Exercise 4D

1 $Z = \dfrac{\bar{X} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$

$= \dfrac{21.2 - 21}{\dfrac{1.5}{\sqrt{20}}}$

$= 0.596$

$0.596 < 1.96$ therefore not significant.

H_0 accepted (two tail 5%).

2 $Z = \dfrac{\bar{X} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$

$= \dfrac{98.5 - 100}{\dfrac{5}{\sqrt{36}}}$

$= -1.8$

$-1.8 < -1.645$

$1.8 > 1.645$ therefore significant.

H_0 rejected (one tail 5%).

3 $Z = \dfrac{\bar{X} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$

$= \dfrac{6.1 - 5}{\dfrac{3}{\sqrt{25}}}$

$= 1.83$

$1.83 < 1.96$ therefore not significant.

H_0 accepted (two tail 5%).

4 $Z = \dfrac{\bar{X} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$

$= \dfrac{16.5 - 15}{\dfrac{3.5}{\sqrt{40}}}$

$= 2.71$

$2.71 > 1.645$ therefore significant.

H_0 rejected (one tail 5%).

5 $Z = \dfrac{\bar{X} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$

$= \dfrac{48.9 - 50}{\dfrac{4.0}{\sqrt{60}}}$

$= -2.13$

$-2.13 < -1.96$

$2.13 > 1.96$ therefore significant.

H_0 rejected (two tail 5%).

6 $\dfrac{\bar{X} - \mu}{\dfrac{\sigma}{\sqrt{n}}} < -1.645$

$\dfrac{\bar{X} - 120}{\dfrac{2}{\sqrt{30}}} < -1.645$

$\bar{X} - 120 < -1.645 \times \dfrac{2}{\sqrt{30}}$

$\bar{X} < 120 - 1.645 \times \dfrac{2}{\sqrt{30}}$

$\bar{X} < 119.399$

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$$\frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} > 2.326$$

$$\frac{\bar{X}-12.5}{\frac{1.5}{\sqrt{25}}} > 2.326$$

$$\bar{X}-12.5 > 2.326\times\frac{1.5}{\sqrt{25}}$$

$$\bar{X} > 12.5+2.326\times\frac{1.5}{\sqrt{25}}$$

$$\bar{X} > 13.198$$

8 $\frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} < -1.282$

$$\frac{\bar{X}-85}{\frac{4}{\sqrt{50}}} < -1.282$$

$$\bar{X}-85 < -1.282\times\frac{4}{\sqrt{50}}$$

$$\bar{X} < 85-1.282\times\frac{4}{\sqrt{50}}$$

$$\bar{X} < 84.275$$

9 $-1.96 > \frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} > 1.96$

$$-1.96\times\frac{3.0}{\sqrt{45}} > \bar{X} > 1.96\times\frac{3.0}{\sqrt{45}} \quad -1.96 > \frac{\bar{X}-0}{\frac{3.0}{\sqrt{45}}} > 1.96$$

$$-0.877 > \bar{X} > 0.877$$

$$\bar{X} > 0.877 \text{ and } \bar{X} < -0.877$$

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$$2.576 < \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} > -2.576$$

$$2.576 < \frac{\bar{X} - (-8)}{\frac{1.2}{\sqrt{20}}} > -2.576$$

$$2.576 \times \frac{1.2}{\sqrt{20}} < \bar{X} + 8 > -2.576 \times \frac{1.2}{\sqrt{20}}$$

$$-8 + 2.576 \times \frac{1.2}{\sqrt{20}} < \bar{X} > -8 + -2.576 \times \frac{1.2}{\sqrt{20}}$$

$$-7.309 < \bar{X} > -8.691$$

$\bar{X} > -7.31$ and $\bar{X} < -8.69$

11 $H_0: \mu = 185$

$H_1: \mu < 185$

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{179 - 185}{\frac{15}{\sqrt{25}}}$$

$$= -2$$

$-2 < -1.645$

$2 > 1.645$ therefore significant.

H_0 rejected (one tail 5%).

There is evidence that the new formula is an improvement.

12 $H_0: \mu = 100$

$H_1: \mu > 100$

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{102.5 - 100}{\frac{15}{10}}$$

$$= 1.667$$

$1.667 > 1.645$ therefore significant.

H_0 rejected (one tail 5%).

There is evidence that eating chocolate improves the IQ score.

13 $H_0: \mu = 9$

$H_1: \mu \neq 9$

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{8.95 - 9}{\frac{0.15}{\sqrt{30}}}$$

$$= -1.826$$

$-1.826 > -1.960$

$1.826 < 1.960$ therefore not significant.

H_0 accepted (two tail 5%).

There is no evidence of a change in diameter.