Edexcel S3 – Ch. 3: Estimation, Confidence Intervals and Tests

1.   A sample of size 5 is taken from a population that is normally distributed with mean 10 and standard deviation 3. Find the probability that the sample mean lies between 7 and 10.

(6)

2.   A computer company repairs large numbers of PCs and wants to estimate the mean time to repair a particular fault. Five repairs are chosen at random from the company's records and the times taken, in seconds, are

205   310   405   195   320.

(*a*)   Calculate unbiased estimates of the mean and the variance of the population of repair times from which this sample has been taken.

(4)

It is known from previous results that the standard deviation of the repair time for this fault is 100 seconds. The company manager wants to ensure that there is a probability of at least 0.95 that the estimate of the population mean lies within 20 seconds of its true value.

(*b*)   Find the minimum sample size required.

(6)

3.   A report on the health and nutrition of a population stated that the mean height of three-year old children is 90 cm and the standard deviation is 5 cm. A sample of 100 three-year old children was chosen from the population.

(*a*)   Write down the approximate distribution of the sample mean height. Give a reason for your answer.

(3)

(*b*)   Hence find the probability that the sample mean height is at least 91 cm.

(3)

4. A biologist investigated whether or not the diet of chickens influenced the amount of cholesterol in their eggs. The cholesterol content of 70 eggs selected at random from chickens fed diet *A* had a mean value of 198 mg and a standard deviation of 47 mg. A random sample of 90 eggs from chickens fed diet *B* had a mean cholesterol content of 201 mg and a standard deviation of 23 mg.

(*a*) Stating your hypotheses clearly and using a 5% level of significance, test whether or not there is a difference between the mean cholesterol content of eggs laid by chickens fed on these two diets.

(7)

(*b*) State, in the context of this question, an assumption you have made in carrying out the test in part (*a*).

(2)

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5. A machine produces metal containers. The weights of the containers are normally distributed. A random sample of 10 containers from the production line was weighed, to the nearest 0.1 kg, and gave the following results

49.7,   50.3,   51.0,   49.5,   49.9

50.1,   50.2,   50.0,   49.6,   49.7.

(*a*) Find unbiased estimates of the mean and variance of the weights of the population of metal containers.

(5)

The machine is set to produce metal containers whose weights have a population standard deviation of 0.5 kg.

(*b*) Estimate the limits between which 95% of the weights of metal containers lie.

(4)

(*c*) Determine the 99% confidence interval for the mean weight of metal containers.

(5)

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6. The time, in minutes, it takes Robert to complete the puzzle in his morning newspaper each day is normally distributed with mean 18 and standard deviation 3. After taking a holiday, Robert records the times taken to complete a random sample of 15 puzzles and he finds that the mean time is 16.5 minutes. You may assume that the holiday has not changed the standard deviation of times taken to complete the puzzle.

Stating your hypotheses clearly test, at the 5% level of significance, whether or not there has been a reduction in the mean time Robert takes to complete the puzzle.

(7)

7. In a trial of diet A a random sample of 80 participants were asked to record their weight loss, x kg, after their first week of using the diet. The results are summarised by

$$\sum x = 361.6 \quad \text{and} \quad \sum x^2 = 1753.95$$

(a) Find unbiased estimates for the mean and variance of weight lost after the first week of using diet A.

(5)

The designers of diet A believe it can achieve a greater mean weight loss after the first week than a standard diet B. A random sample of 60 people used diet B. After the first week they had achieved a mean weight loss of 4.06 kg, with an unbiased estimate of variance of weight loss of 2.50 kg².

(b) Test, at the 5% level of significance, whether or not the mean weight loss after the first week using diet A is greater than that using diet B. State your hypotheses clearly.

(7)

(c) Explain the significance of the central limit theorem to the test in part (b).

(1)

(d) State an assumption you have made in carrying out the test in part (b).

(1)

8. A random sample of the daily sales (in £s) of a small company is taken and, using tables of the normal distribution, a 99% confidence interval for the mean daily sales is found to be

$$(123.5, 154.7)$$

Find a 95% confidence interval for the mean daily sales of the company.

(6)