Q1	$f(t) = kt^3(2-t) \qquad 0 < t \le 2$			
(i)	$\int_0^2 kt^3(2-t)\mathrm{d}t = 1$ $\therefore \left[k\left(\frac{2t^4}{4} - \frac{t^5}{5}\right)\right]_0^2 = 1$ $\therefore k\left(8 - \frac{32}{5}\right) - 0 = 1$ $\therefore k \times \frac{8}{5} = 1 \qquad \therefore k = \frac{5}{8}$	M1 E1	Integral of f(t), including limits (possibly implied later), equated to 1. Convincingly shown. Beware printed answer.	 2
(ii)	$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{5}{8}(6t^2 - 4t^3) = 0$ $\therefore 6t^2 - 4t^3 = 0$ $\therefore 2t^2(3-2t) = 0$ $\therefore t = (0 \text{ or}) \ \frac{3}{2}$	M1 A1	Differentiate and set equal to zero. c.a.o.	 2
(iii)	$\mathrm{E}(T) = \int_0^2 \frac{5}{8}t^4(2-t)\mathrm{d}t$ $= \left[\frac{5}{8}\left(\frac{2t^5}{5} - \frac{t^6}{6}\right)\right]_0^2 = \frac{5}{8} \times \left(\frac{64}{5} - \frac{64}{6}\right) = \frac{4}{3}$ $\mathrm{E}(T^2) = \int_0^2 \frac{5}{8}t^5(2-t)\mathrm{d}t$ $= \left[\frac{5}{8}\left(\frac{2t^6}{6} - \frac{t^7}{7}\right)\right]_0^2 = \frac{5}{8} \times \left(\frac{128}{6} - \frac{128}{7}\right) = \frac{40}{21}$ $\mathrm{Var}(T) = \frac{40}{21} - \left(\frac{4}{3}\right)^2 = \frac{120-112}{63} = \frac{8}{63}$	M1 A1 M1 M1 A1	Integral for E(T) including limits (which may appear later). Integral for E(T^2) including limits (which may appear later). Convincingly shown. Beware printed answer.	 5
(iv)	$\bar{T} \sim \mathrm{N}\left(\frac{4}{3}, \ \frac{8}{63n}\right)$	B1 B1 B1	Normal distribution. Mean. ft c's E(T). Correct variance.	 3

(v)	$n = 100, \quad \bar{t} = \frac{145 \cdot 2}{100} = 1 \cdot 452,$ $s_{n-1}^2 = \frac{223 \cdot 41 - 100 \times 1 \cdot 452^2}{99} = 0 \cdot 12707$	B1	Both mean and variance. Accept sd = 0·3565	
	CI is given by 1·452 ±	M1	ft c's $\bar{t}$ ± .	
	1·96	B1		
	$\times \frac{0 \cdot 3565}{\sqrt{100}}$	M1	ft c's s_{n1}.	
	= 1·452 ± 0·0698 = (1·382, 1·522)	A1	c.a.o. Must be expressed as an interval.	
	Since E(*T*) (= 4/3) lies outside this interval it seems the model may not be appropriate.	E1		6
				18

Q2	$Ca \sim N(60{\cdot}2,\ 5{\cdot}2^2)$ $Co \sim N(33{\cdot}9,\ 6{\cdot}3^2)$ $L \sim N(52{\cdot}4,\ 4{\cdot}9^2)$		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables, penalise the first occurrence only.	
(i)	$P(Co < 40) = P(Z < \frac{40-33\cdot 9}{6\cdot 3} = 0{\cdot}9683)$ $= 0{\cdot}8336$	M1 A1 A1	For standardising. Award once, here or elsewhere. c.a.o.	3
(ii)	Want $P(L > Ca)$ i.e. $P(L - Ca > 0)$ $L - Ca \sim N(52{\cdot}4 - 60{\cdot}2 = -7{\cdot}8,$ $4{\cdot}9^2 + 5{\cdot}2^2 = 51{\cdot}05)$ $P(\text{this} > 0) = P(Z > \frac{0-(-7\cdot 8)}{\sqrt{51\cdot 05}} = 1{\cdot}0917)$ $= 1 - 0{\cdot}8625 = 0{\cdot}1375$	M1 B1 B1 A1	Allow $Ca - L$ provided subsequent work is consistent. Mean. Variance. Accept sd = $\sqrt{51{\cdot}05}$= 7·1449... c.a.o.	4
(iii)	Want $P(Ca_1 + Ca_2 + Ca_3 + Ca_4 > 225)$ $Ca_1 + \ldots \sim N(60{\cdot}2 + 60{\cdot}2 + 60{\cdot}2 + 60{\cdot}2 = 240{\cdot}8,$ $5{\cdot}2^2 + 5{\cdot}2^2 + 5{\cdot}2^2 + 5{\cdot}2^2 = 108{\cdot}16)$ $P(\text{this} > 225) = P(Z > \frac{225-240\cdot 8}{\sqrt{108\cdot 16}} = -1{\cdot}519)$ $= 0{\cdot}9356$ Must assume that the weeks are independent of each other.	M1 B1 B1 A1 B1	Mean. Variance. Accept sd=$\sqrt{108{\cdot}16}$=10·4. c.a.o.	5
(iv)	$R \sim N(0{\cdot}05 \times 60{\cdot}2 + 0{\cdot}1 \times 33{\cdot}9 + 0{\cdot}2 \times 52{\cdot}4 = 16{\cdot}88,$ $0{\cdot}05^2 \times 5{\cdot}2^2 + 0{\cdot}1^2 \times 6{\cdot}3^2 + 0{\cdot}2^2 \times 4{\cdot}9^2 = 1{\cdot}4249)$ $P(R > 20) = P(Z > \frac{20-16\cdot 88}{\sqrt{1\cdot 4249}} = 2{\cdot}613)$ $= 1 - 0{\cdot}9955 = 0{\cdot}0045$	M1 A1 M1 M1 A1 A1	Mean. For $0{\cdot}05^2$ etc. For $\times 5{\cdot}2^2$ etc. Accept sd = $\sqrt{1{\cdot}4249}$ = 1·1937. c.a.o.	6
				18

Q3				
(a) (i)	$H_0 : \mu_D = 0$ $H_1 : \mu_D > 0$ Where μ_D is the (population) mean reduction in absenteeism. Must assume Normality … … of differences.	B1 B1 B1 B1	Both. Accept alternatives e.g. $\mu_D < 0$ for H_1, or $\mu_A - \mu_B$ etc provided adequately defined. Allow absence of "population" if correct notation μ is used, but do NOT allow " $\bar{X} = ...$ " or similar unless $\bar{X}$ is clearly and explicitly stated to be a population mean. Hypotheses in words only must include "population".	4
(ii)	Differences (reductions) (before – after) 1·7, 0·7, 0·6, –1·3, 0·1, –0·9, 0·6, –0·7, 0·4, 2·7, 0·9 $\bar{x} = 0 \cdot 4364$, $s_{n-1} = 1{\cdot}1518$ ($s_{n-1}^2 = 1{\cdot}3265$) Test statistic is $\dfrac{0 \cdot 4364 - 0}{\left(\dfrac{1 \cdot 1518}{\sqrt{11}}\right)}$ $= 1{\cdot}256(56…)$ Refer to t_{10}. Upper 5% point is 1·812. 1·256 < 1·812, ∴ Result is not significant. Seems there has been no reduction in mean absenteeism.	 B1 M1 A1 M1 A1 E1 E1	Allow "after – before" if consistent with alternatives above. Do not allow $s_n = 1{\cdot}098$ ($s_n^2 = 1{\cdot}205$). Allow c's $\bar{x}$ and/or s_{n-1}. Allow alternative: 0 ± (c's 1·812) × $\frac{1{\cdot}1518}{\sqrt{11}}$ (= –0·6293, 0·6293) for subsequent comparison with $\bar{x}$. (Or $\bar{x}$ ± (c's 1·812) × $\frac{1{\cdot}1518}{\sqrt{11}}$ (= –0·1929, 1·0657) for comparison with 0.) c.a.o. but ft from here in any case if wrong. Use of $0 - \bar{x}$ scores M1A0, but ft. No ft from here if wrong. No ft from here if wrong. For alternative H_1 expect –1·812 unless it is clear that absolute values are being used. ft only c's test statistic. ft only c's test statistic. Special case: (t_{11} and 1·796) can score 1 of these last 2 marks if either form of conclusion is given.	7

(b)	For "days lost after" $\bar{x} = 4\cdot6182$, $s_{n-1} = 1\cdot4851$ ($s_{n-1}^2 = 2\cdot2056$)	 B1	 Do not allow $s_n = 1\cdot4160$ ($s_n^2 = 2\cdot0051$).	
	CI is given by $4\cdot6182 \pm$ $2\cdot228$ $\times \frac{1\cdot4851}{\sqrt{11}}$	M1 B1 M1	ft c's $\bar{x}$ ±. ft c's s_{n-1}.	
	$= 4\cdot6182 \pm 0\cdot9976 = (3\cdot620(6), 5\cdot615(8))$	A1	c.a.o. Must be expressed as an interval. ZERO if not same distribution as test. Same wrong distribution scores maximum M1B0M1A0. Recovery to t_{10} is OK.	
	Assume Normality of population of "days lost after".	E1		
	Since 3·5 lies outside the interval it seems that the target has not been achieved.	E1		7
				18

Q4

(i)

Obs	21	24	12	15	13	9	6
Exp	26·53	17·22	20·25	11·00	10·94	8·74	5·32

Working	Mark	Comment	Total
	M1	Probabilities × 100.	
	A1	All Expected frequencies correct.	
$\therefore X^2 = \frac{(21-26\cdot53)^2}{26\cdot53} + \text{etc}$	M1		
= 1·1527 + 2·6695 + 3·3611 + 1·4545 + 0·3879 + 0·0077+ 0·0869	A1	At least 4 values correct.	
= 9·1203	A1		
d.o.f. = 7 – 1 = 6			
Refer to χ^2_6.	M1	No ft from here if wrong.	
Upper 5% point is 12·59	A1	No ft from here if wrong.	
9·1203 < 12·59 ∴ Result is not significant.	E1	ft only c's test statistic.	
Evidence suggests the model fits the data at the 5% level.	E1	ft only c's test statistic.	9

(ii)

Data	Diff = data –124	Rank of |diff|
239	115	9
77	–47	3
179	55	4
221	97	7
100	–24	2
312	188	10
52	–72	5
129	5	1
236	112	8
42	–82	6

Working	Mark	Comment	Total
	M1	For differences.	
	M1	For ranks of |difference|.	
	A1	All correct. ft from here if ranks wrong.	
$W_- = 3 + 2 + 5 + 6 = 16$	B1	Or $W_+ = 9 + 4 + 7 + 10 + 1 + 8 = 39$	
Refer to Wilcoxon single sample (/paired) tables for $n = 10$.	M1	No ft from here if wrong.	
Lower two-tail 10% point is … … 10.	M1A1	Or, if 39 used, upper point is 45. No ft from here if wrong.	
16 > 10 ∴ Result is not significant.	E1	Or 39 < 45. ft only c's test statistic.	
Seems there is no evidence against the median length being 124.	E1	ft only c's test statistic.	9
			18