OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS **4734**

Probability & Statistics 3

MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

This mark scheme consists of 4 printed pages.

1	Model for call-outs is Poisson Mean is $\frac{1}{5}(6+2)$ $=1.6$ Probability is $1-0.9212$ $=0.0788$	B1 M1 A1 M1 A1 **5** **[5]**	For any implication of Poisson For summing two relevant parameters For correct mean of 1.6 For relevant use of tables For correct answer
2	Assume $F = E + M_1 + M_2 + \ldots + M_{50}$, where the masses of the 50 matches in a box are independent the mass of the empty box is independent of the masses of the matches $20.0 = 12.5 + 50\mu$ Hence mean mass of a match is 0.15 grams $0.4^2 = 0.2^2 + 50\sigma^2$ Hence standard deviation is 0.049 grams	 B1 B1 M1 A1 M1 A1 A1 **7** **[7]**	(The relation itself may be implied) For one relevant valid assumption For another relevant valid assumption For attempting $\mathrm{E}(F)$ in terms of μ For correct value 0.15 For attempting $\mathrm{Var}(F)$ as a sum For correct equation For correct value 0.049
3	**(i)** $\bar{x} = 25.0055$ $s^2 = \frac{1}{79}\left(0.2287 - \frac{0.44^2}{80}\right)$ $= 0.00286...$ Interval is $25.0055 \pm 2.576\sqrt{\frac{0.00286}{80}}$ Hence $24.99(0) < \mu < 25.02(1)$	B1 M1 A1 M1 B1 A1 **6**	For correct sample mean, or equivalent; the 25 may be taken into account later For correct unsimplified expression For correct unbiased estimate For calculation of the form $\bar{x} \pm z\sqrt{(s^2/n)}$ For relevant use of $z = 2.576$ For correct interval, stated to an appropriate degree of accuracy
	(ii) The sample size of 80 is sufficient large for the Central Limit Theorem to apply, so it is not necessary to assume a normal distribution	 M1 A1 **2** **[8]**	 For mention of sample size and CLT For the correct conclusion and reason
4	**(i)** $f_e = 100 \times \int_5^{10} 0.1\mathrm{e}^{-0.1x}\,\mathrm{d}x$ $= 100\left[-\mathrm{e}^{-0.1x}\right]_5^{10}$ $= 100(\mathrm{e}^{-0.5} - \mathrm{e}^{-1}) = 23.87$	M1 A1 M1 M1 A1 **5**	For attempting to integrate $\mathrm{f}(x)$ For correct indefinite integral For multiplying by total frequency For use of correct limits For obtaining given answer correctly
	(ii) Combining: f_o: 49, 22, 20, 9; f_e: 39.35, 23.87, 23.25, 13.53 Test statistic is $\frac{9.65^2}{39.35} + \frac{1.87^2}{23.87} + \frac{3.25^2}{23.25} + \frac{4.53^2}{13.53}$ $= 4.484$ This is less than 6.251 Hence there is a satisfactory fit	M1 M1 A1 M1 A1√ **5** **[10]**	For combining the last two classes For correct calculation process For correct value 4.48 For comparison with the correct critical value For correct conclusion, in terms of the fit

5	(i) $P(\lvert X\rvert < a) = P(-a < X < a)$	M1		For consideration of two areas, or equiv
	$= \int_{-a}^{0} (1+x)\,dx + \int_{0}^{a} (1-x)\,dx$	A1		For integrals or equivalent trapezia
	$= \left[x + \frac{1}{2}x^2\right]_{-a}^{0} + \left[x - \frac{1}{2}x^2\right]_{0}^{a} = 2a - a^2$	A1	3	For showing the given answer correctly
	(ii) $P(Y \leqslant y) = P(X^2 \leqslant y) = P\left(\lvert X\rvert \leqslant \sqrt{y}\right) = 2\sqrt{y} - y$	M1		For expression of $P\left(X^2 \leqslant y\right)$ in terms of y
		A1		For correct expression $2\sqrt{y} - y$
	Hence the pgf of Y is $\frac{d}{dy}\left(2\sqrt{y} - y\right) = \frac{1}{\sqrt{y}} - 1$	M1		For differentiation of previous expression
		A1	4	For showing the given answer correctly
	(iii) $E(Y) = \int_0^1 y^{\frac{1}{2}} - y\,dy = \left[\frac{2}{3}y^{\frac{3}{2}} - \frac{1}{2}y^{\frac{1}{2}}\right]_0^1 = \frac{1}{6}$	M1		For the correct integral in terms of y
		A1		For correct answer $\frac{1}{6}$
	$E(X^2) = \int_{-1}^{0} (x^2 + x^3)\,dx + \int_{0}^{1} (x^2 - x^3)\,dx$	M1		For the correct integrals in terms of x
	$= \left[\frac{1}{3}x^3 + \frac{1}{4}x^4\right]_{-1}^{0} + \left[\frac{1}{3}x^3 - \frac{1}{4}x^4\right]_{0}^{1} = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$	A1	4	For the correct answer correctly obtained
	(iv) $E(\sqrt{Y}) = \int_0^1 y^{\frac{1}{2}}\ g(y)\,dy = \int_0^1 (1 - y^{\frac{1}{2}})\,dy$	M1		For forming the correct integral
	$= \left[y - \frac{2}{3}y^{\frac{3}{2}}\right]_0^1 = \frac{1}{3}$	A1	2	For the correct answer $\frac{1}{3}$
			13	
6	(i) H_0 : shoppers' views and age are independent,			
	H_1 : shoppers' views and age are not independent	B1		For stating both hypotheses
	Exp frequencies under H_0 are 163.56 184.44 / 306.44 345.56	M1		For correct method for expected frequencies
		A1		For all four correct
	Test statistic is $\frac{22.94^2}{163.56} + \frac{22.94^2}{184.44} + \frac{22.94^2}{306.44} + \frac{22.94^2}{345.56}$	M1		For correct calculation process, inc Yates
	$= 9.31...$	A1		For correct value of the test statistic
	This is greater than the critical 0.5% value of 7.879	M1		For a relevant (1 df) comparison
	Hence there is very strong evidence to reject H_0 and conclude that views about changing to metric units are not independent of age	A1√	7	For correctly justifying the given answer (the final two marks remain available if Yates' correction is omitted)
	(ii) $H_0 : p_1 = p_2$, $H_1 : p_1 \neq p_2$	B1		For both hypotheses stated
	Under H_0 the sample value of the common proportion is $\frac{187+161}{1000} = 0.348$	B1		For correct value of estimated p
	Test statistic is $\frac{\frac{187}{470} - \frac{161}{530}}{\sqrt{0.348 \times 0.652 \times \left(\frac{1}{470} + \frac{1}{530}\right)}}$	M1		For num $p_1 - p_2$ and denom using attempted s.d. based on a common estimate of p
		A1		For completely correct expression
	$= 3.118$	A1		For correct value of the test statistic
	This is greater than the 0.2% (two-tail) critical value of 3.090	M1		For a relevant comparison using the normal distribution
	Hence this test supports the conclusion of part **(i)**	A1√	7	For any relevant comparison or comment
			14	

[Turn over

7 (i) (a)	$H_0: \mu_d = 0,\ H_1: \mu_d \neq 0$	B1		For both hypotheses stated
	$\bar{d} = 4.1667$	B1		For correct mean difference (subtraction can be either way round)
	$s^2 = \frac{486}{11} - \frac{50^2}{11 \times 12} = 25.2424$	M1		For calculation of unbiased variance estimate
		A1		For correct value 25.24...
	Test statistic is $\frac{4.1667 - 0}{\sqrt{(25.2424/12)}}$	M1		For correct standardising process
	$= 2.873$	A1		For correct value of test statistic
	This is greater than the critical value 2.718	M1		For a relevant comparison using *t* tables
	Hence there is enough evidence to reject H_0 and conclude that there is a difference between the times for the two methods	A1√	**8**	For correctly stated conclusion in context
(b)	Population of differences is normal	B1	**1**	For correct statement
(c)	Interval is $4.1667 \pm 2.201\sqrt{\frac{25.2424}{12}}$	M1		For calculation of the form $\bar{d} \pm t\sqrt{(s^2/n)}$
		B1		For relevant use of $t = 2.201$
	Hence $0.97 < \mu_d < 7.36$	A1	**3**	For correct interval
(ii) (a)	Variation in the speed of individual workers is not eliminated, and may be large compared with the difference between the methods that is being tested	B1	**1**	For any relevant valid statement
(b)	Both samples are from normal populations	B1		For a correct statement about normality
	The population variances are equal	B1	**2**	For a correct statement about the variances
			15	