



GCE

Mathematics (MEI)

Unit **4768**: Statistics 3

Advanced GCE

Mark Scheme for June 2014

4768

Mark Scheme

June 2014

1. Annotations and abbreviations

Annotation in scoris	Meaning
BP	Blank Page – this annotation must be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓ and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Statistics strand

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

Candidates are expected to give numerical answers to an appropriate degree of accuracy. 3 significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting probabilities from Normal tables, we generally expect *some* evidence of interpolation and so quotation to 4 decimal places will often be appropriate. But even this does not always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice, especially if the calculated value of a test statistic is nowhere near any of these values. Sensible discretion *must* be exercised in such cases.

Discretion must also be exercised in the case of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a problem

demands it) but only 2 are given, loss of an accuracy ("A") mark is likely to be appropriate; but if 4 significant figures are given, this should not normally be penalised. Likewise, answers which are slightly deviant from what is expected in a very minor manner (for example a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected) should not be penalised. However, answers which are *grossly* over- or under-specified should normally result in the loss of a mark. This includes cases such as, for example, insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happened to come off the candidate's calculator. Note that this applies to answers that are given as final stages of calculations; intermediate working should usually be carried out, and quoted, to a greater degree of accuracy to avoid the danger of premature approximation.

The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object and/or difficulty of the question being considerably changed, it is likely that all the marks for that question, or section of the question, will be lost. However, misreads are often such that the object and/or difficulty remain substantially unaltered; these cases are considered below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but at least some accuracy ("A") marks do not. It is difficult to legislate in an overall sense beyond this global statement because misreads, even when the object and/or difficulty remains unchanged, can vary greatly in their effects. For example, a misread of 1.02 as 10.2 (perhaps as a quoted value of a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.6746 may have so slight an effect as to be almost unnoticeable in the candidate's work.

A misread should normally attract *some* penalty, though this would often be only 1 mark and should rarely if ever be more than 2. Commonly in sections of questions where there is a numerical answer either at the end of the section or to be obtained and commented on (eg the value of a test statistic), this answer will have an "A" mark that may actually be designated as "cao" [correct answer only]. This should be interpreted *strictly* – if the misread has led to failure to obtain this value, then this "A" mark must be withheld even if all method marks have been earned. It will also often be the case that such a mark is implicitly "cao" even if not

explicitly designated as such.

On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow-through of the candidate's value of a test statistic is generally allowed (and often explicitly stated as such within the marking scheme), so that the candidate may exhibit knowledge of how to compare it with a critical value and draw conclusions. Such "fresh starts" are not affected by any earlier misreads.

A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expression. Such misreads are more likely to bring about a considerable change in the object and/or difficulty of the question; but, if they do not, they should be treated as far as possible in the same way as numerical misreads, *mutatis mutandis*. This also applied to misreads of text, which are fairly rare but can cause major problems in fair marking.

The situation regarding any particular cases that arise while you are marking for which you feel you need detailed guidance should be discussed with your Team Leader.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

4768

Mark Scheme

June 2014

Question	Answer	Marks	Guidance
1 (i)	$\text{Var}(X_1 + X_2) = 2\sigma^2$ $\text{Var}(2X) = 4\sigma^2$ $X_1 + X_2 \text{ means two independent values from } X \text{ are added together.}$ $2X \text{ means that one value from } X \text{ is multiplied by 2.}$	B1 B1 E1 [3]	Allow $2\text{Var}(X)$ and $4\text{Var}(X)$ Any comment explaining why $X_1 + X_2$ is different from $2X$
1 (ii)	$P(X < 22) = P\left(Z < \frac{22 - 30.1}{5.1}\right)$ $= P(Z < -1.5882)$ $= 0.0561$	M1 A1 A1 [3]	For standardising. Award once, here or elsewhere Correct z value cao
1 (iii)	$X + Y + W \sim N(3.7, 58.86)$ $P(X + Y + W > 100) = P\left(Z > \frac{100 - 83.7}{\sqrt{58.86}}\right)$ $P(Z > 2.1246) = 0.0168$	B1 B1 M1 A1 [4]	Mean Variance (or sd = 7.67) Correct set up cao
1 (iv)	$2X \sim N(0.2, 104.04)$ $\rightarrow 2X + Y \sim N(5.6, 121.68)$ $P(2X + Y > b) = 0.02$ $\rightarrow \frac{b - 85.6}{\sqrt{121.68}} = 2.054$ $\rightarrow b = 108.26$ Score exceeded by 2% is 108.3	B1 B1 M1 A1 [4]	Variance 2.054 seen Correct set up cao

4768

Mark Scheme

June 2014

Question	Answer	Marks	Guidance
1 (v)	$P(W < 0.6X) = P(W - 0.6X < 0)$ $W - 0.6X \sim N(0.14, 24.5736)$ $P(W - 0.6X < 0) = P(Z < -2.0455)$ $= 0.0204$	M1 B1 A1 [3]	Either way round Mean and variance Cao. Allow convincing recovery
2 (i)	A simple random sample is one where every sample of the same size has an equal probability of being selected.	E2,1,0 [2]	Allow E1 for every item has the same probability of being selected
2 (ii)	$H_0 : \mu = 1.02 \quad H_1 : \mu < 1.02$ <p>Where μ is the population mean volume</p> $\bar{x} = 1.0131 \quad s = 0.009146$ <p>Test statistic is $\frac{1.0131 - 1.02}{0.009146/\sqrt{10}}$</p> $= -2.3857 \quad \text{value between } -2.38 \text{ and } -2.39$ <p>Refer to t_9 5% point is ± 1.833</p> $-2.3857 < -1.833 \quad \text{reject } H_0$ <p>Conclude mean appears to be below 1.02 and so machine will be cleaned.</p>	B1 B1 B1 M1* A1 M1 A1 M1 dep E1 dep [9]	Both hypotheses. Hypotheses in words only must include "population". Do NOT allow " $\bar{X} = \dots$ " or similar unless \bar{X} is clearly and explicitly stated to be a <u>population</u> mean. For adequate verbal definition. Allow absence of "population" if correct notation μ is used. Do not allow $s_n = 0.00868$ here or in construction of test statistic For method, allow candidates \bar{x}, s . Allow confidence interval approach. cao For t_9 , No FT from here For matching 1.833 seen. No FT from here For rejection. Must compare test statistic with <u>matching</u> 1.833 unless absolute values are being compared. No ft from here if wrong. Needs context

4768

Mark Scheme

June 2014

Question	Answer										Marks	Guidance																					
2	(iii)	H_0 : median of population volumes = 1.05 H_1 : median of population volumes \neq 1.05										B1	Both Allow location parameter or mean (if explanation is given)																				
<table border="1"> <tr> <td>Difference</td> <td>0.005</td> <td>0.014</td> <td>0.013</td> <td>-0.007</td> <td>0.012</td> <td>0.020</td> <td>0.009</td> <td>-0.006</td> <td>0.004</td> <td>0.003</td> </tr> <tr> <td>Rank</td> <td>3</td> <td>9</td> <td>8</td> <td>5</td> <td>7</td> <td>10</td> <td>6</td> <td>4</td> <td>2</td> <td>1</td> </tr> </table>														Difference	0.005	0.014	0.013	-0.007	0.012	0.020	0.009	-0.006	0.004	0.003	Rank	3	9	8	5	7	10	6	4
Difference	0.005	0.014	0.013	-0.007	0.012	0.020	0.009	-0.006	0.004	0.003																							
Rank	3	9	8	5	7	10	6	4	2	1																							
$W_- = 4 + 5 = 9$ Refer to tables of Wilcoxon paired (/single sample) statistic for $n = 10$ Critical value = 8. (47 for upper) Cannot reject H_0 and conclude median volume appears to be 1.05												M1 M1 A1 B1 M1 A1 E1 [8]	For differences from 1.05. ZERO from this point if differences not used. For ranks. ft from here if ranks wrong. Or $W_+ = 1 + 2 + 3 + 6 + 7 + 8 + 9 + 10 = 46$ No FT from here if wrong. No FT from here if wrong (i.e. 1 tail) Must have context.																				

4768

Mark Scheme

June 2014

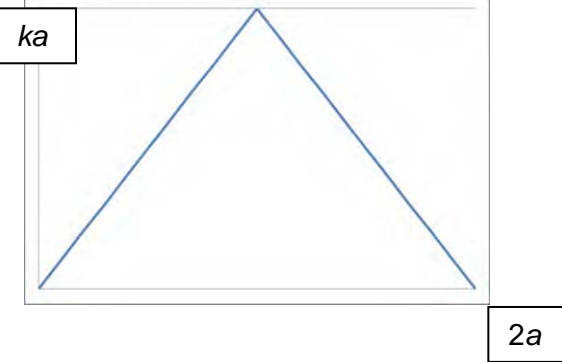
Question	Answer	Marks	Guidance
3 (a)	$H_0 : \mu_D = 0 \quad H_1 : \mu_D < 0$ Where μ_D is the population mean of the differences. MUST BE PAIRED COMPARISON t test. Differences are -2.2, -6.3, -5.7, 1.1, 0.3, -6.3, -6.0, -3.1, -3.4, -9.1, -2.4, 0.7 $\bar{D} = -3.533 \quad s_D = 3.225$ Test statistic is $\frac{-3.533 - 0}{3.225/\sqrt{12}}$ $= -3.79$ Refer to t_{11} Single-tailed 1% point is -2.718 $-3.79 < -2.718$ so reject H_0 Conclude mean time appears reduced	 M1 A1 M1* A1 M1 A1 A1dep A1dep [8]	 Do not allow $s_n = 3.088$ For method, allow their \bar{D}, s . Allow confidence interval approach. cao No FT from here if wrong No FT from here if wrong FT their -3.79 if relevant M1 earned In context. FT their -3.79 if relevant M1 earned

4768

Mark Scheme

June 2014

Question	Answer	Marks	Guidance																																		
3 (b)	<p>H_0 : The Poisson model fits the data H_1 : The Poisson model does not fit the data</p> <table border="1" data-bbox="365 368 958 799"> <thead> <tr> <th>r</th> <th>$P(X=r)$</th> <th>Expected value</th> <th>Combined</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0.03688</td> <td>2.213</td> <td rowspan="2">9.516</td> </tr> <tr> <td>1</td> <td>0.12171</td> <td>7.303</td> </tr> <tr> <td>2</td> <td>0.20083</td> <td>12.050</td> <td></td> </tr> <tr> <td>3</td> <td>0.22091</td> <td>13.255</td> <td></td> </tr> <tr> <td>4</td> <td>0.18225</td> <td>10.935</td> <td></td> </tr> <tr> <td>5</td> <td>0.12029</td> <td>7.217</td> <td></td> </tr> <tr> <td>6</td> <td>0.06616</td> <td>3.969</td> <td rowspan="2">7.027</td> </tr> <tr> <td>≥ 7</td> <td>0.05097</td> <td>3.058</td> </tr> </tbody> </table> $X^2 = \frac{2.516^2}{9.516} + \frac{2.050^2}{12.050} + \frac{3.745^2}{13.255} + \frac{3.065^2}{10.935}$ $+ \frac{0.217^2}{7.217} + \frac{2.027^2}{7.027}$ $= 0.6652 + 0.3488 + 1.0581 + 0.8591 + 0.0065 + 0.5847$ $= 3.522 \text{ awrt } 3.52$ <p>Refer to χ^2_4 Upper 5% point is 9.49 $3.522 < 9.49$ cannot reject H_0 Poisson model appears to fit data.</p>	r	$P(X=r)$	Expected value	Combined	0	0.03688	2.213	9.516	1	0.12171	7.303	2	0.20083	12.050		3	0.22091	13.255		4	0.18225	10.935		5	0.12029	7.217		6	0.06616	3.969	7.027	≥ 7	0.05097	3.058	<p>B1 M1 M1 A1 M1 M1* A1 M1 B1 A1dep A1dep [11]</p>	<p>Both hypotheses. Must be the right way round. Do not accept “data fits model” or equivalent.</p> <p>At least 3 probabilities to 3dp or better or 3 expected values to 3sf or better</p> <p>Multiply by 60 to obtain expected values</p> <p>All correct to 3sf or better</p> <p>Merge first 2 and last 2 cells</p> <p>Calculation of X^2</p> <p>cao</p> <p>Allow correct df from wrongly grouped table.</p> <p>No FT from here if wrong.</p> <p>FT candidates 3.522 if relevant M1 earned.</p> <p>FT candidates 3.522 if relevant M1 earned. Do not accept “data fits Poisson model” or equivalent.</p>
r	$P(X=r)$	Expected value	Combined																																		
0	0.03688	2.213	9.516																																		
1	0.12171	7.303																																			
2	0.20083	12.050																																			
3	0.22091	13.255																																			
4	0.18225	10.935																																			
5	0.12029	7.217																																			
6	0.06616	3.969	7.027																																		
≥ 7	0.05097	3.058																																			

Question	Answer	Marks	Guidance
4 (i)	 <p data-bbox="360 598 952 678">E(K) = a because the distribution is symmetrical about $x = a$</p>	<p data-bbox="1037 220 1081 244">G1</p> <p data-bbox="1037 411 1081 435">G1</p> <p data-bbox="1037 627 1081 651">B1</p> <p data-bbox="1037 691 1081 715">[3]</p>	<p data-bbox="1126 220 1205 244">Shape</p> <p data-bbox="1126 411 1294 435">Scales on axes</p> <p data-bbox="1126 627 1507 651">Do not allow integration method</p>
4 (ii)	<p data-bbox="360 743 616 815">Total area = $\frac{1}{2} 2a \cdot ka$</p> <p data-bbox="360 831 728 887">Or $\int_0^a kx dx + \int_a^{2a} k(2a-x) dx$</p> <p data-bbox="360 903 683 999">$k \left[\frac{x^2}{2} \right]_0^a + k \left[2ax - \frac{x^2}{2} \right]_a^{2a}$</p> <p data-bbox="360 1015 705 1110">$k \left(\frac{a^2}{2} - 0 \right) + k \left(2a^2 - \frac{3a^2}{2} \right)$</p> <p data-bbox="360 1126 436 1166">$= ka^2$</p> <p data-bbox="360 1182 459 1222">$ka^2 = 1$</p> <p data-bbox="360 1238 459 1310">$k = \frac{1}{a^2}$</p>	<p data-bbox="1037 930 1081 954">M1</p> <p data-bbox="1037 1137 1081 1161">A1</p> <p data-bbox="1037 1233 1081 1257">A1</p> <p data-bbox="1037 1329 1081 1353">[3]</p>	<p data-bbox="1126 874 1529 898">Attempting to find area of triangle</p> <p data-bbox="1126 954 2011 978">or setting up the correct integrals including limits (which may appear later).</p> <p data-bbox="1126 1129 1619 1153">oe Correctly finding area in terms of k, a</p> <p data-bbox="1126 1209 1753 1233">Equating area to 1 and convincingly obtaining result.</p> <p data-bbox="1126 1249 1294 1273">Answer given</p>

Question	Answer	Marks	Guidance
<p>4 (iii)</p>	$\text{Var } X = k \int_0^a x^3 dx + k \int_a^{2a} 2ax^2 - x^3 dx - a^2$ $k \left[\frac{x^4}{4} \right]_0^a + k \left[\frac{2ax^3}{3} - \frac{x^4}{4} \right]_a^{2a} - a^2$ $\frac{a^2}{4} + \frac{16a^2}{3} - 4a^2 - \frac{2a^2}{3} + \frac{a^2}{4} - a^2$ $\text{Var } X = \frac{a^2}{6}$	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>Correct integral for $E(X^2)$ including limits (which may appear later).</p> <p>Correctly integrated (dependent on M1 above)</p> <p>Using $E(X^2) = E(X)^2$</p> <p>cao</p>
<p>4 (iv)</p>	<p>$\bar{x} = 1.92 \quad s = 0.8352$</p> <p>Interval is $1.92 \pm 1.96 \frac{0.8352}{\sqrt{50}}$</p> <p>$= (1.69, 2.15)$</p> <p>The distribution of \bar{X} is <i>approximately</i> Normal (CLT) or s is only an estimate.</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>E1</p> <p>[5]</p>	<p>Given</p> <p>1.96 seen</p> <p>Correct SE</p> <p>Centred on 1.92</p> <p>cao</p>
<p>4 (v)</p>	<p>This statement is incorrect</p> <p>The value of a either lies within this particular interval or it does not – so the probability is either 0 or 1. If a large number of such intervals were constructed, then a would lie in 95% of them.</p>	<p>E1</p> <p>E1</p> <p>[2]</p>	<p>A comment either about $p=0$ or 1, or about a large number of intervals</p>