GCE Examinations

Advanced Subsidiary / Advanced Level

Statistics

Module S3

Paper C

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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S3 Paper C – Marking Guide

1. *(a)* 72, 65, 36, 61, 12, 17 — M1 A2

(b) e.g. advantage – avoids bias — B1
disadvantage – time consuming — B1 **(5)**

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2. *(a)* $\overline{T} \sim N(28.5, \frac{7.2^2}{8}) = \sim N(28.5, 6.48)$ — M1 A1

(b) $P(25 < \overline{T} < 30) = P(\frac{25-28.5}{\sqrt{6.48}} < Z < \frac{30-28.5}{\sqrt{6.48}})$ — M1 A1
$= P(^{-}1.37 < Z < 0.59) = 0.7224 - (1 - 0.9147) = 0.637$ — M1 A1 **(6)**

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3. *(a)* $E(X) = (2 \times 0.05) + (4 \times 0.15) + (7 \times 0.3) + (k \times 0.5)$ — M1
$= 2.8 + 0.5k$ — A1

(b) $E(2\overline{X} - 5) = 2(2.8 + 0.5k) - 5 = k + 0.6$ — M1
$\therefore$ bias = 0.6 — M1 A1

(c) unbiased est. of $k = 2\overline{X} - 5.6 = (2 \times 8.34) - 5.6 = 11.08$ — M1 A1 **(7)**

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4. let T = total mass of waste
$\therefore T \sim N(8{\times}6.8 + 3{\times}3.2, 8{\times}1.5^2 + 3{\times}0.6^2) = \sim N(64, 19.08)$ — M2 A2
$P(T > 70) = P(Z > \frac{70-64}{\sqrt{19.08}})$ — M1
$= P(Z > 1.37) = 1 - 0.9147 = 0.0853$ — M1 A1 **(7)**

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5. $H_0 : \mu_A = \mu_N \quad H_1 : \mu_A < \mu_N$ — B1
5% level $\therefore$ C.R. is $z < {}^{-}1.6449$ — B1
test statistic = $\frac{32.8-35.1}{\sqrt{\frac{4.6^2}{50}+\frac{8.0^2}{190}}} = {}^{-}2.6382$ — M2 A2
in C.R. $\therefore$ reject H_0 — M1
there is evidence that those in school teams complete task quicker — A1 **(8)**

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6. expected freq. Highfield/English = $\frac{80\times46}{120} = 30.67$ — M1 A1
giving expected freqs 30.67 15.33
49.33 24.67 — M1 A1
H_0 : no difference in proportions at the two schools
H_1 : there is a difference in proportions at the two schools — B1

O	E	$(O-E)$	$\frac{(O-E)^2}{E}$
32	30.67	1.33	0.0577
14	15.33	$^{-}1.33$	0.1154
48	49.33	$^{-}1.33$	0.0359
26	24.67	1.33	0.0717

$\therefore \Sigma \frac{(O-E)^2}{E} = 0.2807$ — M1 A2
$\nu = 1, \chi^2_{crit}(10\%) = 2.705$ — M1 A1
$0.2807 < 2.705 \;\therefore$ not significant
there is no evidence of a difference in proportions at the two schools — A1 **(11)**

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7. *(a)* $S_{pp} = 70932 - \frac{1176^2}{20} = 1783.2$ M1

$S_{tt} = 19213 - \frac{511^2}{20} = 6156.95$ M1

$S_{pt} = 27188 - \frac{1176 \times 511}{20} = {}^{-}2858.8$ M1

$r = \frac{-2858.8}{\sqrt{1783.2 \times 6156.95}} = {}^{-}0.8628$ M1 A1

(b) $H_0 : \rho = 0 \quad H_1 : \rho < 0$ B1

$n = 20$, 1% level $\therefore$ C.R. is $r < {}^{-}0.5155$ M1 A1

${}^{-}0.8628 < {}^{-}0.5155 \therefore$ significant

there is evidence that people with lower rest pulse are fitter A1

(c) variables need to be jointly normally distributed B1

e.g. it seems reasonable that the fitness of those with a given rest pulse should follow a normal dist. and vice versa B1 **(11)**

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8. *(a)* e.g. particles are emitted singly, at random and at a constant rate (for near future given long half-life) so seems suitable B3

(b) $\hat{\mu} = \bar{x} = \frac{\sum fx}{\sum f} = \frac{96}{80} = 1.2$ M1 A1

$\Sigma fx^2 = 32 + 56 + 72 + 48 = 208$ M1

$\hat{\sigma}^2 = s^2 = \frac{80}{79}\left(\frac{208}{80} - 1.2^2\right) = 1.17$ M1 A1

(c) variance $\approx$ mean as would be expected with a Poisson distribution B1

(d) H_0 : Po(1.2) is a suitable model

H_1 : Po(1.2) is not a suitable model B1

$P(0) = e^{-1.2} = 0.3012$

$P(1) = 1.2e^{-1.2} = 0.3614$

$P(2) = \frac{1.2^2 e^{-1.2}}{2} = 0.2169$

$P(3) = \frac{1.2^3 e^{-1.2}}{3 \times 2} = 0.0867$ M1 A2

$P(4) = \frac{1.2^4 e^{-1.2}}{4 \times 3 \times 2} = 0.0260$

$\times$ 80 to give exp. freqs then freq of ≥ 5 = (80 – sum of others)

$\therefore$ exp. freqs are 24.10, 28.91, 17.35, 6.94, 2.08, 0.62 M1 A1

combining groups ≥ 3 M1

O	E	$(O - E)$	$\frac{(O-E)^2}{E}$
23	24.10	${}^{-}1.1$	0.0502
32	28.91	3.09	0.3303
14	17.35	${}^{-}3.35$	0.6468
11	9.64	1.36	0.1919

$\therefore \Sigma \frac{(O-E)^2}{E} = 1.219$ M1 A1

$\nu = 4 - 2 = 2$, $\chi^2_{\text{crit}}(5\%) = 5.991$ M1

$1.219 < 5.991 \therefore$ do not reject H_0

Po(1.2) is a suitable model A1 **(20)**

Total **(75)**

Performance Record – S3 Paper C

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	sampling	dist. of sample mean	bias	linear comb. of Normal r.v.	diff. of means hyp. test	conting. table	pmcc, hyp. test	goodness of fit, Poisson	
Marks	5	6	7	7	8	11	11	20	75
Student									