1. A total of 228 items are collected from an archaeological site. The distance from the centre of the site is recorded for each item. The results are summarised in the table below.

Distance from the centre of the site (m)	0–1	1–2	2–4	4–6	6–9	9–12
Number of items	22	15	44	37	52	58

Test, at the 5% level of significance, whether or not the data can be modelled by a continuous uniform distribution. State your hypotheses clearly.

(Total 12 marks)

2. The number of goals scored by a football team is recorded for 100 games. The results are summarised in Table 1 below.

Number of goals	**Frequency**
0	40
1	33
2	14
3	8
4	5

Table 1

(a) Calculate the mean number of goals scored per game.

(2)

The manager claimed that the number of goals scored per match follows a Poisson distribution. He used the answer in part (a) to calculate the expected frequencies given in Table 2.

Number of goals	**Expected Frequency**
0	34.994
1	r
2	s
3	6.752
≥ 4	2.221

Table 2

(b) Find the value of r and the value of s giving your answers to 3 decimal places.
(3)

(c) Stating your hypotheses clearly, use a 5% level of significance to test the manager's claim.
(7)
(Total 12 marks)

3. Five coins were tossed 100 times and the number of heads recorded. The results are shown in the table below.

Number of heads	0	1	2	3	4	5
Frequency	6	18	29	34	10	3

(a) Suggest a suitable distribution to model the number of heads when five unbiased coins are tossed.
(2)

(b) Test, at the 10% level of significance, whether or not the five coins are unbiased. State your hypotheses clearly.
(11)
(Total 13 marks)

4. An area of grass was sampled by placing a 1 m × 1 m square randomly in 100 places.

The numbers of daisies in each of the squares were counted.

It was decided that the resulting data could be modelled by a Poisson distribution with mean 2.

The expected frequencies were calculated using the model.

The following table shows the observed and expected frequencies.

Number of daisies	Observed frequency	Expected frequency
0	8	13.53
1	32	27.07
2	27	r
3	18	s
4	10	9.02
5	3	3.61
6	1	1.20
7	0	0.34
≥ 8	1	t

(a) Find values for r, s and t.
(4)

(b) Using a 5% significance level, test whether or not this Poisson model is suitable. State your hypotheses clearly.
(7)

An alternative test might be to estimate the population mean by using the data given.

(c) Explain how this would have affected the test.
(2)
(Total 13 marks)

5. The number of times per day a computer fails and has to be restarted is recorded for 200 days. The results are summarised in the table.

Number of restarts	Frequency
0	99
1	65
2	22
3	12
4	2

Test whether or not a Poisson model is suitable to represent the number of restarts per day. Use a 5% level of significance and state your hypothesis clearly.

(Total 12 marks)

6. Three six-sided dice, which were assumed to be fair, were rolled 250 times. On each occasion the number X of sixes was recorded. The results were as follows.

Number of sixes	0	1	2	3
Frequency	125	109	13	3

(a) Write down a suitable model for X.

(2)

(b) Test, at the 1% level of significance, the suitability of your model for these data.

(11)

(c) Explain how the test would have been modified if it had not been assumed that the dice were fair.

(2)

(Total 15 marks)

1.

Distance from centre of site (m)	0–1	1–2	2–4	4–6	6–9	9–12	
$b-a$	1	1	2	2	3	3	M1
No of artefacts	22	15	44	37	52	58	
$\mathrm{P}(a \le X < b)$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{4}$	A1
$228 \times \mathrm{P}(a \le X < b)$	19	19	38	38	57	57	A1

Class	O_i	E_i	$\frac{(O_i - E_i)^2}{E_i}$	$\frac{O_i^2}{E_i}$
0–1	22	19	$\frac{9}{19} = 0.4736...$	25.57…
1–2	15	19	$\frac{16}{19} = 0.8421...$	11.84…
2–4	44	38	$\frac{36}{38} = 0.9473...$	50.94…
4–6	37	38	$\frac{1}{38} = 0.0263...$	36.02…
6–9	52	57	$\frac{25}{57} = 0.4385...$	47.43…
9–12	58	57	$\frac{1}{57} = 0.0175...$	59.01…

M1

A1

H_0: continuous uniform distribution is a good fit B1
H_1: continuous uniform distribution is not a good fit

$$\sum \frac{(O_i - E_i)^2}{E_i} = \frac{313}{114} = 2.75$$

or

$$\sum \frac{O_i^2}{E_i} - 228 = 230.745... - 228 = \ldots$$

(awrt **2.75**) dM1 A1

$\nu = 6 - 1 = 5$ B1

(ft their ν i.e. $\chi_\nu^2(0.05)$) B1ft

$\chi_5^2(0.05) = 11.070$

2.75 < 11.070, insufficient evidence to reject H_0 M1

Continuous uniform distribution is a suitable model A1

Note

1st M1 for calculation of at least 3 widths and attempting proportions/probs.
or for 1:2:3 ratio seen

1st A1 for correct probabilities

2nd A1 for all correct expected frequencies

2nd M1 for attempting $\frac{(O-E)^2}{E}$ or $\frac{O^2}{E}$, at least 3 correct expressions or values.
Follow through their E_i provided they are not all = 38

3rd A1 for a correct set of calcs – 3rd or 4th column. (2 dp or better and allow e.g. 0.94…)

3rd dM1 **dependent on 2nd M1** for attempting a correct sum or calculation (must see at least 3 terms and +)
The first three Ms and As can be implied by a test statistic of awrt 2.75

4th M1 for a correct statement based on their test statistic (> 1) and their cv (> 3.8) Contradictory statements score M0 e.g. "significant" do not reject H_0.

5th A1 for a correct comment suggesting that continuous uniform model is suitable. No ft

[12]

2. (a) $\lambda = \frac{0\times40+1\times33+2\times14+3\times8+4\times5}{100} = 1.05$ M1 A1 2

Note

M1 for an attempt to find the mean– at least 2 terms on numerator seen Correct answer only will score both marks

(b) Using Expected frequency = 100 × P(X = x)

= 100 × $\frac{e^{-1.05}1.05^x}{x!}$ gives M1

r = 36.743 awrt 36.743 or 36.744 A1

s = 19.290 19.29 or awrt 19.290 A1 3

Note

M1 for use of correct formula (ft their mean).
1st A1 for r, 2nd A1 for s (19.29 OK)

(c) H_0 : Poisson distribution is a suitable model B1

H_1 : Poisson distribution is not a suitable model

Number of goals	Frequency	Expected frequency		
0	40	34.994		
1	33	36.743		
2	14	19.290		
3	8	6.752	8.972443	
≥ 4	5	2.221		M1

$\upsilon = 4 - 1 - 1 = 2$ B1ft

CR : $\chi^2_2(0.05) > 5.991$ B1

$$\sum \frac{(O-E)^2}{E} = \frac{(40-34.9937)^2}{34.9937} + \ldots\ldots + \frac{(13-8.972443)^2}{8.972443}$$ M1

[=0.7161…+0.3813…+1.4508…+1.80789..] A1

= 4.356. (ans in range 4.2 – 4.4)

Not in critical region

Number of goals scored can follow a Poisson distribution / managers claim is justified A1 ft 7

Note

1st B1	Must have both hypotheses and mention Poisson at least once inclusion of their value for mean in hypotheses is B0 but condone in conclusion
1st M1	for an attempt to pool ≥ 4
2nd B1ft	for $n - 1 - 1 = 2$ i.e realising that they must subtract 2 from their n
3rd B1	for 5.991 only
2nd M1	for an attempt at the test statistic, at least 2 correct expressions/values (to 3sf)
1st A1	for answers in the range 4.2~4.4
2nd A1	for correct comment in context based on their test statistic and their cv that mentions goals or manager. Dependent on 2nd M1
	Condone mention of Po(1.05) in conclusion
	Score A0 for inconsistencies e.g. "significant" followed by "manager's claim is justified"

[12]

3. (a) B,(5, 0.5) M1, A1 2

(b) H_0: B(5 ,0.5) is a suitable model (good fit)

H_1: B(5,0.5) is a not a suitable model (not a good fit) B1ft

ft for $\hat{p}$ =0.466

No of heads	0	1	2	3	4	5	
Expected	3.125	15.625	31.25	31.25	15.625	3.125	
Actual	6	18	29	34	10	3	M1 A1 A1

1 correct = A1
All correct = A1
3 s.f. or better

	O	E	$\frac{(O-E)^2}{E}$	
0 or 1	24	18.75	1.47	
2	29	31.25	0.162	
3	34	31.25	0.242	
4 or 5	13	18.75	1.76	M1 A1

grouped O and E
all want 3 s.f
or better

$\sum \frac{(O-E)^2}{E} = 3.637\dot{3}$ $\sum$ required, awrt 3.64 M1 A1

$\nu = (4-1)(2-1) = 2, \chi_3^2(0.10) = 6.251$ B1ftB1ft

Insufficient evidence to reject H_0

B(5 ,0.5) is a suitable model

No evidence that coins are biased A1ft 11

Ungrouped gives awrt 5.44, $\nu = 5, \chi_5^2 = 6.236$

[13]

4. (a) r = 27.07, M1 A1
s = 18.04, B1
t = 0.11 using tables or 0.12 using totals B1ft 4

(b) H_0 : A Poisson model Po(2) is a suitable model. Both B1
H1 : A Poisson model Po(2) is not a suitable model.

Amalgamate data M1

$\sum \frac{(O-E)^2}{E}$ = 3.28 (awrt) M1 A1

= 6 – 1 = 5 B1

χ_5^2 (5 %) = 11.070 (follow through their degrees of freedom) B1ft

3.25 < 11.070 There is insufficient evidence to reject H_0,
Po(2) is a suitable model. A1ft 7

(c)	The expected values, and hence $\sum \frac{(O-E)^2}{E}$ would be different,	B1	
	and the degrees of freedom would be 1 less.	B1	2

[13]

5. H_0: Poisson distribution is a suitable model — both B1

H_1: Poisson distribution is not a suitable model

$\hat{\lambda} = \frac{(0 \times 99) + (1 \times 65) + ... + (4 \times 2)}{200} = \frac{153}{200}$ = $\underline{0.765}$ — M1 A1

Using P($X = x$) = $\frac{0.765^x e^{-0.765}}{200}$ where X represents the 200 × P($X = x$) — M1

Number of restarts gives

X	Observed Frequency	Expected Frequency		
0	99	93.06678…		
1	65	71.19604…	0, 1, 2	A1, A1
2	22	27.23250…		(–1 e.e.)
3	12 } 14	6.94428... }	8.50468	A1
≥ 4	2	1.56040...		

$\propto = 4 - 1 - 1 = 2$; CR: $X_2^2 > 5.991$ from Poisson — B1; B1ft

$\propto = 4 - 1 = 3$; CR: $X^2 > 7.815$ from Poisson (0.765) — M1

$\sum \frac{(O-E)^2}{E} = 5.47368…$ — A1

5.40 – 5.50

Use of $\sum \frac{(O-E)^2}{E}$

5.47 is not in the critical region.
Number of computer failures per day can be modelled by a
Poisson distribution — A1ft 12

[12]

6. (a) $X \sim B(3,1/6)$

bino — B1

3, 1/6 — B1 2

(b)

X	Prob	Expected freq
0	$\left(\frac{5}{6}\right)^3$	144.68
1	$3 \times \left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)$	86.81
2	$3 \times \left(\frac{5}{6}\right)\left(\frac{1}{6}\right)^2$	17.36
3	$\left(\frac{1}{6}\right)^3$	1.15 (1.16)

prob – must show working and use B(3,p) or may be implied by correct answer M1

expected M1

awrt 145,86.8,17.4,1.15/1.16 B2 (–1 ee)

H_0: Binomial model is a good fit B1
H_1: Binomial model is not a good fit

both, no ditto

Amalgamate 3 with another group M1

$\alpha = 0.01$ $\nu = 2$; CR $\chi^2 >$ <u>9.210</u> B1; B1ft

$\sum \frac{(O-E)^2}{E}$ *OR* $\sum \frac{O^2}{E} - N = 8.6894\ldots$ M1A1

answers in range 8.67 – 8.70 or

Evidence that Binomial is a good model. A1ft 11

(c)	Estimate p	B1	
	Degrees of freedom reduced by 1	B1	2

[15]

Special case

Use of B(3,0.192) in part (b)

Expected frequencies

131.8785	M1	
94.01242	M1	
22.339		
1.769	B0	
H_0: Binomial model is a good fit	B1	
H_1: Binomial model is not a good fit		
both, no ditto		
Amalgamate 3 with another group	M1	
$\alpha = 0.01$ $\nu = 1$; CR $\chi^2 > 6.635$	B1; B1ft	
$\sum \frac{(O-E)^2}{E}$ *OR* $\sum \frac{O^2}{E} - N$ in range 5.45 – 5.50	M1 A1	
Evidence that Binomial is a good model.	A1ft	11

1. Some of the weaker candidates assumed that the expected frequencies would all equal 38 and they did not score many marks. Most though handled the unequal class widths correctly and were able to calculate a correct test statistic. Some thought the degrees of freedom should be 4 not 5 but for many candidates this was another good source of marks.

2. Parts (a) and (b) were answered very well and most scored full marks on these two parts but part (c) proved more challenging. Many insisted on including the mean of 1.05 in their hypotheses even though this was incompatible with their correct treatment of the degrees of freedom. The pooling of the last two groups was usually carried out and the calculation of the test statistic was often correct. There was some confusion over the calculation of the degrees of freedom though: many subtracted 2 but others only 1 and some were not sure whether to subtract from the number of classes before or after the pooling. A number failed to score the final mark because their conclusion was not given in context: comments such as “there is evidence to support the manager’s claim” or “there is evidence that the number of goals scored in football matches does follow a Poisson distribution” are fine; “the data follows a Poisson distribution” is not.

3. Most candidates answered this question very well and high scores were common. Errors crept in though through a failure to pool, flimsy hypotheses, incorrect critical values, and, to a lesser extent, an inability to state a correct conclusion. Only a small minority of candidates failed to read the question properly and used an estimate for the probability. Weaker candidates attempted a Poisson distribution which did not score well.

4. Most candidates could use the Poisson tables to find the expected frequencies r and s but simply found $100 \times P(X = 8)$ rather than ensuring that their expected frequencies added to 100. In part (b) some candidates failed to mention that the mean of 2 was part of the hypotheses but most candidates realized that there was a need to amalgamate the final 4 classes and the test statistic was often correct. The calculation for the degrees of freedom was usually correct and the rest of the test was carried out appropriately. In part (c) many realized that the degrees of freedom would be reduced by 1, but they often failed to mention that the expected frequencies, and therefore the value of the test statistic, would be different.

5. Ill-defined hypotheses often resulted in lost marks at the start of the question and when calculating the degrees of freedom. Many candidates did not work sufficiently accurately when calculating the expected frequencies with consequent loss of marks but generally the question was well answered.

6. This was by far the most badly answered question. Candidates who recognised this as a binomial question often did well but unfortunately these were few and far between. Candidates tried to fit normal, uniform and Poisson distributions and some times strange mixtures of these. Candidates should have used B (3, 1/6) but some used B (3, 0.192). The penalty for this understandable error was very small. Even those candidates who did well on the first part of the question fell down at the final hurdle and were unable to explain how the test would have been modified with biased dice.