Exercise 2A

1 a $E(W) = E(X) + E(Y) = 80 + 50 = 130$
$Var(W) = Var(X) + Var(Y) = 9 + 4 = 13$
$W \sim N(130, 13)$

b $E(W) = E(X) - E(Y) = 80 - 50 = 30$
$Var(W) = Var(X) + Var(Y) = 9 + 4 = 13$
$W \sim N(30, 13)$

2 $E(R) = E(X) + E(Y) + E(W) = 45 + 54 + 49 = 148$
$Var(R) = Var(X) + Var(Y) + Var(W) = 6 + 4 + 8 = 18$
$R \sim N(148, 18)$

3 a $T = 3X$, so $T \sim N(3 \times 60, 3^2 \times 25)$
$T \sim N(180, 225)$

b $T = 7Y$, so $T \sim N(7 \times 50, 7^2 \times 16)$
$T \sim N(350, 784)$

c $T = 3X + 7Y$, so $T \sim N(180 + 350, 225 + 784)$
$T \sim N(530, 1009)$

d $T = X - 2Y$, so $T \sim N(60 - 2 \times 50, 25 + 2^2 \times 16)$
$T \sim N(-40, 89)$

4 a Let $D = A + B$, then $D \sim N(50 + 60,\ 6 + 8)$, so $D \sim N(110,\ 14)$.
Then using the normal distribution function on a calculator gives:
$P(A + B < 115) = P(D < 115) = 0.9093$ (4 d.p.)

b Let $D = A + B + C$, then $D \sim N(50 + 60 + 80, 6 + 8 + 10)$, so $D \sim N(190, 24)$
$P(A + B + C > 198) = 1 - P(D < 198) = 1 - 0.9488 = 0.0512$ (4 d.p.)

c Let $D = B + C$, then $D \sim N(60 + 80, 8 + 10)$, so $D \sim N(140, 18)$
$P(B + C < 138) = P(D < 138) = 0.3187$ (4 d.p.)

d Let $D = 2A + B - C$, then $D \sim N(2 \times 50 + 60 - 80, 4 \times 6 + 8 + 10)$, so $D \sim N(80, 42)$
$P(2A + B - C < 70) = P(D < 70) = 0.0614$ (4 d.p.)

e Let $D = A + 3B - C$, then $D \sim N(50 + 3 \times 60 - 80, 6 + 9 \times 8 + 10)$, so $D \sim N(150, 88)$
$P(A + 3B - C > 140) = 1 - P(D < 140) = 1 - 0.1432 = 0.8578$ (4 d.p.)

f Let $D = A + B$, then $D \sim N(50 + 60, 6 + 8)$, so $D \sim N(110, 14)$
$P(105 < A + B < 116) = P(D < 116) - P(D < 105) = 0.9456 - 0.0907 = 0.8549$ (4 d.p.)

5 **a** Let $A = Y - X$, then $A \sim N(80 - 76, 10 + 15)$, i.e. $A \sim N(4, 25)$
$P(Y > X) = P(Y - X > 0) = P(A > 0) = 1 - P(A < 0) = 1 - 0.2119 = 0.7881$ (4 d.p.)

b $P(X > Y) = P(Y - X < 0) = P(A < 0) = 0.2119$ (4 d.p.)

c **i** The probability that X and Y differ by less than 3 = $P(-3 < A < 3)$
$P(-3 < A < 3) = P(A < 3) - P(A < -3) = 0.42074 - 0.08076 = 0.3400$ (4 d.p.)

ii The probability that X and Y differ by more than 7 = $P(A < -7) + P(A > 7)$
$P(A < -7) + P(A > 7) = P(A < -7) + 1 - P(A < 7) = 0.0139 + 1 - 0.7257 = 0.2882$ (4 d.p.)

6 **a** Runner $A \sim N(13.2, 0.9^2)$, Runner $B \sim N(12.9, 1.3^2)$
Let $D = A - B$, then $D \sim N(13.2 - 12.9,\ 0.9^2 + 1.3^2)$, so $D \sim N(0.3,\ 2.5)$.
$P(A - B > 0.5) = P(D > 0.5) = 1 - P(D < 0.5) = 1 - 0.5503 = 0.4497$ (4 d.p.)

b $P(\text{photo finish}) = P(-0.1 < D < 0.1) = P(< 0.1) - P(< -0.1)$
$= 0.44967 - 0.40014 = 0.0495$ (4 d.p.)

7 Let R be the diameter of a steel rod and T be the internal diameter of a steel tube, then
$R \sim N(3.55,\ 0.02^2), T \sim N(3.60,\ 0.02^2)$
Let $A = T - R$, then $A \sim N(3.60 - 3.55,\ 0.02^2 + 0.02^2)$, so $A \sim N(0.05,\ 0.0008)$.
$P(T - R < 0) = P(A < 0) = 0.0385$ (4 d.p.)

8 Let T be the mass of a randomly selected jar of jam, B be the mass of a randomly selected box and then Y be the mass of a box of 6 jars, then
$T \sim N(1000,\ 12^2)$, $B \sim N(250,\ 10^2)$, $Y = T_1 + T_2 + T_3 + T_4 + T_5 + T_6 + B$
So $Y \sim N(6 \times 1000 + 250,\ 6 \times 12^2 + 10^2)$, hence $Y \sim N(6250,\ 964)$
Using a calculator gives $P(Y < 6200) = 0.0537$ (4 d.p.)