Exercise 6E

Note: throughout this exercise, your numerical answers may vary slightly from those shown depending on the level of rounding you have used.

1 H_0: the diameters of the discs were sampled from a normal distribution with mean 3.8mm and standard deviation 0.5mm
H_1: the diameters of the discs were sampled from a different distribution.

	$Z=\left(\frac{D-\mu}{\sigma}\right)$	F(*Z*)	P(*Z*)	E_i	O_i	$\frac{(O_i-E_i)^2}{E_i}$
$D<3.5$	−0.6	0.2743	0.2743	8.229	6	0.604
$3.5\le D<4.0$	0.4	0.6554	0.3811	11.433	12	0.028
$D\ge 4.0$		1.0000	0.3446	10.338	12	0.267
				30	X^2	**0.899**

There are 3 cells and 1 restriction therefore, $v = 3 - 1 = 2$

$\chi^2_{\text{crit}}(2)=5.991$

$\chi^2_{\text{test}}(2)=0.899$

$\chi^2_{\text{test}}(2)=0.899<\chi^2_{\text{crit}}(2)=5.991$

Therefore, not significant.
No evidence to reject H_0.

2 H_0: the observations are from a normal distribution with mean 58 g and standard deviation 4 g.
H_1: the observations are from a different distribution.

	$Z=\left(\frac{b-\mu}{\sigma}\right)$	Cum Prob	Prob	Exp	Obs	Exp (after combining)	Obs (after combining)
$X<50.5$	−1.86	0.0314	0.0314	4.71	12	39.645	41
$50.5\le X<55.5$	−0.63	0.2643	0.2329	34.935	29		
$55.5\le X<60.5$	−0.63	0.7357	0.4714	70.71	67	70.71	67
$60.5\le X<65.5$	1.86	0.9686	0.2329	34.935	32	39.645	42
$X\ge 65.5$		1.0000	0.0314	4.71	10		

$$\sum\frac{(O_i-E_i)^2}{E_i}=\frac{(41-39.645)^2}{39.645}+\frac{(67-70.71)^2}{70.71}+\frac{(42-39.645)^2}{39.645}$$
$$=0.3808$$

There are 3 cells and 1 restriction, therefore, $v = 3 - 1 = 2$

$\chi^2_{\text{crit}}(2)=5.991$

$\chi^2_{\text{test}}(2)=0.381$

$\chi^2_{\text{test}}(2)=0.381<\chi^2_{\text{crit}}(2)=5.991$

Therefore, not significant.
No evidence to reject H_0.

3 H_0: the diameters of the apples are from a normal distribution with mean 8cm and standard deviation 0.9cm.

H_1: the diameters of the apples are from a different distribution.

	$Z = \left(\frac{b-\mu}{\sigma}\right)$	Cum Prob	Prob	Exp	Obs	Exp (after combining)	Obs (after combining)
$D < 6.5$	−1.667	0.0478	0.0478	4.78	8	28.9	37
$6.5 \leq D < 7.5$	−0.556	0.2893	0.2415	24.15	29		
$7.5 \leq X < 8.5$	0.556	0.7107	0.4214	42.14	38	42.1	38
$8.5 \leq X < 9.5$	1.667	0.9522	0.2415	24.15	16	28.9	25
$X \geq 9.5$		1.0000	0.0478	4.78	9		

$$\sum \frac{(O_i - E_i)^2}{E_i} = \frac{(37-28.9)^2}{28.9} + \frac{(38-42.1)^2}{42.1} + \frac{(25-28.9)^2}{28.9}$$

$$= 3.20$$

There are 3 cells and 1 restriction therefore, $v = 3 - 1 = 2$

$\chi^2_{\text{crit}}(2) = 5.991$

$\chi^2_{\text{test}}(2) = 3.20$

$\chi^2_{\text{test}}(2) = 3.20 < \chi^2_{\text{crit}}(2) = 5.991$

Therefore, not significant.

No evidence to reject H_0.

4 a H_0: the data can be modelled by a normal distribution.
H_1: the data cannot be modelled by a normal distribution.

Drinks	0–9	10–19	20–29	30–39	40–50
Midpoint (x)	4.5	14.5	24.5	34.5	45.0
f	10	24	45	14	7
fx	45	348	1102.5	483	315
x^2	20.25	210.25	600.25	1190.25	2025
fx^2	202.5	5046	27011.25	16663.5	14175

$$\frac{\sum fx}{\sum f} = \frac{2293.5}{100}$$
$$= 22.9$$
$$s^2 = \frac{1}{\left(\sum f\right) - 1}\left(\sum fx^2 - \frac{\left(\sum fx\right)^2}{\sum f}\right)$$
$$= \frac{1}{99}\left(63098.25 - \frac{2293.5^2}{100}\right)$$
$$= 106.03$$

$s = 10.30$

d	b	$Z = \left(\frac{b-\mu}{\sigma}\right)$	Cumulative Probability	Probability	Expected	Observed
$d < 10$	9.5	-1.304	0.096	0.096	9.6	10
$10 \le d < 20$	19.5	-0.333	0.369	0.273	27.3	24
$20 \le d < 30$	29.5	0.637	0.738	0.369	36.9	45
$30 \le d < 40$	39.5	1.608	0.946	0.208	20.8	14
$d \ge 40$			1.000	0.054	5.4	7

$$\chi^2_{\text{test}} = \frac{(10-9.6)^2}{9.6} + \frac{(24-27.3)^2}{27.3} + \frac{(45-36.9)^2}{36.9} + \frac{(14-20.8)^2}{20.8} + \frac{(7-5.4)^2}{5.4}$$
$$= 4.89$$

There are 5 cells in the table. μ and σ are estimated, therefore 2 restrictions. Expected frequencies must be 100, therefore 1 restriction.

$v = 5 - 2 - 1 = 2$
$\chi^2_{\text{crit}}(2) = 9.210$
$\chi^2_{\text{test}}(2) = 4.89$
$\chi^2_{\text{test}}(2) = 4.89 < \chi^2_{\text{crit}}(2) = 9.210$

Therefore, not significant.
Accept H_0, the data can be modelled by N(22.9, 10.25^2)

b The shop keeper could use this to help with stock control.

5 a H_0: the data can be modelled by N(1.32, 0.042^2)
H_1: the data cannot be modelled by N(1.32, 0.042^2)

h	Z	Prob	Cum Prob	Exp	Obs	Exp (after combining)	Obs (after combining)
$h < 1.225$	−2.26	0.0119	0.0119	1.428	9	7.272	18
$1.225 \le h < 1.255$	−1.55	0.0487	0.0606	5.844	9		
$1.255 \le h < 1.285$	−0.83	0.1427	0.2033	17.124	18	17.124	18
$1.285 \le h < 1.315$	−0.12	0.2489	0.4522	29.868	23	29.868	23
$1.315 \le h < 1.345$	0.60	0.2735	0.7257	32.82	20	32.82	20
$1.345 \le h < 1.375$	1.31	0.1792	0.9049	21.504	19	21.504	19
$1.375 \le h < 1.405$	2.02	0.0734	0.9783	8.808	17	11.412	22
$h > 1.405$		0.0217	1.0000	2.604	5		

$$\chi^2_{\text{test}} = \frac{(18-7.272)^2}{7.272} + \frac{(18-17.124)^2}{17.124} + \frac{(23-29.868)^2}{29.868} + \frac{(20-32.82)^2}{32.82} +$$
$$+ \frac{(19-21.504)^2}{21.504} + \frac{(22-11.412)^2}{11.412}$$
$$= 32.57$$

There are 6 cells less (8 less 2 combined) and one restriction.
$v = 6 - 1 = 5$
$\chi^2_{\text{crit}}(5) = 12.832$
$\chi^2_{\text{test}}(5) = 32.57$
$\chi^2_{\text{test}}(5) = 32.57 > \chi^2_{\text{crit}}(2) = 12.832$

Therefore, significant.
Reject H_0, the data cannot be modelled by N(1.32, 0.042^2)

5 b

h	Midpoint (x)	f	fx	fx^2
$h < 1.225$	1.21	9	10.89	13.18
$1.225 \le h < 1.255$	1.24	9	11.16	13.84
$1.255 \le h < 1.285$	1.27	18	22.86	29.03
$1.285 \le h < 1.315$	1.30	23	29.90	38.87
$1.315 \le h < 1.345$	1.33	20	26.60	35.38
$1.345 \le h < 1.375$	1.36	19	25.84	35.14
$1.375 \le h < 1.405$	1.39	17	23.63	32.85
$h > 1.405$	1.42	5	7.1	10.08
			157.98	208.37

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{157.98}{120} = 1.1365$$

$$s^2 = \frac{1}{\left(\sum f\right) - 1}\left(\sum fx^2 - \frac{\left(\sum fx\right)^2}{\sum f}\right)$$

$$= \frac{1}{119}\left(208.37 - \frac{1.1365^2}{120}\right)$$

$$= 3.235 \times 10^{-3}$$

$s = 0.0569$

When $\bar{x} = 1.1365$ and $s = 0.0569$

h	Z	Cum Prob	Prob	Exp	Obs	$\frac{(O_i - E_i)^2}{E_i}$
$h < 1.225$	−1.609	0.054	0.054	6.48	9	0.980
$1.225 \le h < 1.255$	−1.081	0.140	0.086	10.32	9	0.169
$1.255 \le h < 1.285$	−0.554	0.290	0.150	18.00	18	0.000
$1.285 \le h < 1.315$	−0.026	0.490	0.200	24.00	23	0.042
$1.315 \le h < 1.345$	0.501	0.692	0.202	24.24	20	0.742
$1.345 \le h < 1.375$	1.029	0.848	0.156	18.72	19	0.004
$1.375 \le h < 1.405$	1.556	0.940	0.092	11.04	17	3.218
$h > 1.405$		1.000	0.060	7.20	5	0.672
						$X^2 = 5.826$

There are 8 cells in the table. μ and σ are estimated, therefore 2 restrictions. Expected frequencies must be 120, therefore 1 restriction.

$v = 8 - 2 - 1 = 5$

$\chi^2_{\text{crit}}(5) = 12.832$

$\chi^2_{\text{test}}(5) = 5.826$

$\chi^2_{\text{test}}(5) = 5.826 < \chi^2_{\text{crit}}(5) = 12.832$

Therefore, not significant.

Accept H_0, the data can be modelled by N(1.3165, 3.2 × 10^{-3})

5 c On the basis of the two χ^2 tests, N(1.3165, 3.235 × 10^{-3}) is the best model.

Size	Cumulative Probability	Probability	Number to Order
Size 1	0.140	0.140	168
Size 2	0.490	0.350	420
Size 3	0.848	0.358	430
Size 4	1.000	0.152	182

6 H_0: the data can be modelled by a uniform distribution.
H_1: the data cannot be modelled by a uniform distribution.

Distance	Prob	E_i	O_i	$\frac{(O_i - E_i)^2}{E_i}$
0–1	$\frac{1}{12}$	25	37	5.76
1–2	$\frac{1}{12}$	25	38	6.76
2–4	$\frac{1}{6}$	50	36	0.72
4–6	$\frac{1}{6}$	50	47	0.18
6–9	$\frac{1}{4}$	75	58	3.85
9–12	$\frac{1}{4}$	75	64	1.61
				18.889

There are 6 cells in the table and 1 restriction.
$v = 6 - 1 = 5$
$\chi^2_{\text{crit}}(5) = 11.070$
$\chi^2_{\text{test}}(5) = 18.889$
$\chi^2_{\text{test}}(5) = 18.889 > \chi^2_{\text{crit}}(5) = 11.070$

Therefore, significant.
Reject H_0, the data is not from a uniform distribution.