1. The Director of Studies at a large college believed that students' grades in Mathematics were independent of their grades in English. She examined the results of a random group of candidates who had studied both subjects and she recorded the number of candidates in each of the 6 categories shown.

	Maths grade A or B	Maths grade C or D	Maths grade E or U
English grade A or B	25	25	10
English grade C to U	15	30	15

(a) Stating your hypotheses clearly, test the Director's belief using a 10% level of significance. You must show each step of your working.

(9)

The Head of English suggested that the Director was losing accuracy by combining the English grades C to U in one row. He suggested that the Director should split the English grades into two rows, grades C or D and grades E or U as for Mathematics.

(b) State why this might lead to problems in performing the test.

(1)
(Total 10 marks)

2. A quality control manager regularly samples 20 items from a production line and records the number of defective items x. The results of 100 such samples are given in Table 1 below.

x	0	1	2	3	4	5	6	7 or more
Frequency	17	31	19	14	9	7	3	0

Table 1

(a) Estimate the proportion of defective items from the production line.

(2)

The manager claimed that the number of defective items in a sample of 20 can be modelled by a binomial distribution. He used the answer in part (a) to calculate the expected frequencies given in Table 2.

x	0	1	2	3	4	5	6	7 or more
Expected frequency	12.2	27.0	r	19.0	s	3.2	0.9	0.2

Table 2

(b) Find the value of r and the value of s giving your answers to 1 decimal place.

(3)

(c) Stating your hypotheses clearly, use a 5% level of significance to test the manager's claim.

(7)

(d) Explain what the analysis in part (c) tells the manager about the occurrence of defective items from this production line.

(1)
(Total 13 marks)

1. (a) H_0: Maths grades are independent of English grades or
No association...
H_1: Maths and English grades are dependent or B1
There is an association...

Expected Frequencies e.g. $\frac{60\times 40}{120} = 20$ M1A1

20	27.5	12.5
20	27.5	12.5

$$\sum \frac{(O-E)^2}{E} = 2\times\left(\frac{5^2}{20}+\frac{2.5^2}{27.5}+\frac{2.5^2}{12.5}\right), = 3.9545...$$

AWRT 3.95 or 3.955 M1, A1

$\nu = (3-1)(2-1) = 2$; $\chi_2^2(10\%)$ c.v. = 4.605 B1; B1

3.95 < 4.605 or not significant or do not reject H_0 (allow reject H_1) M1

Insufficient evidence of an association between English and maths grades

or there is support for the Director's belief

or Student's grades in maths and English are independent A1 9

1st B1 for both hypotheses in terms of independence or association and in context.
Must mention Maths and English in at least one of the hypotheses.
"relationship" or "correlation" or "connection" or "link" is B0

1st M1 for some correct calculation seen

1st A1 for all expected frequencies correct. Accept answers without formula seen.

2nd M1 for some evidence seen of attempt to calculate test statistic.
At least one correct term seen. Follow through their expected frequencies.

2nd A1 for AWRT 3.95. Answers only please escalate!

3rd M1 for correct comparison or statement – may be implied by correct conclusion.

3rd A1 for conclusion in context using "association" or "independence" in connection with grades.
Don't insist on seeing English or maths mentioned here.
Use ISW for comments if a false statement and correct statement are seen.

(b) May have some expected frequencies < 5 (and hence need to pool rows/cools) B1 1

B1 If they just say expected frequencies are "small" they must go onto mention need to pool.

[10]

2. (a) $\frac{0\times17+1\times31+...}{17+31+...}=\left(\frac{200}{100}=2\right)$, $\hat{p}=\frac{2}{20}=\underline{0.1}$ (Accept $\frac{2}{20}$ or 2 per 20) M1,A1 2

M1 for attempt to find mean or $\hat{p}$ (as printed or better).
The 0.1 must be seen in part (a).

(b) e.g. $r = 100 \times \binom{20}{2}(0.1)^2\,(0.9)^{18}$ M1

r = 28.5, s = AWRT 9 A1, A1 3

M1 for correct expression for r or s using the binomial distribution.
Follow through their $\hat{p}$.

(c)

x	0	1	2	3	≥ 4
O_i	17	31	19	14	19
E_i	12.2	27.0	28.5	19.0	13.3
$\frac{(O-E)^2}{E}$	1.89	0.59	3.17	1.32	2.44

Pooling M1

$\sum\frac{(O-E)^2}{E}=$ AWRT 9.4 M1A1cao

$\nu = 5 - 2 = 3$, $\chi_3^2(5\%) = 7.815$ B1ft, B1ft

H_0: Binomial distribution is a good/suitable model/fit
[Condone: B(20, 0.1) is...]

H_1: Binomial distribution is not a suitable model both B1

(Significant result) Binomial distribution is not a suitable model A1cao 7

1st M1 for some pooling (accept $x \geq 5$), obs. freq. .... 14, 9, 10 and exp. freq. 19.0, *s*, 4.3)

2nd M1 for calculation of test statistic (N.B. $x \geq 5$ gives 14.5). One correct term seen.

1st B1ft for number of classes – 2 (N.B. $x \geq 5$ will have 6 – 2 = 4)

2nd B1ft for the appropriate tables value, ft their degrees of freedom. (NB $\chi_4^2(5\%) = 9.488$)

3rd B1 (for hypotheses) allow just "$X \sim B(20, 0.1)$" for null etc.

2nd A1 for correctly rejecting Binomial model. No ft and depends on 2nd M1.

(d) defective items do <u>not</u> occur <u>independently</u> **<u>or</u>** <u>not</u> with <u>constant probability</u> B1ft 1

B1ft for independence or constant probability – must mention defective items or defectives
Follow through their conclusion in (c). So if they do not reject they may say "defectives occur with probability 0.1".
Stating the value implies constant probability.

[13]

1. Part (a) posed few difficulties for most candidates. The hypotheses were usually stated correctly in terms of "association" or "independence" and the calculations of the test statistic, and degrees of freedom were handled well. Most quoted the correct critical value (although some used the 5% value) and a correct conclusion generally followed. Once again the simplest interpretation was to remark that there is support for the Director's belief but many candidates gave correct, but more complicated, statements about insufficient evidence of any association between grades in Mathematics and English.

Part (b) was supposed to generate a response about the likelihood of some expected frequencies falling below 5 and the consequent need to amalgamate the groups. A good number of candidates identified this problem but some got side-tracked by the change in degrees of freedom which does not cause problems in performing the test.

2. Although most candidates answered part (a) correctly a surprising number failed to do so some not appreciating the difference between the mean number of defective items and the proportion. Part (b) was usually answered well with only a few using n as 100 instead of 20. Most candidates knew about combining classes in part (c) but they didn't always calculate the degrees of freedom correctly and this sometimes led to them failing to reject the binomial model. There were many good responses to part (d) with candidates showing a sound understanding of the implications of rejecting the binomial model.