1. Some biologists were studying a large group of wading birds. A random sample of 36 were measured and the wing length, x mm of each wading bird was recorded. The results are summarised as follows

$$\sum x = 6046 \qquad \sum x^2 = 1016338$$

(a) Calculate unbiased estimates of the mean and the variance of the wing lengths of these birds.

(3)

Given that the standard deviation of the wing lengths of this particular type of bird is actually 5.1 mm,

(b) find a 99 % confidence interval for the mean wing length of the birds from this group.

(5)
(Total 8 marks)

2. A report on the health and nutrition of a population stated that the mean height of three year old children is 90 cm and the standard deviation is 5 cm. A sample of 100 three year old children was chosen from the population.

(a) Write down the approximate distribution of the sample mean height. Give a reason for your answer.

(3)

(b) Hence find the probability that the sample mean height is at least 91 cm.

(3)
(Total 6 marks)

3. A machine produces metal containers. The weights of the containers are normally distributed. A random sample of 10 containers from the production line was weighed, to the nearest 0.1 kg, and gave the following results

49.7, 50.3, 51.0, 49.5, 49.9

50.1, 50.2, 50.0, 49.6, 49.7.

(a) Find unbiased estimates of the mean and variance of the weights of the population of metal containers.
(5)

The machine is set to produce metal containers whose weights have a population standard deviation of 0.5 kg.

(b) Estimate the limits between which 95% of the weights of metal containers lie.
(4)

(c) Determine the 99% confidence interval for the mean weight of metal containers.
(5)
(Total 14 marks)

4. The drying times of paint can be assumed to be normally distributed. A paint manufacturer paints 10 test areas with a new paint. The following drying times, to the nearest minute, were recorded.

82, 98, 140, 110, 90, 125, 150, 130, 70, 110.

(a) Calculate unbiased estimates for the mean and the variance of the population of drying times of this paint.
(5)

Given that the population standard deviation is 25,

(b) find a 95% confidence interval for the mean drying time of this paint.
(5)

Fifteen similar sets of tests are done and the 95% confidence interval is determined for each set.

(c) Estimate the expected number of these 15 intervals that will enclose the true value of the population mean μ.
(2)
(Total 12 marks)

5. A sample of size 5 is taken from a population that is normally distributed with mean 10 and standard deviation 3. Find the probability that the sample mean lies between 7 and 10.

(Total 6 marks)

6. A computer company repairs large numbers of PCs and wants to estimate the mean time to repair a particular fault. Five repairs are chosen at random from the company's records and the times taken, in seconds, are

205 310 405 195 320.

(a) Calculate unbiased estimates of the mean and the variance of the population of repair times from which this sample has been taken.

(4)

It is known from previous results that the standard deviation of the repair time for this fault is 100 seconds. The company manager wants to ensure that there is a probability of at least 0.95 that the estimate of the population mean lies within 20 seconds of its true value.

(b) Find the minimum sample size required.

(6)
(Total 10 marks)

7. Kylie regularly travels from home to visit a friend. On 10 randomly selected occasions the journey time x minutes was recorded. The results are summarised as follows.

$\sum x = 753, \quad \sum x^2 = 57\ 455.$

(a) Calculate unbiased estimates of the mean and the variance of the population of journey times.

(3)

After many journeys, a random sample of 100 journeys gave a mean of 74.8 minutes and a variance of 84.6 minutes^2.

(b) Calculate a 95% confidence interval for the mean of the population of journey times.

(5)

(c) Write down two assumptions you made in part (b).

(2)
(Total 10 marks)

1.	(a)	$\bar{x} = \left(\frac{6046}{36} = \right)167.94...$	awrt **168**	B1	
		$s^2 = \frac{1016338 - 36 \times \bar{x}^2}{35}$		M1	
		= 27.0253....	awrt **27.0** (Accept 27)	A1	3

M1 for a correct expression for s^2, follow through their mean, beware it is very "sensitive"

$167.94 \rightarrow \frac{999.63..}{35} \rightarrow 28.56...$

$167.9 \rightarrow \frac{1483.24..}{35} \rightarrow 42.37...$

$168 \rightarrow \frac{274}{35} \rightarrow 7.82$

These would all score M1A0
Use of 36 as the divisor (= 26.3…) is M0A0

	(b)	99% Confidence Interval is: $\bar{x} = 2.5758 \times \frac{5.1}{\sqrt{36}}$		M1A1ft	
			2.5758	B1	
		= (165.755…, 170.133…)	awrt (**166,170**)	A1A1	5

M1 for substituting their values in $\bar{x} \pm z \times \frac{5.1 \text{ or } s}{\sqrt{36}}$

where z is a recognizable value from tables

1st A1 follow through their mean and their z (to 2dp) in $\bar{x} \pm z \times \frac{5.1}{\sqrt{36}}$

Beware: $167.94 \pm 2.5758 \times \frac{5.1^2}{36} \rightarrow (166.07..,169.8..)$

but scores B1M0A0A0A0
Correct answer only in (b) scores 0/5
2 nd & 3rd A marks depend upon 2.5758 **and** M mark.

[8]

2.	(a)	$\overline{X} \sim N(90, \frac{5^2}{100})$ i.e. N, (90, 0.25)		M1, A1	
		Application of <u>central limit theorem</u> as (sample large)		B1	3
	(b)	$P(\overline{X} \geq 91) = 1 - P(Z < \frac{91-90}{0.5})$	Stand.	M1 A1	
		$= 1 - P(Z < 2)$			
		$= 1 - 0.9772$			

= 0.0228 A1 3

awrt 0.0228

[6]

3. (a) $\bar{X} = \frac{500}{10} = 50$ M1 A1

$S^2 = \frac{1}{9}(25001.74 - \frac{500^2}{10}) = 0.193$ awrt 0.193 M1 A1 A1 5

(b) Limits are 50 ± 1.966 B1 1.96 M1 B1

= (49.02, 50.98) awrt 49(0), 51(0) A1 A1 4

(c) Confidence interval is

$(50 - 2.5758 \times \frac{0.5}{\sqrt{10}}, 50 + 2.5758 \times \frac{0.5}{\sqrt{10}})$ B1 2.5758 M1 B1 A1ft

= (49.59273, 50.40727…) awrt 49.6, awrt 50.4 A1 A1 5

Use of estimate in (a) in (b) AND (c) assume MISREAD

[14]

4. (a) $\hat{\mu} = \frac{82+98+140+110+90+125+150+130+70+110}{10}$ M1

= 110.5 A1

$\hat{\sigma} = \frac{1}{9}(128153 - 10 \times 110.5^2)$ B1

128153 M1

= 672.28 (AWRT 672) A1 5

(b) 95% confidence limits are (condone use of 5 instead of 25) M1
(for 1.96) B1
A1ft

110.5 $\pm 1.96 \times \frac{25}{\sqrt{10}}$ A1 A1 5

95% conf. lim. = AWRT(95, 126)

(c) Number of intervals = $\frac{95}{100} \times 15$ M1

= 14.25 (allow 14 or 14.3 if method is clear) A1 2

[12]

5.	$X \sim N(10, 3^2)\ \therefore\ \overline{X} \sim N(10, \tfrac{9}{5})$	B1; B1		
	Can be implied 10; $\tfrac{9}{5}$			
	$P(7 \le \overline{X} \le 10) = P\left(\frac{7-10}{\sqrt{\tfrac{9}{5}}} < Z < 0\right)$	M1 A1		
	Standardising with 10 & their σ			
	$= P(-2.236 < Z < 0)$			
	$= \Phi(0) - \{1 - \Phi(2.24)\}$	M1($p < 0.5$)		
	$= \underline{0.4875}$	A1	6	
				[6]

6.	(a)	Let X represent repair true			
		$\therefore \sum x = 1435 \therefore \bar{x} = \frac{1435}{5} = \underline{287}$	B1		
		$\sum x^2 + = 442575\ \therefore s^2 = \frac{1}{4}\left\{442575 - \frac{1435^2}{5}\right\}$	M1 A1		
		$= \underline{7682.5}$	A1	4	
	(b)	$P(\lvert \mu - \hat{\mu} \rvert) < 20 = 0.95$	M1		
		Use of 10,20 as 40 with their σ *&* $\sqrt{n}$			
		$\therefore \frac{20}{\sigma/\sqrt{n}} = 1.96$	B1		
		$\therefore n = \frac{1.96^2\sigma^2}{20^2} = \frac{1.96^2\sigma^2}{400} = \underline{96.04}$	M1 A1		
		Solving for n			
		$\therefore$ <u>Sample size (≥) 97 required</u>	A1	6	
					[10]

7.	(a)	$\bar{x} = 75.3$	B1	
		$s^2 = \frac{1}{9}\left\{57455 - \frac{753^2}{10}\right\}$	M1	
		$= 83.7\dot{8}, 83\frac{71}{90}, 83.8$	A1	3
		awrt 83.8		

(b)	$74.8 \pm 1.96\sqrt{\frac{84.6}{100}}$		
	1.96	B1	
	any z value, may use 75.3, 83.8 for M	M1	
		A1ft on z only	
	(73.0, 76.6)	A1, A1	5
	awrt 73.0,76.6		
(c)	Journey times independent Sample large enough to use central limit theorem Same distribution / population	any 2 B1, B1	2

[10]

1. The mean in part (a) was nearly always correct but premature approximation meant that the answers for s^2 were often not accurate to 3sf. In part (b) the correct formula for a confidence interval was usually used but the full z value of 2.5758 was sometimes truncated or an approximate value used from the "large" table.

2. Very well answered on the whole, but many candidates lost a mark by failing to mention the Central Limit Theorem as required.

3. Part (a) was almost always completely correct, but parts (b) and (c) elicited the usual mistakes involving incorrect variances. In part (b) it was not unusual to see 50 ±1.96 $\frac{0.5}{\sqrt{10}}$

4. The mean was almost always correct and the majority of the candidates knew how to find an unbiased estimate of the variance. Most knew how to find 95% confidence limits although a few used 1.6449 and some thought the formula was $1.96 \pm \frac{\sigma}{\sqrt{n}}$. Occasionally the standard deviation of 25 was misread as a variance. The final part caused some confusion. Many students carried out the required calculation but some started to find the width of the interval and others did not attempt this part.

5. There were many correct answers to this question but there were also some common errors. These were the use of the wrong standard error and being unable to handle a negative z-value.

6. Part (a) was usually well answered by most of the candidates. Many candidates did not really understand how to tackle part (b) and of those that did many did not interpret the phrase 'lies within' correctly.

7. Candidates generally did either very well with this question or very badly. Many managed to find the mean but failed to find an unbiased estimate of the variance. Many different and some times very odd formulae were used but $\frac{1}{10}(57455 - \frac{753^2}{10})$ was fairly common. In part (b) the z value was often correct but candidates then went on to use $\frac{84.6}{10}$ rather than $\sqrt{\frac{84.6}{100}}$. In part (c) it was usually recognised that the central limit theorem could be invoked but few other correct assumptions were given.