ADVANCED GCE UNIT

MATHEMATICS (MEI)

4768/01

Statistics 3

TUESDAY 5 JUNE 2007

Afternoon

Time: 1 hour 30 minutes

Additional Materials:
- Answer booklet (8 pages)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

[Turn over

1 A manufacturer of fireworks is investigating the lengths of time for which the fireworks burn. For a particular type of firework this length of time, in minutes, is modelled by the random variable T with probability density function

$$f(t) = kt^3(2 - t) \quad \text{for } 0 < t \leqslant 2$$

where k is a constant.

(i) Show that $k = \frac{5}{8}$. [2]

(ii) Find the modal time. [2]

(iii) Find $\mathrm{E}(T)$ and show that $\mathrm{Var}(T) = \frac{8}{63}$. [5]

(iv) A large random sample of n fireworks of this type is tested. Write down in terms of n the approximate distribution of $\overline{T}$, the sample mean time. [3]

(v) For a random sample of 100 such fireworks the times are summarised as follows.

$$\Sigma t = 145.2 \qquad \Sigma t^2 = 223.41$$

Find a 95% confidence interval for the mean time for this type of firework and hence comment on the appropriateness of the model. [6]

2 The operator of a section of motorway toll road records its weekly takings according to the types of vehicles using the motorway. For purposes of charging, there are three types of vehicle: cars, coaches, lorries. The weekly takings (in thousands of pounds) for each type are assumed to be Normally distributed. These distributions are independent of each other and are summarised in the table.

Vehicle type	Mean	Standard deviation
Cars	60.2	5.2
Coaches	33.9	6.3
Lorries	52.4	4.9

(i) Find the probability that the weekly takings for coaches are less than £40 000. [3]

(ii) Find the probability that the weekly takings for lorries exceed the weekly takings for cars. [4]

(iii) Find the probability that over a 4-week period the total takings for cars exceed £225 000. What assumption must be made about the four weeks? [5]

(iv) Each week the operator allocates part of the takings for repairs. This is determined for each type of vehicle according to estimates of the long-term damage caused. It is calculated as follows: 5% of takings for cars, 10% for coaches and 20% for lorries. Find the probability that in any given week the total amount allocated for repairs will exceed £20 000. [6]

3 The management of a large chain of shops aims to reduce the level of absenteeism among its workforce by means of an incentive bonus scheme. In order to evaluate the effectiveness of the scheme, the management measures the percentage of working days lost before and after its introduction for each of a random sample of 11 shops. The results are shown below.

Shop	A	B	C	D	E	F	G	H	I	J	K
% days lost before	3.5	5.0	3.5	3.2	4.5	4.9	4.1	6.0	6.8	8.1	6.0
% days lost after	1.8	4.3	2.9	4.5	4.4	5.8	3.5	6.7	6.4	5.4	5.1

(a) The management decides to carry out a t test to investigate whether there has been a reduction in absenteeism.

(i) State clearly the hypotheses that should be used together with any necessary assumptions. [4]

(ii) Carry out the test using a 5% significance level. [7]

(b) Find a 95% confidence interval for the true mean percentage of days lost after the introduction of the incentive scheme and state any assumption needed. The management has set a target that the mean percentage should be 3.5. Do you think this has been achieved? Explain your answer. [7]

4 A machine produces plastic strip in a continuous process. Occasionally there is a flaw at some point along the strip. The length of strip (in hundreds of metres) between successive flaws is modelled by a continuous random variable X with probability density function $\mathrm{f}(x) = \dfrac{18}{(3+x)^3}$ for $x > 0$. The table below gives the frequencies for 100 randomly chosen observations of X. It also gives the probabilities for the class intervals using the model.

Length x (hundreds of metres)	Observed frequency	Probability
$0 < x \leqslant 0.5$	21	0.2653
$0.5 < x \leqslant 1$	24	0.1722
$1 < x \leqslant 2$	12	0.2025
$2 < x \leqslant 3$	15	0.1100
$3 < x \leqslant 5$	13	0.1094
$5 < x \leqslant 10$	9	0.0874
$x > 10$	6	0.0532

(i) Examine the fit of this model to the data at the 5% level of significance. [9]

You are given that the median length between successive flaws is 124 metres. At a later date the following random sample of ten lengths (in metres) between flaws is obtained.

239 77 179 221 100 312 52 129 236 42

(ii) Test at the 10% level of significance whether the median length may still be assumed to be 124 metres. [9]