1. A machine fills jars with jam. The weight of jam in each jar is normally distributed. To check the machine is working properly the contents of a random sample of 15 jars are weighed in grams. Unbiased estimates of the mean and variance are obtained as

$$\hat{\mu} = 560 \qquad s^2 = 25.2$$

Calculate a 95% confidence interval for,

(a) the mean weight of jam,

(4)

(b) the variance of the weight of jam.

(5)

A weight of more than 565 g is regarded as too high and suggests the machine is not working properly.

(c) Use appropriate confidence limits from parts (a) and (b) to find the highest estimate of the proportion of jars that weigh too much.

(5)
(Total 14 marks)

2. A town council is concerned that the mean price of renting two bedroom flats in the town has exceeded £650 per month. A random sample of eight two bedroom flats gave the following results, £x, per month.

705, 640, 560, 680, 800, 620, 580, 760

[You may assume $\sum x = 5345$ $\sum x^2 = 3621025$]

(a) Find a 90% confidence interval for the mean price of renting a two bedroom flat.

(6)

(b) State an assumption that is required for the validity of your interval in part (a).

(1)

(c) Comment on whether or not the town council is justified in being concerned. Give a reason for your answer.

(2)
(Total 9 marks)

3. A machine is filling bottles of milk. A random sample of 16 bottles was taken and the volume of milk in each bottle was measured and recorded. The volume of milk in a bottle is normally distributed and the unbiased estimate of the variance, s^2, of the volume of milk in a bottle is 0.003

(a) Find a 95% confidence interval for the variance of the population of volumes of milk from which the sample was taken.

(5)

The machine should fill bottles so that the standard deviation of the volumes is equal to 0.07

(b) Comment on this with reference to your 95% confidence interval.

(3)
(Total 8 marks)

4. The weights, in grams, of apples are assumed to follow a normal distribution. The weights of apples sold by a supermarket have variance $\sigma_S{}^2$. A random sample of 4 apples from the supermarket had weights

114, 110, 119, 123.

(a) Find a 95% confidence interval for $\sigma_S{}^2$.

(7)

The weights of apples sold on a market stall have variance $\sigma_M{}^{2.}$ A second random sample of 7 apples was taken from the market stall. The sample variance $s_M{}^2$ of the apples was 318.8.

(b) Stating your hypotheses clearly test, at the 1% level of significance, whether or not there is evidence that $\sigma_M{}^2 > \sigma_S{}^2$.

(5)
(Total 12 marks)

5. A diabetic patient records her blood glucose readings in mmol/l at random times of day over several days. Her readings are given below.

5.3 5.7 8.4 8.7 6.3 8.0 7.2

Assuming that the blood glucose readings are normally distributed calculate

(a) an unbiased estimate for the variance σ^2 of the blood glucose readings,

(2)

(b) a 90% confidence interval for the variance σ^2 of blood glucose readings.

(5)

(c) State whether or not the confidence interval supports the assertion that $\sigma = 0.9$.

Give a reason for your answer.

(1)
(Total 8 marks)

6. A tree is cut down and sawn into pieces. Half of the pieces are stored outside and half of the pieces are stored inside. After a year, a random sample of pieces is taken from each location and the hardness is measured. The hardness x units are summarised in the following table.

	Number of pieces sampled	$\sum x$	$\sum x^2$
Stored outside	20	2340	274050
Stored inside	37	4884	645282

(a) Show that unbiased estimates for the variance of the values of hardness for wood stored outside and for the wood stored inside are 14.2 and 16.5, to 1 decimal place, respectively.

(2)

The hardness of wood stored outside and the hardness of wood stored inside can be assumed to be normally distributed with equal variances.

(b) Calculate 95% confidence limits for the difference in mean hardness between the wood that was stored outside and the wood that was stored inside.

(8)

(c) Using your answer to part (b), comment on the means of the hardness of wood stored outside and inside. Give a reason for your answer.

(2)
(Total 12 marks)

7. A supervisor wishes to cheek the typing speed of a new typist. On 10 randomly selected occasions, the supervisor records the time taken for the new typist to type 100 words. The results, in seconds, are given below.

110, 125, 130, 126, 128, 127, 118, 120, 122, 125

The supervisor assumes that the time taken to type 100 words is normally distributed.

(a) Calculate a 95% confidence interval for

(i) the mean,

(ii) the variance

of the population of times taken by this typist to type 100 words.

(13)

The supervisor requires the average time needed to type 100 words to be no more than 130 seconds and the standard deviation to be no more than 4 seconds.

(b) Comment on whether or not the supervisor should be concerned about the speed of the new typist.

(3)
(Total 16 marks)

8. A random sample of 15 tomatoes is taken and the weight *x* grams of each tomato is found. The results are summarised by $\sum x = 208$ and $\sum x^2 = 2962$.

(a) Assuming that the weights of the tomatoes are normally distributed, calculate the 90% confidence interval for the variance σ^2 of the weights of the tomatoes.

(7)

(b) State with a reason whether or not the confidence interval supports the assertion $\sigma^2 = 3$.

(2)
(Total 9 marks)

1. (a) 95% confidence interval for μ is 2.145 B1

$560 \pm t_{14}(2.5\%)\ \sqrt{\frac{25.2}{15}} = 560 \pm 2.145\sqrt{\frac{25.2}{15}} =$

(557.2, 562.8) M1 A1 A1 4

(b) 95% confidence interval for σ^2 is

$5.629 < \frac{14 \times 25.2}{\sigma^2} < 26.119$ B1 M1 B1

$\sigma^2 < 62.675$ $\sigma^2 > 13.507$

$13.507 < \sigma^2 < 62.675$ awrt 13.5, 62.7 A1 A1 5

(c) Require $\mathrm{P}(X > 565) = \mathrm{P}\left(Z > \frac{565 - \mu}{\sigma}\right)$

to be as large as possible OR

$\frac{565 - \mu}{\sigma}$ to be as small as possible; both

imply highest σ and μ. M1

$\frac{565 - 562.8}{\sqrt{62.675}} = 0.28$ M1A1

$\mathrm{P}(Z > 0.28) = 1 - 0.6103 = 0.3897$ awrt 0.39 – 0.40 M1 A1 5

Note

M1 for using their largest σ and μ

M1 for using $\frac{x - \mu}{\sigma}$

M1 1 – their prob

[14]

2. (a) $\bar{x} = 668.125$ $s = 84.428$ M1 M1

$T_7(5\%) = 1.895$ B1

Confidence limits $= 668.125 \pm \frac{1.895 \times 84.428}{\sqrt{8}}$ M1

= 611.6 and 724.7 A1A1 6

Confidence interval = (612, 725)

(b) Normal distribution B1 1

(c) £650 is within the confidence interval. No need to worry. B1ftB1ft 2

[9]

3. Confidence interval = $\left(\dfrac{15\times 0.003}{27.488}, \dfrac{15\times 0.003}{6.262}\right)$ M1 B1B1

= (0.00164, 0.00719) A1A1 5

(b) $0.07^2 = 0.0049$ M1

0.0049 is within the 95% confidence interval. A1

There is no evidence to reject the idea that the standard deviation of the volumes is not 0.07 or The machine is working well. A1 3

[8]

4. (a) $\left(\bar{x} = \dfrac{466}{4} = 116.5\right)$

$S_{x^2} = \dfrac{54386 - 4\bar{x}^2}{3} = 32.\dot{3} \text{ or } \dfrac{97}{3}$ or awrt 32.3 M1 A1

$0.216 < \dfrac{3S_{x^2}}{\sigma^2} < 9.348$ B1 M1 B1

$10.376... < \sigma^2 < 449.07...$ awrt <u>10.4, 449</u> A1 A1 7

(b) $H_0: \sigma_m = \sigma_s$ $H_1: \sigma_m > \sigma_s$

$H_0: \sigma_m{}^2 = \sigma_s{}^2$ $H_1: \sigma_m{}^2 > \sigma_s{}^2$ both B1

$\dfrac{S_{m^2}}{S_{s^2}} = \dfrac{318.8}{32.3} = 9.859...$ awrt 9.86 M1 A1

$F_{6,3}(17_0 \text{c.v.}) = 27.91$ B1

9.15 < 27.91 A1ft 5

Insufficient evidence of an increase in variance

Insufficient evidence to say $\sigma_m{}^2 > \sigma_s{}^2$ is OK

Variance can be assumed to be the same is OK

[NB $\dfrac{32.3}{318.3} = 0.101...$ *only gets M1A1 <u>if</u> appropriate F value attempted.]*

[12]

5. (a) $\Sigma x = 49.6$; $\Sigma x^2 = 362.36$

$s^2 = \frac{1}{6}(362.36 - \frac{49.6^2}{7}) = 1.8180952 ...$ awrt 1.82 M1 A1 2

(b) CI = $\left(\frac{6 \times 1.818..}{12.592}, \frac{6 \times 1.818..}{1.635}\right)$ M1

12.592, 1.635 B1 B1

= (0.866 ... , 6.67...) awrt (0866, 6.67) A1 A1 5

(c) $0.9^2 < 0.866$, interval does not support $\sigma = 0.9$ as out of range. B1 1

6. (a) $\sigma_1 = \frac{1}{19}(274050 - \frac{(2340)^2}{20}) = 14.2$ M1

$\sigma_0 = \frac{1}{36}$ $(645282 - \frac{(4884)^2}{37}$ = 16.5 AG both A1 2

(b) $S_p = \sqrt{\frac{19 \times 14.2 + 36 \times 16.5}{55}} = \sqrt{15.705} = 3.963 ...$ M1 A1

Mean outside = $\frac{2340}{20}$ = 117, Mean inside = 132 B1 B1

Confidence limits = (132 – 117) ± 2.004 × 3.933 ... $\sqrt{\frac{1}{20} + \frac{1}{37}}$ M1 A1ft

= (12.8, 17.2) A1 A1 8

(c) O lies outside confidence interval. The means are different. B1 B1 2

[12]

7. (a) $\bar{x} = 123.1$ B1

$s^2 = 34.544...$, $s = 5.87745….$ B1

(NB: $\Sigma x = 1231$; $\Sigma x^2 = 151847$)

(i) 95% confidence interval is given by

$123.1 \pm 2.262 \times \dfrac{5.87745....}{\sqrt{10}}$

$\bar{x} \pm t_{\alpha/2} \, s/\sqrt{n}$ M1

2.262 B1

i.e: <u>(118.8958…, 127.30418…)</u> A1 ft

AWRT (119, 127) A1 A1 7

(ii) 95% interval is given by

$\dfrac{9 \times 5.87745...}{19.023} < \sigma^2 < \dfrac{9 \times 5.87745...}{2.700}$

use of $\dfrac{(n-1)s^2}{\sigma^2}$ M1

19.023 B1

2.700 B1

<u>i.e:- (16.34336…, 115.14806….)</u> A1ft

AWRT (16.3, 115) A1 A1 6

(b) 130 is just outside confidence interval B1

16 is just outside confidence interval B1

$\therefore$ supervisor should be concerned since both conditions are not met B1 3

[16]

8. (a) $s^2 = \dfrac{2962 - 15 \times \left(\dfrac{208}{15}\right)^2}{14} = 5.55$ or $(n-1)s^2 = 2962 - \dfrac{208^2}{15} = 77.3$ M1 A1

either

$\dfrac{14 \times 5.55}{23.685} < \sigma^2 < \dfrac{14 \times 5.55}{6.571}$ 23.685, 6.571 M1 B1, B1

$3.28 < \sigma^2 < 11.83$ A1 A1 7

(b)	Since 9 lies in the interval, yes	B1, B1 (dep)	2

[9]

1. The most able candidates gained full marks for this question. The most common error was in part (a) when they used $\sqrt{25.2}$ rather than $\sqrt{\frac{25.2}{15}}$. In part (b) many candidates gained the first 3 marks but were then unable to correctly calculate the interval for σ^2. In part (c) many candidates knew that they needed to use the highest values from parts (a) and (b), but then either did not square root the "62.5" or used $\sqrt{\frac{"62.5"}{15}}$ when finding *z*.

2. Most candidates knew the method to use and realised that the *t*-value was required but a significant number used the normal distribution value of 1.6449. In part(c) many candidates thought that because the value was in the confidence interval that the council should be worried showing a lack of understanding of what confidence intervals are.

3. The vast majority of candidates gained full marks for part (a). In part (b) many candidates simply used 0.07 and stated it was not in the interval and a minority of those who did realize they needed to use 0.0049 did not state at any point that there was evidence that the standard deviation of the volumes was 0.07.

4. Most realized that the Chi squared distribution was required to establish the confidence interval in part (a) and there were many correct solutions. The *F* test was usually used in part (b) but sometimes the degrees of freedom were the wrong way around and some used a 5% significance level.

5. Part (a) was well done, but many candidates started with a confidence interval in part (b) that required a significant amount of manipulation and inevitably gave rise to errors. The incorrect comparison of 0.9 with the interval, rather than 0.81 was not unusual in part (c).

6. A reasonable start was made by most to show the calculations leading to the given answers, but accuracy errors usually marred the attempts in part (b) and correct answers in part (c) were rare.

7. It was unusual for a candidate not to score all the marks for part (a) and to gain at least 1 and often 2 of the marks in part (b). The candidates certainly knew how to calculate these confidence intervals.

8. The majority started this question well, but often lost marks due to accuracy errors in part (a). Some thought 11.83 was an answer to 3 significant figures, and that 3 was contained in the interval in part (a).