OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Probability & Statistics 3

4734

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Friday 19 June 2009
Afternoon

Duration: 1 hour 30 minutes

* 4 7 3 4 *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 A continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \dfrac{2x}{5} & 0 \leqslant x \leqslant 1, \\ \dfrac{2}{5\sqrt{x}} & 1 < x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

Find

(i) $E(X)$, **[3]**

(ii) $P(X \geqslant E(X))$. **[3]**

2 The number of bacteria in 1 ml of drug A has a Poisson distribution with mean 0.5. The number of the same bacteria in 1 ml of drug B has a Poisson distribution with mean 0.75. A mixture of these drugs used to treat a particular disease consists of 1.4 ml of drug A and 1.2 ml of drug B. Bacteria in the drugs will cause infection in a patient if 5 or more bacteria are injected.

(i) Calculate the probability that, in a sample of 20 patients treated with the mixture, infection will occur in no more than one patient. **[7]**

(ii) State an assumption required for the validity of the calculation. **[1]**

3 A machine produces circular metal discs whose radii have a normal distribution with mean μ cm. A random sample of five discs is selected and their radii, in cm, are as follows.

6.47 6.52 6.46 6.47 6.51

(i) Calculate a 95% confidence interval for μ. **[6]**

(ii) Hence state a 95% confidence interval for the mean circumference of a disc. **[1]**

4 In order to compare the difficulty of two Su Doku puzzles, two random samples of 40 fans were selected. One sample was given Puzzle 1 and the other sample was given Puzzle 2. Of those given Puzzle 1, 24 could solve it within ten minutes. Of those given Puzzle 2, 15 could solve it within ten minutes.

(i) Using proportions, test at the 5% significance level whether there is a difference in the standard of difficulty of the two puzzles. **[8]**

(ii) The setter believed that Puzzle 2 was more difficult than Puzzle 1. Obtain the smallest significance level at which this belief is supported. **[2]**

5 Each person in a random sample of 15 men and 17 women from a university campus was asked how many days in a month they took exercise. The numbers of days for men and women, x_M and x_W respectively, are summarised by

$$\Sigma x_M = 221, \quad \Sigma x_M^2 = 3992, \quad \Sigma x_W = 276, \quad \Sigma x_W^2 = 5538.$$

(i) State conditions for the validity of a suitable test of the difference in the mean numbers of days for men and women on the campus. **[3]**

(ii) Given that these conditions hold, carry out the test at the 5% significance level. **[10]**

(iii) If in fact the random sample was drawn entirely from the university Mathematics Department, state with a reason whether the validity of the test is in doubt. **[1]**

6 The function $\mathrm{F}(t)$ is defined as follows.

$$\mathrm{F}(t) = \begin{cases} 0 & t < 0, \\ \sin^4 t & 0 \leqslant t \leqslant \frac{1}{2}\pi, \\ 1 & t > \frac{1}{2}\pi. \end{cases}$$

(i) Verify that F is a (cumulative) distribution function. **[2]**

The continuous random variable T has (cumulative) distribution function $\mathrm{F}(t)$.

(ii) Find the lower quartile of T. **[3]**

(iii) Find the (cumulative) distribution function of Y, where $Y = \sin T$, and obtain the probability density function of Y. **[5]**

(iv) Find the expected value of $\dfrac{1}{Y^3 + 2Y^4}$. **[3]**

7 In 1761, James Short took measurements of the parallax of the sun based on the transit of Venus. The mean and standard deviation of a random sample of 50 of these measurements are 8.592 and 0.7534 respectively, in suitable units.

(i) Show that if $X \sim \mathrm{N}(8.592, 0.7534^2)$, then

$$\mathrm{P}(X \leqslant 8.084) = \mathrm{P}(8.084 < X \leqslant 8.592) = \mathrm{P}(8.592 < X \leqslant 9.100) = \mathrm{P}(X > 9.100) = 0.25.$$ **[4]**

The following table summarises the 50 measurements using these intervals.

Measurement (x)	$x \leqslant 8.084$	$8.084 < x \leqslant 8.592$	$8.592 < x \leqslant 9.100$	$x > 9.100$
Frequency	8	22	11	9

(ii) Carry out a test, at the $\frac{1}{2}$% significance level, of whether a normal distribution fits the data. **[7]**

(iii) Obtain a 99% confidence interval for the mean of all similar parallax measurements. **[3]**