Exercise 4E

1 $H_0: \mu_1 = \mu_2$ and $H_1: \mu_1 > \mu_2$ so use a one-tailed test.

$$Z = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{23.8 - 21.5 - 0}{\sqrt{\frac{5.0^2}{15} + \frac{4.8^2}{20}}}$$

$$= 1.370$$

$1.370 < 1.645$

Therefore, not significant so accept H_0, $\mu_1 = \mu_2$

2 $H_0: \mu_1 = \mu_2$ and $H_1: \mu_1 \neq \mu_2$ so use a two-tailed test.

$$Z = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{49.6 - 51.7 - 0}{\sqrt{\frac{4.2^2}{30} + \frac{3.6^2}{25}}}$$

$$= -1.996$$

$-1.996 < -1.96$

Therefore, significant so reject H_0, $\mu_1 \neq \mu_2$

3 $H_0: \mu_1 = \mu_2$ and $H_1: \mu_1 < \mu_2$ so use a one-tailed test.

$$Z = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{3.62 - 4.11 - 0}{\sqrt{\frac{0.81^2}{25} + \frac{0.75^2}{36}}}$$

$$= -2.395$$

$-2.395 < -2.326$

Therefore, significant so reject H_0, $\mu_1 < \mu_2$

4 $H_0 : \mu_1 = \mu_2$ and $H_1 : \mu_1 \neq \mu_2$ so use a two-tailed test.

$$Z = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{112.0 - 108.1 - 0}{\sqrt{\frac{8.2^2}{85} + \frac{11.3^2}{100}}}$$

$$= 2.712$$

$2.712 > 2.576$

Therefore, significant so reject H_0, $\mu_1 \neq \mu_2$

5 $H_0 : \mu_1 = \mu_2$ and $H_1 : \mu_1 > \mu_2$ so use a one-tailed test.

$$Z = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{72.6 - 69.5 - -0}{\sqrt{\frac{18.3^2}{100} + \frac{15.4^2}{150}}}$$

$$= 1.396$$

$1.396 < 1.645$

Therefore, not significant so accept H_0, $\mu_1 = \mu_2$

6 $H_0 : \mu_1 = \mu_2$ and $H_1 : \mu_1 < \mu_2$ so use a one-tailed test.

$$Z = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{0.863 - 0.868 - 0}{\sqrt{\frac{0.013^2}{120} + \frac{0.015^2}{90}}}$$

$$= -2.529$$

$-2.529 < -2.326$

Therefore, significant so reject H_0, $\mu_1 < \mu_2$

7 $H_0 : \mu_1 = \mu_2$ and $H_1 : \mu_1 \neq \mu_2$ so use a two-tailed test.

$$Z = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{6.531 - 6.524 - 0}{\sqrt{\frac{0.011^2}{10} + \frac{0.015^2}{15}}}$$

$$= 1.345$$

$1.345 < 1.960$

Therefore, not significant so accept H_0.

Pipes produced by the two machines have the same mean length.

8 a $H_0: \mu_{\text{new}} = \mu_{\text{old}} + 1.0$ and $H_1: \mu_{\text{new}} > \mu_{\text{old}} + 1.0$ so use a one-tailed test.

$$Z = \frac{\bar{X}_{\text{new}} - \bar{X}_{\text{old}} - (\mu_{\text{new}} - \mu_{\text{old}})}{\sqrt{\frac{\sigma_{\text{new}}^2}{n_{\text{new}}} + \frac{\sigma_{\text{old}}^2}{n_{\text{old}}}}}$$

$$= \frac{6.5 - 5.0 - 1.0}{\sqrt{\frac{0.8}{80} + \frac{0.6}{70}}}$$

$$= 3.669$$

$3.669 > 1.645$

Therefore, significant so reject H_0.

There is evidence that the new seed has a mean yield more than 1.0 tonne greater than the old seed.

b Large samples allow the assumption of normal distributions for $\bar{X}_{\text{old}}$ and $\bar{X}_{\text{new}}$

9 a $H_0: \mu_{\text{g}} = \mu_{\text{g\&g}}$ and $H_1: \mu_{\text{g}} \neq \mu_{\text{g\&g}}$ so use a two-tailed test.

$$Z = \frac{\bar{X}_{\text{g}} - \bar{X}_{\text{g\&g}} - (\mu_{\text{g}} - \mu_{\text{g\&g}})}{\sqrt{\frac{\sigma_{\text{g}}^2}{n_{\text{g}}} + \frac{\sigma_{\text{g\&g}}^2}{n_{\text{g\&g}}}}}$$

$$= \frac{4.1 - 3.7 - 0}{\sqrt{\frac{0.8}{60} + \frac{0.75}{50}}}$$

$$= 2.376$$

$2.376 > 1.960$

Therefore, significant so reject H_0.

There is evidence that a diet of grass and grain affects the fat content.

b It is assumed that $\bar{X}_{\text{g}}$ and $\bar{X}_{\text{g\&g}}$ are normally distributed random variables.

Challenge

a

$$\bar{X} = \frac{\sum_{i=1}^{n_x} x_i}{n_x} \Rightarrow n_x \bar{X} = \sum_{i=1}^{n_x} x_i$$

$$\bar{Y} = \frac{\sum_{i=1}^{n_y} y_i}{n_y} \Rightarrow n_y \bar{X} = \sum_{i=1}^{n_y} y_i$$

$$\sum n = n_x + n_y$$

$$\hat{\mu} = \frac{\sum_{i=1}^{n_x} x_i + \sum_{i=1}^{n_y} y_i}{\sum n}$$

$$= \frac{n_x \bar{X} + n_y \bar{Y}}{n_x + n_y} \text{ as required}$$

b $\sigma_{xy}^2 = \dfrac{n_x\sigma_x^2 + n_y\sigma_y^2}{(n_x + n_y)^2}$

$= \dfrac{100\times 16.0 + 120\times 24.0}{(100+120)^2}$

$= 0.0925...$

$\sigma_{xy} = 0.3042$

$\hat{\mu} = \dfrac{100\times 46.0 + 120\times 47.0}{100+120}$

$= 46.545$

A 99% confidence interval is given by

$(\hat{\mu} - 2.576\sigma_{xy}, \hat{\mu} + 2.576\sigma_{xy}) = (46.535 - 2.576\times 0.3042, 46.535 - 2.576\times 0.3042)$

$= (45.76, 47.33)$