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RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4768**

Statistics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Tuesday 22 June 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

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1 **(i)** The manager of a company that employs 250 travelling sales representatives wishes to carry out a detailed analysis of the expenses claimed by the representatives. He has an alphabetical (by surname) list of the representatives. He chooses a sample of representatives by selecting the 10th, 20th, 30th and so on. Name the type of sampling the manager is attempting to use. Describe a weakness in his method of using it, and explain how he might overcome this weakness. **[3]**

The representatives each use their own cars to drive to meetings with customers. The total distance, in miles, travelled by a representative in a month is Normally distributed with mean 2018 and standard deviation 96.

(ii) Find the probability that, in a randomly chosen month, a randomly chosen representative travels more than 2100 miles. **[3]**

(iii) Find the probability that, in a randomly chosen 3-month period, a randomly chosen representative travels less than 6000 miles. What assumption is needed here? Give a reason why it may not be realistic. **[5]**

(iv) Each month every representative submits a claim for travelling expenses plus commission. Travelling expenses are paid at the rate of 45 pence per mile. The commission is 10% of the value of sales in that month. The value, in £, of the monthly sales has the distribution $N(21\,200,\ 1100^2)$. Find the probability that a randomly chosen claim lies between £3000 and £3300. **[7]**

2 William Sealy, a biochemistry student, is doing work experience at a brewery. One of his tasks is to monitor the specific gravity of the brewing mixture during the brewing process. For one particular recipe, an initial specific gravity of 1.040 is required. A random sample of 9 measurements of the specific gravity at the start of the process gave the following results.

1.046 1.048 1.039 1.055 1.038 1.054 1.038 1.051 1.038

(i) William has to test whether the specific gravity of the mixture meets the requirement. Why might a t test be used for these data and what assumption must be made? **[3]**

(ii) Carry out the test using a significance level of 10%. **[9]**

(iii) Find a 95% confidence interval for the true mean specific gravity of the mixture and explain what is meant by a 95% confidence interval. **[6]**

3 (a) In order to prevent and/or control the spread of infectious diseases, the Government has various vaccination programmes. One such programme requires people to receive a booster injection at the age of 18. It is felt that the proportion of people receiving this booster could be increased and a publicity campaign is undertaken for this purpose. In order to assess the effectiveness of this campaign, health authorities across the country are asked to report the percentage of 18-year-olds receiving the booster before and after the campaign. The results for a randomly chosen sample of 9 authorities are as follows.

Authority	A	B	C	D	E	F	G	H	I
Before	76	98	88	81	86	84	83	93	80
After	82	97	93	77	83	95	91	95	89

This sample is to be tested to see whether the campaign appears to have been successful in raising the percentage receiving the booster.

(i) Explain why the use of paired data is appropriate in this context. **[1]**

(ii) Carry out an appropriate Wilcoxon signed rank test using these data, at the 5% significance level. **[10]**

(b) Benford's Law predicts the following probability distribution for the first significant digit in some large data sets.

Digit	1	2	3	4	5	6	7	8	9
Probability	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046

On one particular day, the first significant digits of the stock market prices of the shares of a random sample of 200 companies gave the following results.

Digit	1	2	3	4	5	6	7	8	9
Frequency	55	34	27	16	15	17	12	15	9

Test at the 10% level of significance whether Benford's Law provides a reasonable model in the context of share prices. **[7]**

[Question 4 is printed overleaf.]

PMT

4 A random variable X has an exponential distribution with probability density function $\mathrm{f}(x) = \lambda e^{-\lambda x}$ for $x \geqslant 0$, where λ is a positive constant.

(i) Verify that $\int_0^{\infty} \mathrm{f}(x)\,\mathrm{d}x = 1$ and sketch $\mathrm{f}(x)$. **[5]**

(ii) In this part of the question you may use the following result.

$$\int_0^{\infty} x^r e^{-\lambda x}\,\mathrm{d}x = \frac{r!}{\lambda^{r+1}} \quad \text{for } r = 0,\ 1,\ 2,\ \ldots\ .$$

Derive the mean and variance of X in terms of λ. **[6]**

The random variable X is used to model the lifetime, in years, of a particular type of domestic appliance. The manufacturer of the appliance states that, based on past experience, the mean lifetime is 6 years.

(iii) Let $\overline{X}$ denote the mean lifetime, in years, of a random sample of 50 appliances. Write down an approximate distribution for $\overline{X}$. **[4]**

(iv) A random sample of 50 appliances is found to have a mean lifetime of 7.8 years. Does this cast any doubt on the model? **[3]**

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