}{10}\right)^2}{9} = 453.433...$ = awrt <u>453</u>	B1
	$H_0 : \sigma = 19.71$ (or $\sigma^2 = ...$) $H_1 : \sigma > 19.71$ (or $\sigma^2 > ...$)	B1
	$\dfrac{(n-1)s^2}{\sigma^2} \sim \chi^2_9$ test statistic = 10.5046… = awrt **<u>10.5</u>**	M1A1
	χ^2_9 (0.05) cv = **16.919**	B1
	Not significant so insufficient evidence that the **<u>scores</u>** of the **<u>students</u>** are more varied than normal. <u>Or</u> **<u>Admission tutor's</u>** claim is not supported	A1 (6)
(b)	$\chi^2_{29}(0.01)$ cv = **49.588**	B1
	Reject H_0 if $\dfrac{29S^2}{19.71^2} > 49.588$	M1
	So critical region is $S^2 > 664.281...$ = awrt **<u>664.281</u>**	A1cso (3)
(c)	P(Type II error) = P($S^2 < 664.281... \mid \sigma = 22.20$) <u>or</u> $P\left(\chi_{29}{}^2 < \dfrac{664.281...\times 29}{22.20^2}\right)$	M1 A1ft
	= P(χ^2_{29} < 39.088…) = 0.90 = awrt **<u>0.90</u>**	A1 (3)
		(12 marks)

	Notes
(a)	M1 for use of the correct test statistic
(b)	M1 for use of a correct expression (LHS) only
(c)	M1 for a correct probability expression involving S^2 or χ^2_{29} . Ft their CR, may be implied by a correct answer 1st A1ft for a correct probability expression with χ^2_{29} but ft their CR, may be implied by a correct answer

Question	Scheme	Marks
5. (a)(i)	$\bar{x} = \left(\frac{880}{15} =\right)$ $58.\dot{6}$ or awrt 58.7	B1
	$s_x^{\,2} = \left(\frac{54892 - 15 \times 58.\dot{6}^2}{14} =\right) 233.238...$	B1
	$t_{14}(0.025)$ cv = 2.145	B1
	95% CI for μ is $58.\dot{6} \pm 2.145 \times \sqrt{\frac{233.238...}{15}}$	M1
	= (50.209…, 67.124…) = awrt **<u>(50.2, 67.1)</u>**	A1, A1
(ii)	$\chi_{14}^{\,2}(0.025) = 5.629$, $\chi_{14}^{\,2}(0.975) = 26.119$	B1, B1
	95% CI for σ^2 is given by: $5.629 < \frac{14 s_x^{\,2}}{\sigma^2} < 26.119$	M1
	= (125.017… , 580.0911…)	A1
	So 95% CI for σ is = (11.1811… , 24.0850…) = awrt **<u>(11.2, 24.1)</u>**	A1
		(11)
(b)	Require P($S > d$) ≤ 0.80 i.e. $\text{P}\left(Z > \frac{d-\mu}{\sigma}\right) \leq 0.80$	
	From tables $\pm$ 0.8416	B1
	So require: $\frac{d-\mu}{\sigma} > -0.8416$	M1
	i.e. $d > \mu - 0.8416\sigma$	A1
	Worst case is when $\mu = \mu_{\max}$ and $\sigma = \sigma_{\min}$	M1
	So $d > 67.1 - 0.8416 \times 11.2$ (= 57.674…) so they should set a **<u>pass mark of 58</u>**	A1 (5)
		(16 marks)
	Notes	
(a)	1st M1 ' their $\bar{x}$ ' $\pm\ t\ value \times \frac{\text{'their } s\text{'}}{\sqrt{15}}$	
	1st A1 for awrt 50.2	
	2nd A1 for awrt 67.1	
	2nd M1 for use of their values in $\chi^2 < \frac{14s^2}{\sigma^2} < \chi^2$	
	3rd A1 for awrt 125 or 580	
	4th A1 for awrt 11.2 and 24.1	
(b)	1st M1 for forming a correct expression in d, μ, σ and their z value	
	2nd M1 for using their top value from CI for μ and lowest value for CI for σ	

Question	Scheme	Marks
6. (a)	$\mathrm{E}(X)=\int_0^a x\frac{2}{a^2}x\,\mathrm{d}x=\left[\frac{2}{a^2}\frac{x^3}{3}\right]_0^a=\underline{\frac{2a}{3}}$	B1cso
	$\mathrm{E}(X^2)=\int_0^a x^2\frac{2}{a^2}x\,\mathrm{d}x=\left[\frac{2}{a^2}\frac{x^4}{4}\right]_0^a=\frac{a^2}{2}$ so $\sigma^2=\frac{a^2}{2}-\frac{4a^2}{9}=\underline{\frac{a^2}{18}}$	M1 A1
	So $\mathrm{E}(\bar{X})=\mu=\frac{2a}{3}$ and $\mathrm{Var}(\bar{X})=\frac{\sigma^2}{n}=\frac{a^2}{18n}$	A1cso (4)
(b)	$p=\underline{\frac{3}{2}}$ and $\mathrm{Var}(S)=\frac{9}{4}\mathrm{Var}(\bar{X})=\underline{\frac{a^2}{8n}}$	B1, B1ft (2)
(c)	$\mathrm{E}(M)\to a$ as $n\to\infty$, and $\mathrm{Var}(M)\to 0$ as $n\to\infty$ So *M* is a consistent estimator of *a*	B1, B1 dB1 (3)
(d)	$q=\underline{\frac{2n+1}{2n}}$, $\mathrm{Var}(T)=\frac{\cancel{(2n+1)^2}}{4n^{\cancel{2}}}\times\frac{\cancel{n}}{(n+1)\cancel{(2n+1)^2}}a^2,=\underline{\frac{a^2}{4n(n+1)}}$	B1, M1, A1 (3)
(e)	$\frac{a^2}{4n(n+1)}<\frac{a^2}{8n}\iff 2<n+1\iff 1<n$ So $\mathrm{Var}(T)<\mathrm{Var}(S)$ So (since both are unbiased) choose *T* since it has the lower variance	M1 A1 A1cso. (3)
(f)	m = 7.8 so using t gives estimate of $\frac{11}{10}\times 7.8,=8.58$ [NB $\bar{x}=6$ and *s* gives 9]	M1, A1ft (2)
(g)	Using Var(*T*) = $\frac{a^2}{120}$; so standard error is $\frac{8.58}{\sqrt{120}}$, = awrt **<u>0.78</u>** [NB *s* gives $\frac{a}{\sqrt{40}}$ = 1.42]	M1;A1 (2) **(19 marks)**
	Notes	
(a)	1st B1 for some working to establish μ. Allow median of triangle for example. 1st M1 for correct method for σ^2	
(b)	2nd B1ft ft their value of *p*	
(c)	3rd dB1 dependent on both of first 2 Bs in (c) for concluding that *M* is consistent	
(d)	M1 for correct use of Var(*T*) = q^2 Var(*M*) for their *q*.	
(e)	M1 for attempt to compare Var(*T*) and Var(*S*) 1st A1 for clearly establishing that Var(*T*) < Var(*S*) 2nd A1 for choosing *T* and stating variance is smaller SC M0 A0 B1 for T because it has a smaller variance	
(f)	M1 for using their estimator chosen in (e)	
(g)	M1 for using their Variance formula to calculate std. error. subst in *n*=4 and their (f)	

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Mark Scheme (Results)

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Summer 2014
Publications Code UA040129

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper or ag- answer given
   - ∟ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

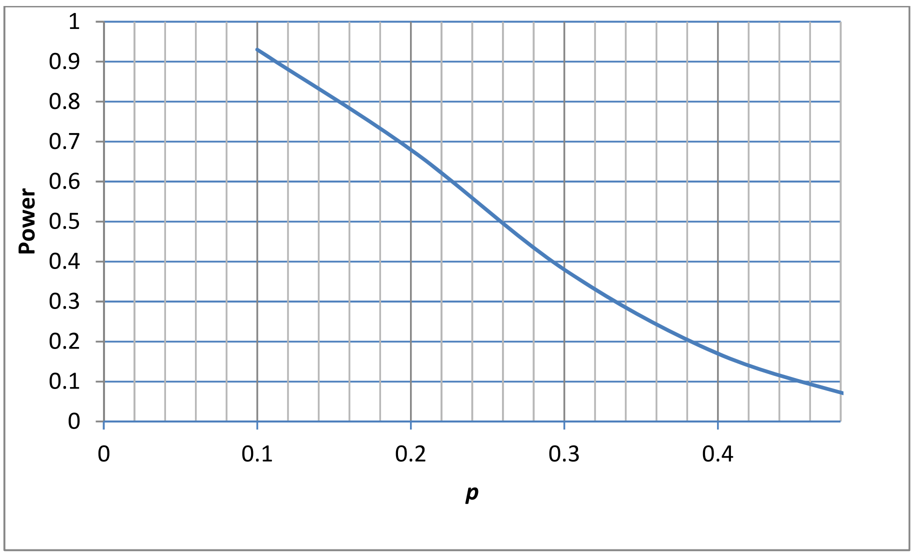
7. Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme	Marks
1.	$H_0 : \mu = 100 \quad H_1 : \mu < 100$	B1
	$t = \frac{\lvert\bar{x}-\mu\rvert}{s/\sqrt{n}} = \frac{\lvert 92.875-100\rvert}{8.3055/\sqrt{8}} = 2.4264\ldots$ or $\frac{c-100}{8.3055/\sqrt{8}} = -1.895 \therefore$ CR $c < 94.435$	M1A1
	$t_7(5\%) = \pm\ 1.895$	B1
	There is evidence to reject H_0. Malcolm's belief is supported or there is evidence that the amount of oil placed in bottles is less than 100mm	A1ft
		(5)
	Notes	
	B1 both hypotheses	
	M1 either $\frac{\lvert 92.875-100\rvert}{8.3055/\sqrt{8}}$ or $p = 0.0228$ or $\frac{c-100}{8.3055/\sqrt{8}} = -(\text{a } t \text{ value})$	
	A1 awrt 2.43 or awrt 94.4 or awrt 0.0228	
	B1 ± 1.895 or $0.0228 < 0.05$ (must have correct comparison for hypotheses) A1ft Do Not allow contradictions	

Question Number	Scheme	Marks
2(a)(i)	Type I - H_0 rejected when it is true	B1
(ii)	Type II - H_0 is accepted when it is false	B1
		(2)
(b)	$P(X< c \mid \lambda = 6) \approx 0.05$	M1
	$P(X \leq 2) = 0.0620$	
	$P(X \leq 1) = 0.0174$	
	Critical region = $X \leq 2$	A1
	P(Type 1 error) = $P(X \leq 2 \mid \lambda = 6) = 0.062$	A1cao
		(3)
(c)	P(Type 2 error) = $P(X \geq 3 \mid \lambda = 4)$	M1
	$= 1 – 0.2381$	
	$= 0.7619$	A1
		(2)
	Notes	
(b)	M1 use of Po(6) A1 correct CR. May be implied by correct probability. Alow if written as part of a 2 tailed CR A1 awrt 0.062	
(c)	M1 using Po(4) and 1 - $P(X \leq 2)$,ft their CR in (b) if one tail A1awrt 0.762	

Question Number	Scheme	Marks
3(a) (i)	$\bar{x} = \frac{181}{9} = 20.111 ...$	B1
	$s_x^2 = \left(\frac{3913 - 9\times\bar{x}^2}{8} = \right) 34.1111 \quad (s_x = 5.84)$	B1
	$t_8(0.025)$ cv = 2.306	B1
	95% CI for μ is $= 20.111 \pm 2.306 \times \frac{5.84}{\sqrt{9}}$	M1
	= (15.6, 24.6) awrt **<u>(15.6, 24.6)</u>**	A1, A1
(ii)	$\chi_8^2(0.025) = 2.18(0)$, $\chi_8^2(0.975) = 17.535$	B1B1
	95% CI for σ^2 is given by $2.180 < \frac{8s_x^2}{\sigma^2} < 17.535$	M1
	So 95% CI for σ^2 is = **<u>awrt (15.6, 125)</u>**	A1
		(10)
(b)	Require P(X < 16) = P$\left(Z < \frac{16-\mu}{\sigma}\right)$ to be as small as possible OR	
	$\frac{16-\mu}{\sigma}$ to be as large as possible but negative; **imply lowest σ and largest μ.**	M1
	P($Z < \frac{16-24.6}{\sqrt{15.6}}$); = 1 – 0.9854 = **<u>0.0146 or 0.0147</u>**	M1A1ft;A1
		(4)
	Notes	
(a)(i)	1st M1 ' their $\bar{x}$ ' $\pm$ t $value \times \frac{\text{'their } s\text{'}}{\sqrt{9}}$ 1st A1 awrt 15.6 2nd A1 awrt 24.6	
(ii)	2nd M1 $\chi^2 < \frac{8s^2}{\sigma^2} < \chi^2$ A1 awrt 15.6 and 125	
(b)	M1 Identify must use **lowest σ and largest μ** M1 standardising and finding correct area use either limit for μ and σ A1 ft their **lowest σ and largest μ** A1 awrt 0.0146 or 0.0147	

Question Number	Scheme	Marks
4(a)	The <u>differences</u> in the mean number of hours sleep are <u>normally</u> distributed	B1
		(1)
(b)	Differences are 0.8, 0.7, -0.3, 1.2, 0.7, 2.9, 1.3, 0.8	M1
	$\bar{d} = = \frac{8.1}{8} = 1.0125$	M1
	$s_d = \sqrt{\frac{13.89-8\times1.0125^2}{7}} = 0.901\ldots$ both $\bar{d}$ and s	M1
	$H_0: \mu_D = 1/6 \quad H_1: \mu_D > 1/6$	B1
	$t = \frac{1.0125-1/6}{0.901/\sqrt{8}}$ = awrt 2.65 or $\frac{c-1/6}{0.901/\sqrt{8}} = 2.988 \therefore$ CR $c >$ awrt 1.12	M1A1
	$t_7(1\%) = 2.998$ (or prob. = awrt 0.0164)	B1
	There is insufficient evidence to suggest the <u>drug</u> <u>increases</u> the mean number of hours <u>slept</u> by <u>more</u> than <u>10</u> minutes.	A1ft
		(8)
	Notes	
(a)	B1 for a comment that mentions "differences" and "normal" distribution	
(b)	1st M1 for attempting the *d*s 2nd M1 for attempting $\bar{d}$ 1st M1 for s_d or $s_d^{\,2}$ 1st B1 for both hypotheses correct in terms of μ or μ_d .(allow a defined symbol) Do not allow 10 instead of 1/6(awrt 0.167) unless working in minutes throughout 3rd M1 for attempting the correct test statistic $\frac{\bar{d}-1/6}{s_d/\sqrt{8}}$ or p = awrt 0.016 or $\frac{c-1/6}{0.901/\sqrt{8}} = t$ value 2nd A1 awrt 2.65 /2.655 or awrt 1.12 or awrt 0.016 2nd B1 2.988 or 0.0164 3rd A1ft for a correct comment in context based on their test statistic and their cv. Do not allow contradictions.	

Question Number	Scheme	Marks
5 (a)	$X \sim B(10, 0.5)$ Size = P(reject $H_0 \mid p = 0.5$) $= P(X < 3 \mid p = 0.5)$ = 0.0547	 B1 (1)
(b)	Power = $P(X = 2) + P(X = 1) + P(X = 0)$ $= 45p^2(1 - p)^8 + 10p(1 - p)^9 + (1 - p)^{10}$ $= (1 - p)^8(45p^2 + 10p(1 - p) + (1 - p)^2)$ $= (1 - p)^8(36p^2 + 8p + 1)$	M1 A1 A1cso (3)
(c)	$r = 0.68$ $s = 0.17$	B1 B1 (2)
(d)		B1 points B1 curve (2)
(e)	P(Type II error) $\le$ 0.4 1 – power ≤ 0.4 Power ≥ 0.6 $p <$ 0.23	M1 A1 A1 (3)
	Notes	
(b) (c) (e)	M1 for a correct expression/selection of probabilities A1 for a fully correct expression SC B1 B0 both correct but not given to 2 dp M1 may be implied by Power ≥ 0.6 or correct value or by correct answer A1 may be implied by correct answer A1 allow number between 0.22 and 0.23 inclusive and either $<$ or $\le$	

Question Number	Scheme	Marks
6(a)	It is the probability distribution of T.	B1 (1)
(b)	An estimator is biased if $\mathrm{E}(T) \neq \theta$	B1 (1)
(c)	$\mathrm{E}(\hat{\mu}_1) = \frac{\mathrm{E}(X_3)+\mathrm{E}(X_5)+\mathrm{E}(X_7)}{3} = \frac{\mu+\mu+\mu}{3} = \mu \quad \therefore \quad \text{Bias} = 0$	M1A1
	$\mathrm{E}(\hat{\mu}_2) = \frac{5E(X_1)+2E(X_2)+E(X_9)}{6} = \frac{5\mu+2\mu+\mu}{6} = \frac{4\mu}{3} \therefore \text{ Bias} = \frac{\mu}{3}$	A1
	$\mathrm{E}(\hat{\mu}_3) = \frac{3\mathrm{E}(X_{10}) - \mathrm{E}(X_1)}{3} = \frac{3\mu-\mu}{3} = \frac{2\mu}{3} \quad \therefore \quad \text{Bias} = -\frac{\mu}{3}$	A1 (4)
(d)	$\mathrm{Var}(\hat{\mu}_1) = \frac{1}{9}(\mathrm{Var}(X_3) + \mathrm{Var}(X_5) + \mathrm{Var}(X_7))$	M1
	$= \frac{1}{9}(\sigma^2 + \sigma^2 + \sigma^2)$	
	$= \frac{\sigma^2}{3}$	A1
	$\mathrm{Var}(\hat{\mu}_2) = \frac{1}{36}(25\mathrm{Var}(X_1) + 4\mathrm{Var}(X_2) + \mathrm{Var}(X_9))$	M1
	$= \frac{1}{36}(25\sigma^2 + 4\sigma^2 + \sigma^2)$	
	$= \frac{5}{6}\sigma^2$	A1
	$\mathrm{Var}(\hat{\mu}_3) = \frac{1}{9}(9\mathrm{Var}(X_{10}) + \mathrm{Var}(X_1))$	M1
	$= \frac{1}{9}(9\sigma^2 + \sigma^2)$	
	$= \frac{10\sigma^2}{9}$	A1 (6)
(e)(i)	$\hat{\mu}_1$ is the best estimator. It has no bias	B1
(ii)	It has <u>same magnitude of bias as $\hat{\mu}_2$</u> but it has the <u>largest variance</u> $\hat{\mu}_3$ is the worst estimator.	B1ft B1dcao (3)
	Notes	
(c)	M1 finding $\mathrm{E}(\hat{\mu})$ A1 bias 0 A1 $\pm\frac{\mu}{3}$ A1 $\pm\frac{\mu}{3}$	
(d)	For method marks allow an incorrect variance, M1 squaring 9, M1 Squaring 5 and 2, M1 adding variances. Do not penalise same mistake twice.	
(e)(ii)	Must have idea that its bias is the same as another ($\hat{\mu}_2$) and state it has largest variance for first B1 . ft their values of Var. Second B1 dependent on first B1cao SC $\hat{\mu}_3$ because <u>largest variance</u> B1 B0	

Question Number	Scheme	Marks
7(a)	The variance of the two group's marks must be the same.	B1
	$H_0 : \sigma_1^2 = \sigma_2^2 \quad H_1 : \sigma_1^2 \neq \sigma_2^2$	B1
	$s_1^2 = 16.25$	B1
	$(F_{8,6} =) \frac{16.25}{12.9} = (1.2597...) \quad \left(\frac{1}{F_{8,6}} = \frac{12.9}{16.25} = 0.7938... \right)$	M1A1
	$F_{8,6}$ 5% (two-tail) cv = **4.15** (0.241) (or prob. = awrt 0.39)	B1
	Not significant so can accept the assumption that variances are equal.	A1
		(7)
(b)	$H_0 : \mu_1 = \mu_2 \quad H_1 : \mu_1 \neq \mu_2$	B1
	$s_p^2 = \frac{8 \times 16.25 + 6 \times 12.9}{14}, = 14.814...$ or $s_p = 3.8489...$	M1
	$(t_{14} =)(\pm) \frac{30.33 - 31.29}{s_p \sqrt{\frac{1}{9} + \frac{1}{7}}} = (\pm)0.494927..$ = awrt **<u>0.49</u>**	B1 M1A1
	$t_{14}(0.025)$ two-tail cv = **2.145**	B1
	There is insufficient evidence to reject H_0. There is no evidence of a significant difference between the **<u>mean marks</u>** of the two groups	A1
		(7)
	Notes	
(a)	2nd B1allow σ or σ^2 3rd B1 allow awrt 16.3 or s_1= awrt 4.03 M1 for use of the correct test statistic 5th B1 allow "assumption is correct"	
(b)	1st M1 for attempting s_p or s_p^2 1st B1 for 30.33 2nd M1 for use of a correct test statistic 2nd A1 for awrt 0.49 (accept $\pm$) or 0.495 2nd B1 for 2.145 (allow $\pm$ 1.761 for one-tailed H_1)	

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Mark Scheme (Results)

Summer 2015

Pearson Edexcel GCE in Statistics 4 (6686/01)

Summer 2015
Publications Code UA042714

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misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

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June 2015
6686 S4
Mark Scheme

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>1. (a)</td><td>

Store	A	B	C	D	E	F	G	H
Difference *July-Jan*	33	63	121	–60	–54	24	–19	33

</td><td>B1</td></tr>
<tr><td></td><td>$\bar{d} = \frac{141}{8} = (\pm)17.625$</td><td>M1</td></tr>
<tr><td></td><td>$s_d{}^2 = \frac{8}{7}\left(\frac{28241}{8} - 17.625^2\right) = 3679.4\ldots$
or
$s_d{}^2 = \frac{1}{7}\left(28241 - \frac{141^2}{8}\right) = 3679.4\ldots$</td><td>M1</td></tr>
<tr><td></td><td>To test $H_0 : \mu_d = 0$ against $H_1 : \mu_d > 0$ (o.e.)</td><td>B1</td></tr>
<tr><td></td><td>Test stat
$t = \frac{17.625 - 0}{\sqrt{\frac{3679.4\ldots}{8}}} = 0.8218\ldots$</td><td>M1A1cso</td></tr>
<tr><td></td><td>Critical value, $t_7 = 1.895$</td><td>B1</td></tr>
<tr><td></td><td>Not in critical region therefore insufficient reason to reject H_0
No significant evidence that on average stores sell more lottery **tickets** in **July** than in **January**</td><td>A1ft
(8)</td></tr>
<tr><td>(b)</td><td>Need assumption that the underlying distribution of the **difference** in sales in July and in January is **normally distributed**.</td><td>B1 (1)
Total 9</td></tr>
<tr><td></td><td>**Notes**</td><td></td></tr>
<tr><td>(a)</td><td>1st B1 for differences all correct (o.e.)
1st M1 attempt to find $\bar{d} = \frac{\sum \text{"their } d\text{"}}{8}$
2nd M1 attempting s_d or $s_d{}^2$ $\frac{1}{7}\left(\sum \text{"their } d^2\text{"} - \frac{(\sum \text{"their } d\text{"})^2}{8}\right)$
2nd B1 both correct in terms of μ or μ_d (allow a defined symbol) condone $\mu_{July-Jan}$
3rd M1 for attempting the correct test statistic $\frac{\bar{d}}{s_d/\sqrt{8}}$
1st A1cso awrt 0.822 with no errors.
3rd B1 alternate method, p value of 0.219. Allow 2.365 for 2-tail test
Final A1 need conclusion in context, need tickets July and January, ft their test stat and critical value
NB difference of 2 means test gains no marks</td><td></td></tr>
<tr><td>(b)</td><td>B1 need **differences** to be normally distributed, not just normal distribution</td><td></td></tr>
</table>

Question Number	Scheme	Marks
2. (a)	$n=8 \quad \sum x = 843 \quad \sum x^2 = 89211$	
	$\therefore \quad \bar{x} = 105.375$	
	$s^2 = \frac{8}{7}\left(\frac{89211}{8} - 105.375^2\right) = 54.2678\ldots$	
	or	M1A1
	$s^2 = \frac{1}{7}\left(89211 - \frac{843^2}{8}\right) = 54.2678\ldots$	
	Confidence interval is given by	
	$\frac{7\times 54.267\ldots}{14.067} < \sigma^2 < \frac{7\times 54.267\ldots}{2.167}$	M1B1
	$\therefore \quad 27.004\ldots < \sigma^2 < 175.299\ldots$	
	$5.1966\ldots < \sigma < 13.240\ldots$	M1d A1
		(6)
(b)	Need to assume underlying **Normal** distribution for **weights** of blocks of cheese.	B1
		(1)
(c)	Lower limit of CI is $> 5\ g$ suggests that **Fred** needs **training**.	B1ft
		(1)
(d)	To test $\quad H_0 : \mu = 100\ , \quad H_1 : \mu \neq 100 \quad (\mu > 100)$	B1
	where μ is the mean weight of blocks of cheese	
	Test statistic $\quad t = \frac{102.6-100}{\sqrt{\frac{19.4}{20}}} = 2.6399\ldots$	M1A1
	Critical value(s): $\quad t_{19} = (\pm)1.729\ (1.328)$	B1
	In critical region, therefore significant evidence to reject H_0 and accept H_1	A1ft
	Significant evidence that the **mean weight** of the **blocks** of cheese is not 100 g (more than 100g)	B1cso (6)
		Total 14
	Notes	
(a)	1st M1 attempting s or s^2 1st A1 awrt 54.3	
	2nd M1 for $\frac{7s^2}{\chi^2}$	
	B1 14.067 & 2.167	
	3rd M1d Dept on previous M mark. Rearranging leading to interval for σ- must square root	
	A1 awrt 5.20 and 13.2 (allow 5.2)	
	NB a correct interval gains full marks	
(c)	B1ft on their CI must have **Fred/He/employee (do not allow empoloyees)** and **training**. They must have an interval in part(a)	
(d)	1st B1 Both hypotheses with μ. Allow one-tail	
	1st M1 $\frac{102.6-100}{\frac{s \text{ or } s^2}{\sqrt{20}}}$ 1st A1 awrt 2.64	
	2nd B1 allow p value of 0.0161 in place of critical value. CV must follow from H_1	
	2nd A1ft a correct statement – do not allow contradicting non context statement.	
	3rd B1cso need correct conclusion in context containing the words in bold from a fully correct solution. For one tail need “more than 100g”	

Question Number	Scheme	Marks
3. (a)	$s_p^{\,2} = \dfrac{12\times161+9\times48}{13+10-2} = \dfrac{2364}{21} = 112.571\ldots = 112.6\ \text{(1dp)}$	M1A1cso
		(2)
(b)	To test $\text{H}_0 : \mu_s = \mu_a$ against $\text{H}_1 : \mu_s \neq \mu_a$ (o.e.)	B1
	Test stat, $t = \pm\dfrac{195-186}{\sqrt{112.57\ldots\left(\frac{1}{10}+\frac{1}{13}\right)}} = \pm2.016\ldots$ (awrt2.02)	M1A1
	Critical values, $t_{21} = (\pm)1.721$	B1
	In critical region, therefore significant evidence to reject H_0 and accept H_1	
	Evidence of difference in mean **arm span** of adult male **swimmers** and adult male **athletes** or No evidence to support **Ali's claim**.	A1
		(5)
(c)	To test $\text{H}_0 : \sigma_s^{\,2} = \sigma_a^{\,2}$ against $\text{H}_1 : \sigma_s^{\,2} \neq \sigma_a^{\,2}$	B1
	Test stat, $F_{12,9} = \dfrac{161}{48} = 3.354\ldots \quad \left(\dfrac{1}{\text{F}_{12,9}} = \dfrac{48}{161} = 0.2981...\right)$	M1A1
	Critical value, $F_{12,9} = 3.07\ (0.3257\ldots)$	B1
	In critical region, therefore significant evidence to reject H_0 and accept H_1	
	Evidence of difference in **variance** of arm span of adult male swimmers and adult male athletes or the data supports **Bea's belief**	A1cso
		(5)
(d)	Should do test for variance first as **equal variances** is necessary assumption for t test for means	B1
	but is **not supported** in (c), so result in **(b) is invalid**.	B1d
		(2)
		Total 14
	Notes	
(a)	M1 for $\dfrac{12\times161+9\times48}{13+10-2}$	
	A1cso need to get awrt112.57 **or** $\dfrac{2364}{21}$ then write 112.6	
(b)	M1 $\dfrac{195-186}{\sqrt{112.6\left(\frac{1}{10}+\frac{1}{13}\right)}}$	
	2nd B1 alternate method, p value of 0.0566 in place of critical value	
	Final A1 requires correct conclusion in context	
(c)	1st B1 allow $\text{H}_0 : \sigma_s = \sigma_a$ against $\text{H}_1 : \sigma_s \neq \sigma_a$	
	M1 allow $\dfrac{161^2}{48^2}$ if they write the formula down	
	Final A1 requires correct conclusion	
(d)	1st B1 equal variances is necessary assumption (may be implied by saying not equal)	
	2nd B1d but not supported in (c)/(variances not equal) therefore (b) result invalid	

Question Number	Scheme	Marks
4. (a)	Power function $= \text{P}(\text{H}_0 \text{ rejected}) = \text{P}(X_1 \geq 2) + \text{P}(X_1 = 1) \times \text{P}(X_2 \geq 1)$	
	$= 1-(1-p)^6 - 6p(1-p)^5 + 6p(1-p)^5 \times (1-(1-p)^6)$	
	$= 1-(1-p)^6 - 6p(1-p)^5 + 6p(1-p)^5 - 6p(1-p)^{11}$	M1A1
	$= 1-(1-p)^6 - 6p(1-p)^{11}$	A1cso
		(3)
(b)	Size of test is value of power function when $p = 0.05$	
	Size of test $= 1-0.95^6 - 6\times 0.05 \times 0.95^{11} = 0.094268\ldots$ (awrt 0.0943)	M1A1
		(2)
(c)	E[number of eggs inspected] $= 12 \times \text{P}(X_1 = 1) + 6 \times \text{P}(X_1 \neq 1)$	M1
	$= 12 \times 6 \times 0.1 \times 0.9^5 + 6 \times (1-(6\times 0.1 \times 0.9^5))$	A1
	$= 8.1257\ldots$(awrt 8.13)	A1
		(3)
(d)	P(Type II error $\mid p = 0.1) = 1-$(value of power function when $p = 0.1$)	M1
	P(Type II error $\mid p = 0.1) = 1-(1-0.9^6 - 6\times 0.1 \times 0.9^{11}) = 0.7197\ldots$ (awrt 0.720)	A1
		(2)
(e)	Prob of Type II error, accepting $p = 0.05$ when it is actually 0.1, unacceptably high, is large, therefore not a good **test**.	B1
		(1)
		Total 11
	Notes	
(a)	M1 for $\text{P}(X_1 \geq 2) + \text{P}(X_1 = 1) \times \text{P}(X_2 \geq 1)$ or $1 - \left(\text{P}(X_1 = 0) + \text{P}(X_1 = 1) \times \text{P}(X_2 = 0)\right)$ oe or a correct line of working A1 a correct line of working before the final answer A1 fully correct solution no errors.	
(b)	M1 attempt to subst 0.05 into (a)	
(c)	M1 for $12 \times \text{P}(X_1 = 1) + 6 \times \text{P}(X_1 \neq 1)$ A1 $12 \times 6 \times p \times 0.9(1-p)^5 + 6 \times (1-(6\times p \times (1-p)^5)$	
(d)	M1 $1-(1-(1-p)^6 - 6\times p \times (1-p)^{11})$	
(e)	B1 idea that the Probability of a Type II error is **too high** or the power is too low so the **test is not good/powerful or test needs changing**	

Question Number	Scheme	Marks
5 **(a)**	$\bar{x} = \frac{\sum x}{n} = \frac{1116}{9} = 124$	B1
	$s^2 = \frac{9}{8}\left(\frac{138728}{9} - 124^2\right) = 43$	
	Or	
	$s^2 = \frac{1}{8}\left(138728 - \frac{1116^2}{9}\right) = 43$	B1
		(2)
(b)	Test stat	
	$\chi^2 = \frac{8 \times 43}{25} = 13.76$	M1A1
	Critical value	
	$\chi^2 = 15.507$	B1
	Therefore not in critical region, insufficient evidence to reject H_0	
	There is evidence at the 5% level that the **company's** claim is supported	B1d
		(4)
(c)	CI given by	
	$\frac{11 \times 8.17}{21.920} < \sigma^2 < \frac{11 \times 8.17}{3.816}$	M1
	Therefore	
	$4.0999\ldots < \sigma^2 < 23.55\ldots$ awrt 4.10 and 23.6	A1
		(2)
(d)	$\sigma^2 = 25$ is not in CI which suggests **Gurdip's(his)** claim may not be true.	B1ft
		(1)
		Total 9
	Notes	
(a)	B1 124 B1 43	
(b)	M1 $\frac{8 \times \text{their } 43}{25}$ A1 awrt 13.8 B1 15.507 B1 dep on previous M1 being awarded. Allow the standard deviation of the **IQ** scores is 5 oe. Must have IQ	
(c)	M1 $\frac{11 \times 8.17}{3.816 \text{ or } 21.92}$ A1 both correct	
(d)	B1ft their interval from part(c). Gurdip's claim may not be true NB, no interval in (c) then B0	

Question Number	Scheme	Marks
6. (a)	$\mathrm{E}[A]=\frac{1}{2}(\mathrm{E}[X_1]+\mathrm{E}[X_2]+\mathrm{E}[X_3]+\mathrm{E}[Y_1]+\mathrm{E}[Y_2]=\frac{1}{2}\left(3\times\frac{\mu}{3}+2\times\frac{\mu}{2}\right)=\mu$	M1
	Therefore A is an unbiased estimator	A1
	$\mathrm{E}[B]=\frac{3\mathrm{E}[X_1]}{2}+\frac{2\mathrm{E}[Y_1]}{3}=\frac{3}{2}\times\frac{\mu}{3}+\frac{2}{3}\times\frac{\mu}{2}=\frac{5\mu}{6}$	A1
	Therefore B is biased with bias $(-)\frac{\mu}{6}$	B1ft
	$\mathrm{E}[C]=\frac{1}{3}(3\mathrm{E}[X_1]+4\mathrm{E}[Y_1])=\frac{1}{3}\left(\frac{3\mu}{3}+\frac{4\mu}{2}\right)=\mu$	
	Therefore C is an unbiased estimator	A1
		(5)
(b)	Best estimator is unbiased estimator with least variance	
	$\mathrm{Var}\,(A)=\frac{1}{4}(\mathrm{Var}\,X_1+\mathrm{Var}\,X_2+\mathrm{Var}\,X_3+\mathrm{Var}\,Y_1+\mathrm{Var}Y_2)$	M1
	$=\frac{1}{4}\left(3\times3\sigma^2+2\times\frac{\sigma^2}{2}\right)=\frac{5\sigma^2}{2}$	A1
	$\mathrm{Var}\,(C)=\frac{1}{9}(9\mathrm{Var}\,X_1+16\mathrm{Var}\,Y_1)=\frac{1}{9}\left(9\times3\sigma^2+16\times\frac{\sigma^2}{2}\right)=\frac{35\sigma^2}{9}$	A1
	Therefore A is a better estimator of μ (smaller variance)	B1dft
		(4)
(c)	$\mathrm{E}[D]=\frac{1}{k}\left(2n\times\frac{\mu}{3}+n\times\frac{\mu}{2}\right)=\mu$	M1A1
	$k=\frac{2n}{3}+\frac{n}{2}=\frac{7n}{6}$	A1
		(3)
(d)	$\mathrm{Var}\,(D)=\frac{1}{k^2}\left(2n\times3\sigma^2+n\times\frac{\sigma^2}{2}\right)=\frac{1}{k^2}\times\frac{13n\sigma^2}{2}$	M1
	$\mathrm{Var}\,(D)=\frac{36}{49n^2}\times\frac{13n\sigma^2}{2}=\frac{234\sigma^2}{49n}$	M1d A1
	Therefore $\mathrm{Var}\,D\to0$ as $n\to\infty$, therefore D is a consistent estimator	A1dd
		(4)
(e)	Want	
	$\frac{234\sigma^2}{49n}<\frac{5\sigma^2}{2}$	M1
	Therefore	
	$\frac{234}{49}\times\frac{2}{5}<n$	
	$n>1.910\ldots$	
	So minimum value is $n=2$	A1cso
		(2)
		Total 18

	Notes
(a)	M1 for a correct method for E(A) or E(B) or E(C) A1 for each correct expectation with a correct method B1ft bias of B, condone missing – sign. Do not allow a bias of 0
(b)	M1 Use of Var(aX) = a^2Var (X) and subst $3\sigma^2$ for Var(X) and $\frac{\sigma^2}{2}$ for Var(Y) A1 for each correct variance B1dft their variances. Dep on m1 being awarded. If no variances given then B0
(c)	M1 attempts E(D) and puts = to μ (may be implied) A1 for E(D)
(d)	M1 for $\frac{1}{k^2}\left(2n\times 3\sigma^2 + n\times\frac{\sigma^2}{2}\right)$ or $\frac{1}{k^2}\times\frac{13n\sigma^2}{2}$ M1d for subst in k A1 Correct Var (D) A1dd Need correct reason for being a consistent estimator dep on previous method marks being awarded
(e)	M1 for forming an inequality with their Var(D) < their best estimator leading to n

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