Exercise 3B

1 $n=9,\ \sigma^2=36,\ \bar{x}=128$

a 95% C.I. for μ is $128 \pm 1.96 \times \frac{6}{\sqrt{9}} = (124.08, 131.92\ldots)$

$= (124, 132)$ (3 s.f.)

b 99 % C.I. for μ is $128 \pm 2.5758 \times \frac{6}{\sqrt{9}} = (122.84\ldots, 133.15\ldots)$

$= (123, 133)$ (3 s.f.)

2 $n=25,\ \sigma=4,\ \bar{x}=85$

a 90% C.I for μ is $85 \pm 1.6449 \times \frac{4}{\sqrt{25}} = (83.684\ldots, 86.315\ldots)$

$= (83.7, 86.3)$ (3 s.f.)

b 95% C.I. for μ is $85 \pm 1.96 \times \frac{4}{\sqrt{125}} = (83.432, 86.568)$

$= (83.4, 86.6)$ (3 s.f.)

3 $\bar{x} + 1.96 \times \frac{\sigma}{\sqrt{n}} = 27.19$

$\bar{x} - 1.96 \times \frac{\sigma}{\sqrt{n}} = 25.61$

$2\bar{x} = 52.8$

$\bar{x} = 26.4$

$26.4 + 1.96 \times \frac{\sigma}{\sqrt{n}} = 27.19$

$\frac{\sigma}{\sqrt{n}} = 0.403...$

A 99% confidence interval is

$\bar{x} \pm 2.5758 \times \frac{\sigma}{\sqrt{n}}$

$26.4 \pm 2.5758 \times 0.403...$

$26.4 \pm 1.038...$

The confidence interval is (25.36, 27.44) (3 s.f.)

4 $\sigma = 15$

C.I. is $\bar{x} \pm z \times \frac{\sigma}{\sqrt{n}}$

width $= \frac{2z\sigma}{\sqrt{n}}$

a

$90\% \Rightarrow z = 1.6449 \quad \therefore \frac{2 \times 1.6449 \times 15}{\sqrt{n}} < 2$

$\Rightarrow \sqrt{n} > 24.67\ldots \qquad \therefore n > 608.78\ldots$

So $n = 609$

b

$95\% \Rightarrow z = 1.96 \therefore \frac{2 \times 1.96 \times 15}{\sqrt{n}} < 2$

$\Rightarrow \sqrt{n} > 1.96 \times 15 \therefore n > 864.36\ldots$

So $n = 865$

c

$99\% \Rightarrow z = 2.5758 \therefore \frac{2 \times 2.5758 \times 15}{\sqrt{n}} < 2$

$\Rightarrow \sqrt{n} > 2.5758 \times 15 \therefore n > 1492.817\ldots$

So $n = 1493$

5 a

$\sigma = 50 \quad n = 200 \quad \bar{x} = 310$

90% C.I is $\bar{x} \pm 1.6449 \times \frac{\sigma}{\sqrt{200}}$

$= \left(310 \pm 1.6449 \times \frac{50}{\sqrt{200}}\right)$

$= (304.184\ldots, 315.815\ldots)$

$= (304, 316) \quad$ (3 s.f.)

b First we calculate the probability that μ is contained in exactly 3 **specific** 90% confidence intervals out of the total 5.

The probability that this happens is:

$90\% \times 90\% \times 90\% \times 10\% \times 10\%$

$= 0.9 \times 0.9 \times 0.9 \times 0.1 \times 0.1$

$= 0.00729$

Now we calculate that there are 5C3 = 10 ways we may choose 3 out of 5 (using the binomial expansion or nCr button on a calculator). Therefore there are 10 specific examples of μ being contained in exactly 3 of the 5 90% confidence intervals and so we have a probability of 0.0729.

6

$\sigma = 15\ 000 \quad n = 80 \quad \bar{x} = 75\ 872$

$90\%\text{ C.I is } \bar{x} \pm 1.6449 \times \frac{\sigma}{\sqrt{80}}$

$= \left(75\ 872 \pm 1.6449 \times \frac{15\ 000}{\sqrt{200}}\right)$

$= (73\ 113.41\ldots,\ 78\ 630.58\ldots)$

$= (73\ 113,\ 78\ 631) \quad \text{(nearest integer)}$

or $(73\ 100,\ 78\ 600) \quad \text{(3 s.f.)}$

7 $\sigma = 13.5 \quad n = 250 \quad \bar{x} = 68.4$

a Must assume that these students form a random sample or that they are representative of the population.

b

$95\%\text{ C.I is } 68.4 \pm 1.96 \times \frac{13.5}{\sqrt{250}}$

$= (66.726\ldots,\ 70.073\ldots)$

$= (66.7,\ 70.1)\ \text{(3 s.f.)}$

c If $\mu = 65.3$ that is outside the C.I. so the examiner's sample was not representative. The examiner marked more 'better than average' candidates.

8 **a** (23.2, 26.8) is 95% C.I. since it is the narrower interval.

b

$\bar{x} = \frac{1}{2}(23.2 + 26.8) = 25$

$\therefore\ 1.96\frac{\sigma}{\sqrt{n}} = 25 - 23.2 = 1.8$

$\therefore\ \frac{\sigma}{\sqrt{n}} = 0.9183\ldots = 0.918 \quad \text{(3 s.f.)}$

c $\hat{\mu} = \bar{x} = 25$ (mid-point of the intervals)

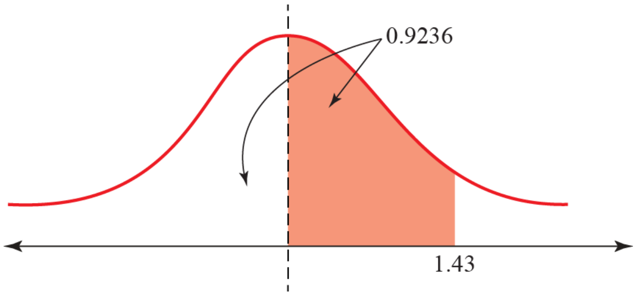
9 **a** $\bar{x} = \frac{1}{2}(128.14 + 141.86) = \frac{270}{2} = 135$

$\therefore$ C.I. will be (130, 140)

9 b

$z \times \frac{\sigma}{\sqrt{n}} = 5$ but $1.96 \frac{\sigma}{\sqrt{n}} = 6.86$

$\therefore z = \frac{5}{\left(\frac{6.86}{1.96}\right)} = 1.4285\ldots$ Use 1.43



$\therefore$ C.I. is $2 \times (0.9236 - 0.5)$

$= 0.8472$ (tables)

or $= 0.846872\ldots$ (calculator)

$\therefore$ C.I. is 85%

c Now we know $1.96 \frac{\sigma}{\sqrt{100}} = 6.86$

$\therefore \sigma = \frac{6.86 \times 10}{1.96} = 35$

and require $z \times \frac{\sigma}{\sqrt{n}} = 5$ where $z = 1.96$

$\therefore \frac{1.96 \times 35}{5} = \sqrt{n}$

$\Rightarrow n = 188.23\ldots$

$\therefore$ Need $n = 189$ or more

10

$W \sim N(\mu, 2.4^2)$ $n = 36$ $\bar{w} = 31.4$

99% C.I. is $31.4 \pm 2.5758 \times \frac{2.4}{\sqrt{36}}$

$= (30.369\ldots, 32.430\ldots)$

$= (30.4, 32.4)$ (3 s.f.)

11

$\sigma = 20,\ n = 40,\ \bar{x} = 266$

99% C.I. is $266 \pm 2.5758 \times \frac{20}{\sqrt{40}}$

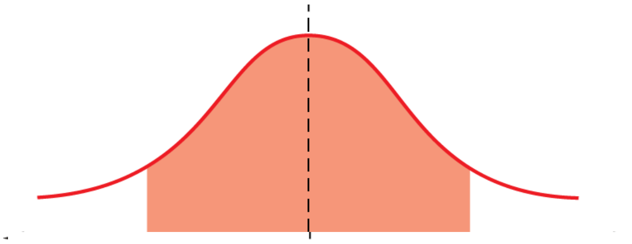
$= (257.854\ldots, 274.145\ldots)$

$= (258, 274)$ (3 s.f.)

12 $E \sim \text{N}(0, 1^2)$

a $\text{P}(|E| < 0.4) = (0.6554 - 0.5) \times 2$

$= 0.311$



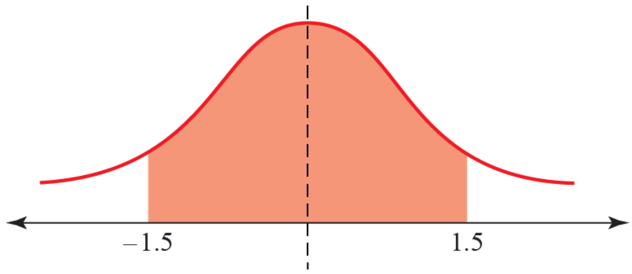
b

$\bar{E} \sim \text{N}(0, \tfrac{1}{9})$

$\text{P}(|\bar{E}| < 0.5) = \text{P}\left(|Z| < \frac{0.5}{\sqrt{\frac{1}{9}}}\right)$

$= (0.9332 - 0.5) \times 2$

$= 0.866$



c

98% C.I. is $22.53 \pm 2.3263 \times \frac{1}{\sqrt{9}}$

$= (21.754\ldots, 23.305\ldots)$

$= (21.8, 23.3)$ (3 s.f.)