4734 Probability & Statistics 3

Penalise 2 sf instead of 3 once only. Penalise final answer ≥ 6 sf once only.

1 (i)	$\int_0^1 \frac{2}{5}x^2 dx + \int_1^4 \frac{2}{5}\sqrt{x}\, dx$	M1		Attempt to integrate xf(x), both parts added, limits
	$= \left[\frac{2x^3}{15}\right]_0^1 + \left[\frac{4x^{3/2}}{15}\right]_1^4 = 2$	A1		Correct indefinite integrals
		A1	**3**	Correct answer
(ii)	$\int_2^4 \frac{2}{5\sqrt{x}} dx = \left[\frac{4\sqrt{x}}{5}\right]_2^4 = \frac{4}{5}(2-\sqrt{2})$ or 0.4686	M1		Attempt correct integral, limits; needs "1 –" if $\mu < 1$
		A1		Correct indefinite integral, √ on their μ
		A1	**3**	Exact aef, or in range [0.468, 0.469]
2 (i)	Po(0.5), Po(0.75)	M1		0.5, 0.75 scaled
	Po(0.7) and Po(0.9)	A1		These
	$A + B \sim$ Po(1.6)	M1		Sum of Poissons used, can have wrong parameters
	P($A + B \geq 5$) = 0.0237	A1		0.0237 from tables or calculator
	B(20, 0.0237)	M1		Binomial (20, their p), soi
	$0.9763^{20} + 20\times0.9763^{19}\times0.0237$	A1√		Correct expression, their p
	= 0.9195	A1	**7**	Answer in range [0.919, 0.92]
(ii)	Bacteria should be independent in drugs; *or* sample should be random	B1	**1**	Any valid relevant comment, must be contextualised
3 (i)	Sample mean = 6.486	B1		
	s^2 = 0.00073	B1		0.000584 if divided by 5
	$6.486 \pm 2.776 \times \sqrt{\frac{0.00073}{5}}$	M1		Calculate sample mean ± $ts/\sqrt{5}$, allow 1.96, s^2 etc
		B1		t = 2.776 seen
	(6.45, 6.52)	A1A1	**6**	Each answer, cwo (6.45246, 6.5195)
(ii)	2π × above [= (40.5, 41.0)]	M1	**1**	
4 (i)	H$_0$: $p_1 = p_2$; H$_1$: $p_1 \neq p_2$, where p_i is the proportion of all solvers of puzzle i	B1		Both hypotheses correctly stated, allow eg $\hat{p}$
	Common proportion 39/80	M1A1		[= 0.4875]
	$s^2 = 0.4875 \times 0.5125 / 20$	B1		[= 0.01249, σ = 0.11176]
	$(\pm)\frac{0.6-0.375}{0.1117} = (\pm)2.013$	M1		$(0.6 - 0.375)/s$
		A1√		Allow 2.066√ from unpooled variance, p = 0.0195
	2.013 > 1.96, or 0.022 < 0.025	M1		Correct method and comparison with 1.96 or 0.025, allow unpooled, 1.645 from 1-tailed *only*
	Reject H$_0$. Significant evidence that there is a difference in standard of difficulty	A1√	**8**	Conclusion, contextualised, not too assertive
(ii)	One-tail test used	M1		One-tailed test stated or implied by
	Smallest significance level 2.2(1)%	A1	**2**	Φ("2.013"), OK if off-scale; allow 0.022(1)

5 (i)	Numbers of men and women should have normal dists; with equal variance; distributions should be independent	B1 B1 B1 **3**	Context & 3 points: 2 of these, B1; 3, B2; 4, B3. [Summary data: 14.73 49.06 52.57 16.24 62.18 66.07]
(ii)	H_0: $\mu_M = \mu_W$; H_1: $\mu_M \neq \mu_W$ $3992 - \frac{221^2}{15} + 5538 - \frac{276^2}{17}$ [≈ 1793] 1793/(14 + 16) = 59.766 $(\pm)\frac{221/15 - 276/17}{\sqrt{59.766(\frac{1}{15}+\frac{1}{17})}} = (-)0.548$ Critical region: $\lvert t \rvert \geq 2.042$ Do not reject H_0. Insufficient evidence of a difference in mean number of days	B1 M1 A1 A1 M1 A1√ A1 B1 M1 A1√ **10**	Both hypotheses correctly stated Attempt at this expression (see above) Either 1793 or 30 Variance estimate in range [59.7, 59.8] (or √ = 7.73) Standardise, allow wrong (but not missing) $1/n$ Correct formula, allow $s^2(\frac{1}{15}+\frac{1}{17})$ or $(\frac{s_1^2}{15}+\frac{s_2^2}{17})$, allow 14 & 16 in place of 15, 17; 0.548 or –0.548 2.042 seen Correct method and comparison type, must be t, allow 1-tail; conclusion, in context, not too assertive
(iii)	Eg Samples not indep't so test invalid	B1 **1**	Any relevant valid comment, eg "not representative"

6	(i)	F(0) = 0, F(π/2) = 1 Increasing	B1 B1 **2**	Consider both end-points Consider F between end-points, can be asserted
	(ii)	$\sin^4(Q_1) = ¼$ $\sin(Q_1) = 1/\sqrt{2}$ $Q_1 = \pi/4$	M1 A1 A1 **3**	 Can be implied. Allow decimal approximations Or 0.785(4)
	(iii)	$G(y) = P(Y \le y) = P(T \le \sin^{-1} y)$ $= F(\sin^{-1} y)$ $= y^4$ $g(y) = \begin{cases} 4y^3 & 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$	M1 A1 A1 M1 A1 **5**	 Ignore other ranges Differentiate G(y) Function and range stated, allow if range given in G
	(iv)	$\int_0^1 \frac{4}{1+2y}\,dy = \left[2\ln(1+2y)\right]_0^1$ = **2 ln 3**	M1 A1 A1 **3**	Attempt $\int \frac{g(y)}{y^3+2y^4}\,dy$; $\int_0^1 \frac{4}{1+2y}\,dy$ Or 2.2, 2.197 or better
7	(i) α	$\Phi\left(\frac{8.084-8.592}{0.7534}\right) = \Phi(-0.674) = 0.25$ $\Phi(0) - \Phi(\text{above}) = 0.25$ $P(8.592 \le X \le 9.1)$ = same by symmetry	M1 A1 A1 A1 **4**	Standardise once, allow $\sqrt{}$ confusions, ignore sign Obtain 0.25 for one interval For a second interval, justified, eg using $\Phi(0) = 0.5$ For a third, justified, eg "by symmetry"
	or β	$\frac{x-8.592}{0.7534} = 0.674$ $x = 8.592 \pm 0.674 \times 0.7534$ = (8.084, 9.100)	 M1A1 A1A1	[from probabilities to ranges] A1 for art 0.674
	(ii)	H_0: normal distribution fits data All E values 50/4 = 12.5 $X^2 = \frac{4.5^2+9.5^2+1.5^2+3.5^2}{12.5} = 10$ 10 > 7.8794 Reject H_0. Significant evidence that normal distribution is not a good fit.	B1 B1 M1 A1 B1 M1 A1$\sqrt{}$ **7**	*Not* N(8.592, 0.7534). Allow "it's normally distributed" [Yates: 8.56: A0] CV 7.8794 seen Correct method, incl. formula for χ^2 and comparison, allow wrong ν Conclusion, in context, not too assertive
	(iv)	$8.592 \pm 2.576 \times \frac{0.7534}{\sqrt{49}}$ (8.315, 8.869)	M1 A1 A1 **3**	Allow $\sqrt{}$ errors, wrong σ or z, allow 50 Correct, including $z = 2.576$ or $t_{49} = 2.680$, *not* 50 In range [8.31, 8.32] and in range (8.86, 8.87], even from 50, or (8.306, 8.878) from t_{49}